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Decision Procedure to Minimize Marginal Production Cost in a System Dynamics Model.

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Abstract

A decision procedure is described which has been employed to control the evolution of a system dynamics societal model by minimization of the marginal production cost for food. Although explicitly discussed within this framework, the procedure should have more general applicability when optimization of the marginal value of some controlling figure of merit is desired. The basic decision procedure is supplemented by several refinements in order to modify the dynamical behavior of the model.

1. Introduction

In a separate report (1), a societal model is described in which food production is governed by a production function, R, that depends on three mechanisms, the agricultural labor YA, the land devoted to agriculture YB, and the technological plant in support of agriculture YC. Costs, PA, PB, and PC, respectively, are assigned to the production mechanisms. The system levels corresponding to the three production mechanisms are described by differential equations of the form $\dot{YI}=FI$, where I=A, B, and C. (In the societal model (1), the variables YA, YB, and YC are denoted y_2, y_7, and y_8, respectively).

At each incremental step in the integration of the equations, the food stock is compared with the desired food stock. The right sides of the three equations are adjusted by a decision procedure that is intended to minimize the marginal production cost. The costs and the production function depend upon model conditions determined both by the three production mechanism levels and by other levels in the societal model. The decision procedure should be adaptable to many modelling situations that are characterized by price indices, or other value indices, and where an optimization of the net marginal index is desired.

2. Basic Decision Procedure

The basic procedure will be set forth as a series of six steps.

(A) The status of the food supply is appraised by monitoring the food stock level, Z. Consumption and decay of food lessen the stock while production of food replenishes the stock. Corrective action is based on the food imbalance S=(desired stock)-(stock). A food stock index is defined as VZ=(stock)/(desired stock). Thus, S=Z(1/VZ - 1). The decision logic is symmetrical with regard to a shortage or an excess of food.

(B) Coefficients may be introduced which are measures of the change in the production function, ΔR, due to a change in a production mechanism, ΔYI, according to ΔR=M(I,I)*ΔYI. Judgement must be exercised as to how the real society operates. For example, food production may be increased by an increase in the amount of land devoted to agriculture, YB. If the other production mechanisms YA and YC are maintained constant, the intensities of agricultural labor and technology will decrease, and these decreases cause secondary effects on the production. (The intensity of a production mechanism is the amount of that mechanism employed per unit of agricultural land). Alternatively, one may stipulate that with increase in the agricultural land there must be concomitant increases in labor and technology in order to maintain the intensities constant. This latter stipulation is used in the societal model.

The production function may be expressed in terms of indices in such a way that each index can be varied independently of the others. In the societal model, the indices are: VA=YA/YB the intensity of agricultural labor, VB the ratio of the land in use to land available, and VC=YC/YB the intensity of technology. Then, for example, ΔR=M(B,B)*ΔYB for a change in the agricultural land with VA and VC fixed.

(C) The stipulation that a primary production mechanism be changed while holding certain indices fixed may require concomitant changes in other production mechanisms. If production mechanism YI is changed by ΔYI, the concomitant change in YJ is expressed as ΔYJ=M(I,J)*ΔYI. For example, mechanism YB may be changed by ΔYB with the stipulation that the indices VA and VC be unchanged; from the definitions of the indices given in (B), one has M(B,A)=VA and M(B,C)=VC. In the societal model, the other M(I,J) are zero.

(D) The costs to produce a given change in food production are calculated for each production mechanism. Three categories of cost contributions must be considered. The cost of increasing the primary mechanism itself is PI*ΔYI. From the definition of M(I,I) given in (B), one may write ΔYI=ΔR/M(I,I). The cost due to mechanism YI itself is then PI*ΔR/M(I,I). The cost of the concomitant increases in other mechanisms, as noted in (C), may be expressed as PJ*ΔYJ = PJ*M(I,J)*ΔYI = PJ*M(I,J)*ΔR/M(I,I).

There may be costs due to other implied adjustments in the economy. Again, the matter requires judgement as to the extent to which such costs are utilized in the decision making process. Two examples may be cited from the societal model. Increase in technology implies an increase in the labor force in the technology sector. One defines an index M(I,K) according to ΔYK=M(I,K)*ΔYI, where I denotes the primary production mechanism that is changed (technology) and K is some level (technological labor) other than those associated with the three production mechanisms. The contribution to the cost is PK*M(I,K)*ΔR/M(I,I). In the societal model, there is a category of labor denoted as "loading" workers, which includes labor not directly associated with food production. Costs associated with this labor category are not considered in the decision procedure.

The cost index TA for changing the food production by employment of primary production mechanism YA is finally defined according to: (cost for primary mechanism YA)=TA*ΔR. This yields

TA = (PA + PB * M(A,B) + PC * M(A,C) + PK * M(A,K)) / M(A,A).

The cost indices TB and TC are defined similarly. Some of the terms may be zero, and there may be additional terms in the category of the K term.

(E) The changes in production mechanisms are chosen so as to minimize the marginal production cost. In the societal model, one simply rank orders the TI cost indices. For a food shortage, the least expensive mechanism is increased. For a food excess, the most expensive mechanism is decreased. In earlier studies on the societal model, constraints were placed on the variables which necessitated use of a simplex routine at this step. In a societal model, it is probably more realistic not to impose hard constraints, but instead to introduce structure into the feedback loops so that the "constraints" tend to be obeyed in the long term.

(F) Next, the right sides of the three differential equations for the production mechanisms are set. The desired rate of change in the food production function is $\dot{R}=K1*s$, where K1 is a model parameter and S is the food imbalance defined in (A). The definition of M(I,I) from (B) is used as $\dot{YI}=\dot{R}/M(I,I)$. The principal mechanism to be adjusted, as determined in (E), is governed by

$$\dot{YI} = K1 * S \;/\; M(I,I).$$

The concomitant adjustments in the other mechanisms, as explained in (C), are governed by

$$\dot{YJ} = M(I,J) * \dot{YI}.$$

3. Dynamical Refinements

Nine modifications, which affect the dynamical behavior of the system, are made to the basic decision procedure.

(a) The food stock index, introduced in step (A) of the preceding section, is redefined in terms of a lagged version of the actual food stock according to VZ=(lagged food stock)/(desired food stock).

(b) The right sides of the differential equations governing the three food production mechanisms, as set in step (F) of the preceding section, are applied to the differential equations only after a delay.

The delays in (a) and here simulate inevitable delays in gathering and acting on information, respectively. These delays may cause profound dynamical effects, including oscillations and instabilities.

(c) The food imbalance S, employed in step (F) to set the right sides of the differential equations governing the food production mechanisms, is proportional to the food stock Z. This factor of Z is replaced by the current estimate of the annual food consumption, denoted as ZZ. The latter tends to be stable while the food stock tends to fluctuate. It is reasonable to suppose that real-world decisions take cognizance of both the longer term, anticipated consumption and the short term status of the food stock.

(d) The food imbalance S also contains the factor (1/VZ - 1), which effectively gives "proportional control" in the alleviation of the food imbalance. This factor is replaced by the function G1, which gives a similar effect for a small food imbalance but shows a saturation for a large imbalance. The function G1 is depicted in Fig. 1A. The saturation of the corrective action reflects the limited ability of a real system to adapt itself to rapidly changing demands.

BRADEN, "Decision Procedure" Fig. 1.

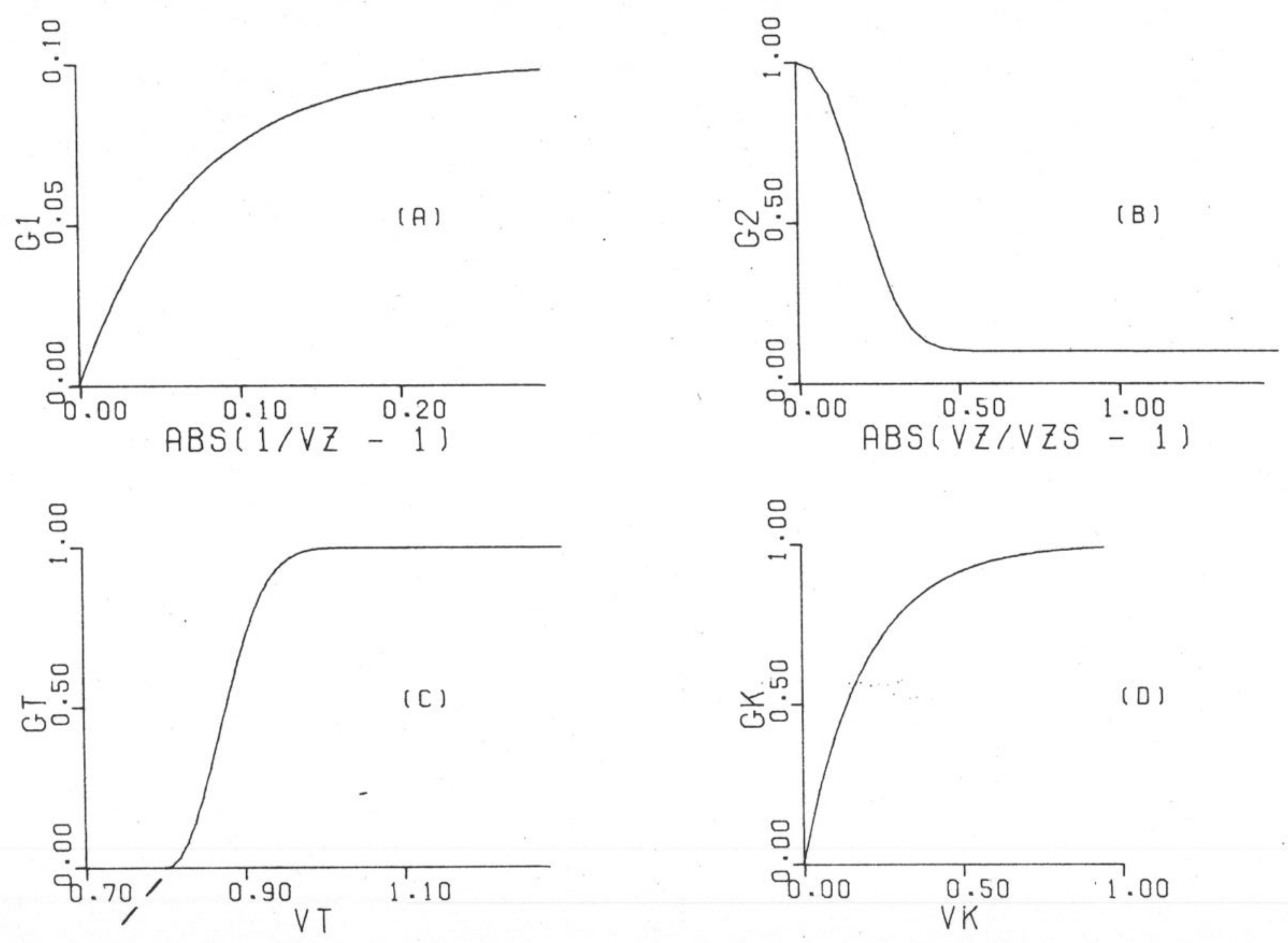

FIG. 1. FUNCTIONS DISCUSSED IN DYNAMICAL REFINEMENTS SECTION.

(e) Because of the instabilities induced by the delays described in (a) and (b), it is necessary to introduce a stabilizing mechanism into the model. This is done by the use of "derivative feedback" in the differential equations that govern the three food production mechanisms. The equation given for $\dot{YI}$ in step (F) of the preceding section is replaced by

$$\dot{YI} = (K1 * S - K2 * \dot{Z}) / M(I,I),$$

where K2 is a damping parameter. The time derivative of the food stock, $\dot{Z}$, is determined from the current and just previous values of the lagged food stock. In earlier studies, $\dot{Z}$ was averaged over the recent history of the lagged food stock, but no significant difference in behavior was noted.

Although "derivative feedback" is hardly an explicitly defined concept in real-world socioeconomic decisions, it is inherent in the decision processes. The prodent manager is reluctant to drastically change extant policies even in the face of apparently rapidly changing conditions.

In Fig. 2 are shown typical approaches to equilibrium in the societal model. In the case shown with no damping, the model is marginally stable with the oscillations continuing at about the same amplitude. Moderately strong damping generally produces a stable but sluggish behavior, as depicted (there is a transient peculiarity due to the action of delays). The use of a very large damping parameter does not necessarily contribute to model stability but can lead to violent oscillations for certain model conditions.

(f) In order to simulate the vicissitudes of climate, etc., "noise" may be introduced into the food production. The response of the model to this noise is stabilized by introduction of a factor G2 into the right sides of the differential equations governing the food production mechanisms. The G2 function is intended to discriminate between genuine changes in the societal system and temporary fluctuations. In addition to the food stock index VZ, defined in (a), a smoothed version of this index, VZS, is introduced. The dependence of the function G2 on the indices VZ and VZS is depicted in Fig. 1B. If the two indices differ considerably, the indicated changes in the food production mechanisms are severely attenuated.

(g) It is desired to study the societal model behavior when the technology level YC is near zero. At this limit, a reduction in technology cannot be accomplished if it is called for. In order to alleviate this problem, the decision procedure is modified. If the technology level is near zero and if the technology mechanism proves to be the most expensive of the three food production mechanisms, the most expensive and the next most expensive mechanisms are interchanged.

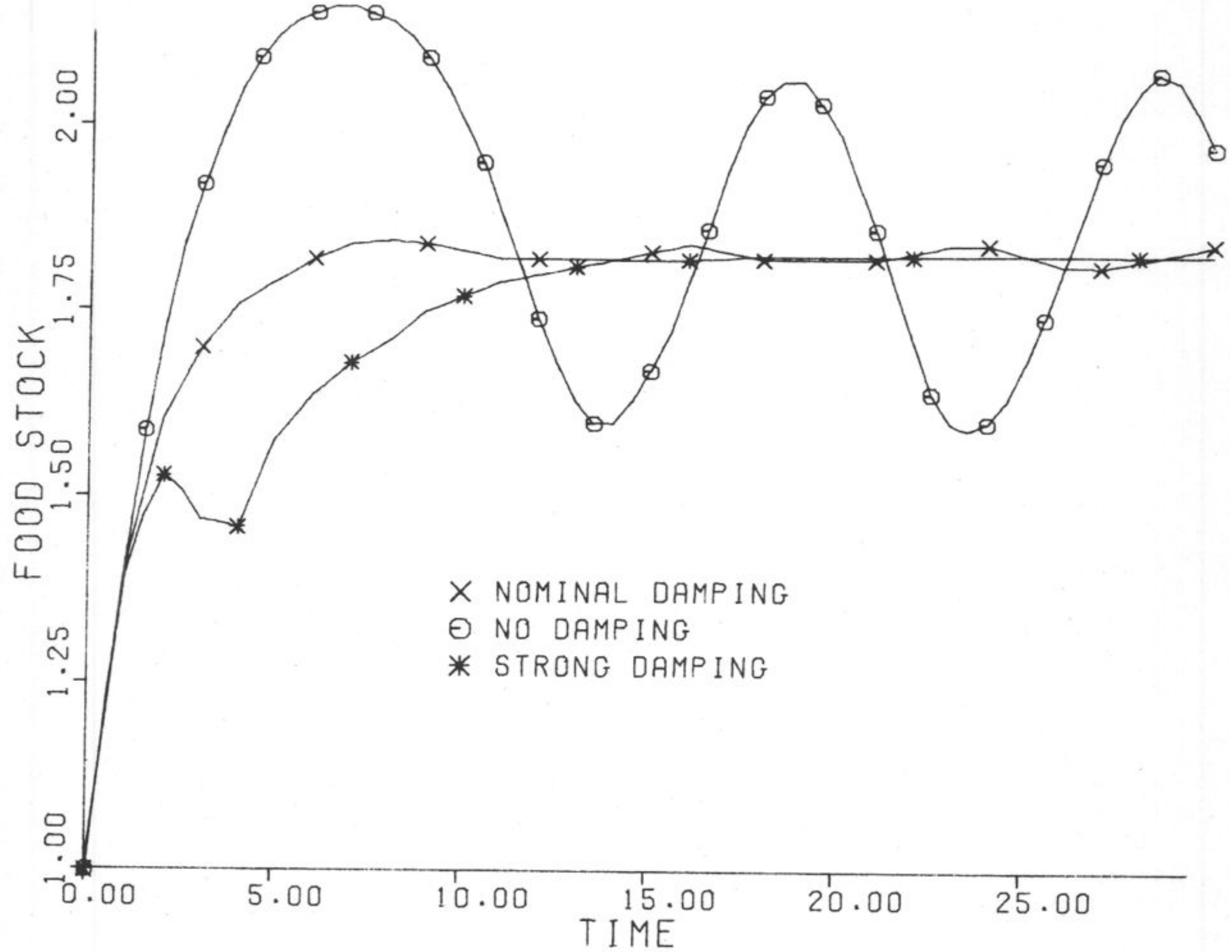

FIG. 2. APPROACH TO EQUILIBRIUM.

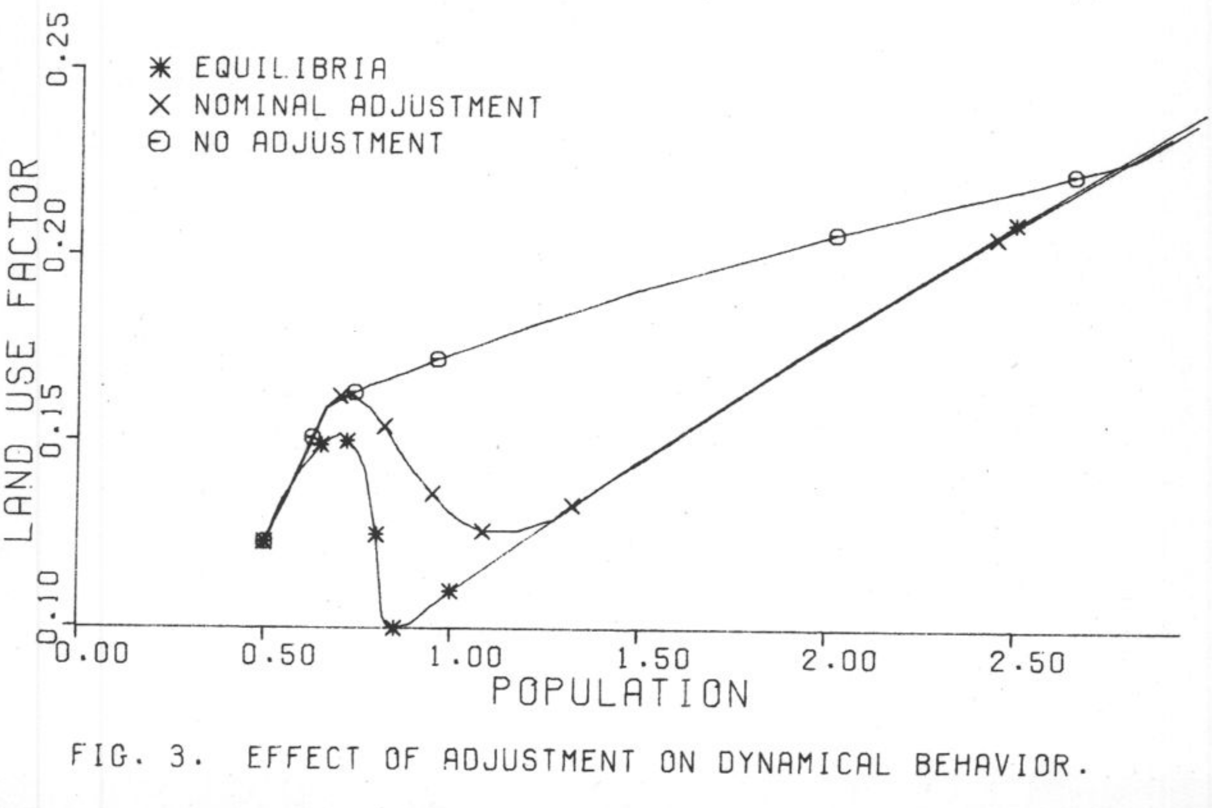

FIG. 3. EFFECT OF ADJUSTMENT ON DYNAMICAL BEHAVIOR.

(h) In a few instances, "inhibitors" are introduced in order to constrain certain variables as they approach their reasonable limits. For example, when the society is near full employment, the right sides of all differential equations governing labor are multiplied by a factor which attenuates any attempt to increase the labor.

Another inhibitor attenuates the growth of the technological plant if the labor in that sector is inadequate. Fig. 1C shows this inhibitor (GT) as a function of the adequacy of the technological labor (VT). If the labor is grossly inadequate, the technological plant decreases even though the decision process calls for an increase.

The effect of an inhibitor might be incorporated into the main structure of the model, as opposed to being introduced in an _ad hoc_ manner. However, it may be more practical and conceptually clearer to introduce the inhibiting action explicitly. In a real system, one may expect analogous, strong constraints to come into play as the system nears some natural limit.

(i) An "adjustment" process better enables the society to adjust for optimum cost conditions. With the basic decision procedure, if there is no food imbalance there will be no adjustment of the mix of the production mechanisms. Furthermore, if there is an imbalance which calls for increase in the least costly production mechanism, the most expensive mechanism will be retained at its current level. The basic decision procedure tends to find an optimum mix if the system oscillates between food shortage and food excess, but such a situation may not occur.

In a real system, one expects uneconomical production mechanisms to be discarded gradually in favor of more economical ones even though the net productivity need not change. In order to simulate such behavior, the right sides of the differential equations governing the three production mechanisms have "adjustment" terms introduced into them. An index VK measures the disparity in costs between the most expensive and least expensive production mechanisms. The adjustment terms are proportional to a factor GK, which is plotted against this cost disparity index in Fig. 1D. The adjustment process reduces the most expensive mechanism and increases the least expensive so as to produce no net change in the food production rate. Changes dictated by the adjustment process are accompanied by the concomitant changes in other variables, in accordance with step (C) of the preceding section.

The effect of the adjustment process on the dynamical behavior of the system is depicted in Fig. 3. The fraction of available land that is in use, the land use factor, is plotted. The curve denoted "nominal adjustment" shows the dynamical behavior of the societal model as the population increases at a stipulated rate. The curve denoted "no adjustment" is similar, except the adjustment process is inoperative. The curve denoted (equilibria" represents an idealised behavior in which equilibrium conditions, that correspond to the optimum mix of production mechanisms, are attained at each population level. Only if the population increased very slowly with time could the system track this curve.

In the growth of the society depicted here, there is a transition from a low-technology to a high-technology society. When the population is about unity, it becomes economical to utilize relatively less land but with a much more intense use of technology. However, the monotonically increasing population gives the model no opportunity to reduce land usage unless the adjustment process is operative. With inclusion of the adjustment process, the dynamical behavior more nearly resembles the idealized behavior. There is a limited period over which the society operates well away from its most economical state. A real society would be expected to show a similar behavior while new technology is being implemented.

4. Discussion

Adoption of a minimal marginal production cost as the criterion in the decision procedure does not imply that the "best" society is achieved thereby. Numerous alternative uses of a decision process in the structure of a societal model suggest themselves. One may base the decision on a figure of merit that is believed to reflect the propensity of decision makers in a real society, and the society may then be evaluated in terms of independently defined quality indices. Alternatively, one may base the decision on a quality index itself. Or, one may adopt a scheme whereby the decision procedure adjusts the system in accordance with criteria thought to be preeminent in actual decision making (criteria based on economic indices, most likely); then, independently defined quality indices may be introduced into the feedback loop structure in such a way as to drive the system towards "higher quality". An example relevant to the societal model studies would be to employ essentially the present decision criteria but introduce control on the birth rate which drives the population so as to improve some quality index. An interesting line of study would be to modify the economics of the society in various ways (perhaps through modifications in the price structure) and see if a congruence can be achieved, in the evolution of the system, between use of an economically based figure of merit versus use of a more abstractly defined quality index.

The decision procedure aims to minimize the marginal production cost, not necessarily the production cost itself. One should like to have a society which achieves the best possible state at each step in its evolution. This implies an optimization of the controlling figure of merit, from scratch, at each incremental step in the evolution of the society. In the dynamical evolution of a real society this is not practicable. However, one may ask to know the relationship between the state attained by whatever controlling procedure is in use and the absolute optimum. This is not an easy matter to discuss, and no generally valid statements can be made on the efficacy of the decision procedure towards the attainment of absolute optimization. Considerable model study has been devoted to the matter. With the inclusion of the refinements, notably the adjustment process, it does appear that the equilibrium conditions attained for a fixed population represent the most favorable production situation for the stipulated societal structure.

Occurrences of discrepancies between actual and optimal conditions during the evolution of the society, as indicated in Fig. 3, are also reflected in approach to equilibrium studies. When there exist two quite different but almost equally economical states, the system may be given initial conditions near the less economical state, and the system will move gradually to the more economical state. In such a situation, it proves difficult to find precisely the equilibrium state because the cost disparity over a wide range of states is almost zero, and the adjustment process is not very effective. (Improvement in this respect was made by adjustment of the slope of the GK versus VK curve, shown in Fig. 1D. near the origin). A real system is expected to reflect similar considerations. If there is little economic incentive to change things, the society will tend to remain in whatever state it happens to be.

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References

(1) Braden, Charles H. (1978) Societal Model to Study a Geeignet Population, Dynamica, Vol. 4, No. 3.