SPECIFYING DYNAMIC OBJECTIVE FUNCTIONS: PROBLEMS AND POSSIBILITIES

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1. THE SEARCH FOR OBJECTIVE FUNCTIONS

The creation of workable and reliable objective functions has been an elusive goal of economists, policy analysts, and dynamic modelers since the beginning of quantitative analyses of social policy. The development of such functions would produce several dramatic benefits for the formation of social policy. Not only would objective functions provide a precise index of system performance, but they would also clarify and explicate the criteria being used in the process of policy formation. Perhaps more importantly, objective functions would allow analysts to rank order preferred sets of policy alternatives.

From a technical perspective, as well, the creation of workable and reliable objective functions would be extremely valuable in dealing with questions related to parameter sensitivity. The testing of sensitivity would be greatly simplified if analysts could evaluate the reaction of the overall system performance to changes in parameters, rather than focusing on trajectories, characteristic modes, or recommended policies. Furthermore, the field of automatic control is immediately available with a host of optimization techniques which would be applicable to dynamic models upon the development of workable and reliable objective functions. Unfortunately, severe conceptual and technical problems currently appear to inhibit their possible use.

2. PROBLEMS WITH OBJECTIVE FUNCTIONS

The problems associated with developing workable and reliable objective functions may be classified as cross-sectional problems and dynamic problems. Cross-sectional problems refer to those associated with devising a function that will allow a decision maker to express a preference for any single point in the system's state-space. Dynamic problems refer to those associated with evaluating the performance of a system over time.

2.1 Cross-sectional Problems

In order to form an objective function, each n-tuple in an n-state model should map into a single value. The resulting unidimensional measure allows for the preferential ranking of all possible states of the system. The following technical problems exist in constructing such a cross-sectional objective function:

1. Defining criteria to be used by a decision maker in assessing the performance of a particular system state.
2. Establishing a hierarchy of system objectives from the myriad of detailed criteria at the bottom to a refined set of superordinate goals at the top.
3. Assessing consistent trade-offs between the various criteria for a particular decision maker.
4. Allowing the separation of expressed preference for an ideal system state from preferences for optimal feasible conditions within the constraints of the system's structure.

2.2 Dynamic Problems

Even if the above problems associated with formulating cross-sectional objective functions could be solved, a second set of dynamic problems exist in evaluating the performance of a system over time. In short, how could a decision maker evaluate the overall time path for the validated cross-sectional objective function? That is, is it possible to describe precisely the characteristics of dynamic objective functions that would make one trajectory preferable to another? One could imagine that some decision makers might be interested in a comparison of the initial and end-states of the overall trajectory (assuming that the system is stable), while other decision makers might be more interested in the monotenasticity of the overall curve. Others might even prefer the curve that produces a maximum when integrated over time. Each of these methods for evaluating the overall dynamic objective function trajectory could yield different judgments concerning what the overall preferred policy would be.

3. AN APPROACH TO SPECIFYING OBJECTIVE FUNCTIONS

Typically, engineers and economists have been able to sidestep many of the difficult problems involved in the construction of dynamic objective functions by evoking notions of minimum energy (or cost), maximum efficiency, or minimum total energy. In general, the cross-sectional problem has been solved by specifying a quadratic objective of the form

$$0(t) = X(t)\,Q\,X(t)^T$$

where $0(t)$ is the objective function at some specified time t, $X(t)$ is the vector of system states at time t, and Q is a matrix of weights applied to the various quadratic terms. The evaluation of dynamic trajectories typically has been handled by taking the integral of the quadratic objective function defined above with an added term for the evaluation of the system's end-state with the form

$$0(t_f) = \int_{t_0}^{t_f} X(\sigma)\,Q\,X(\sigma)\,d\sigma + f(X(T_f))$$

where $O(t_f)$ is the overall evaluation of the objective function at the final time t_f and $f(X(T_f))$ is the relative weight given to the system's final state (assuming that the system is stable). The justification for the use of such a simplified dynamic objective function has typically rested on **a priori** deductions concerning what the proper objective should be rather than upon detailed empirical investigation of what is the actual preference structure for an individual decision maker.

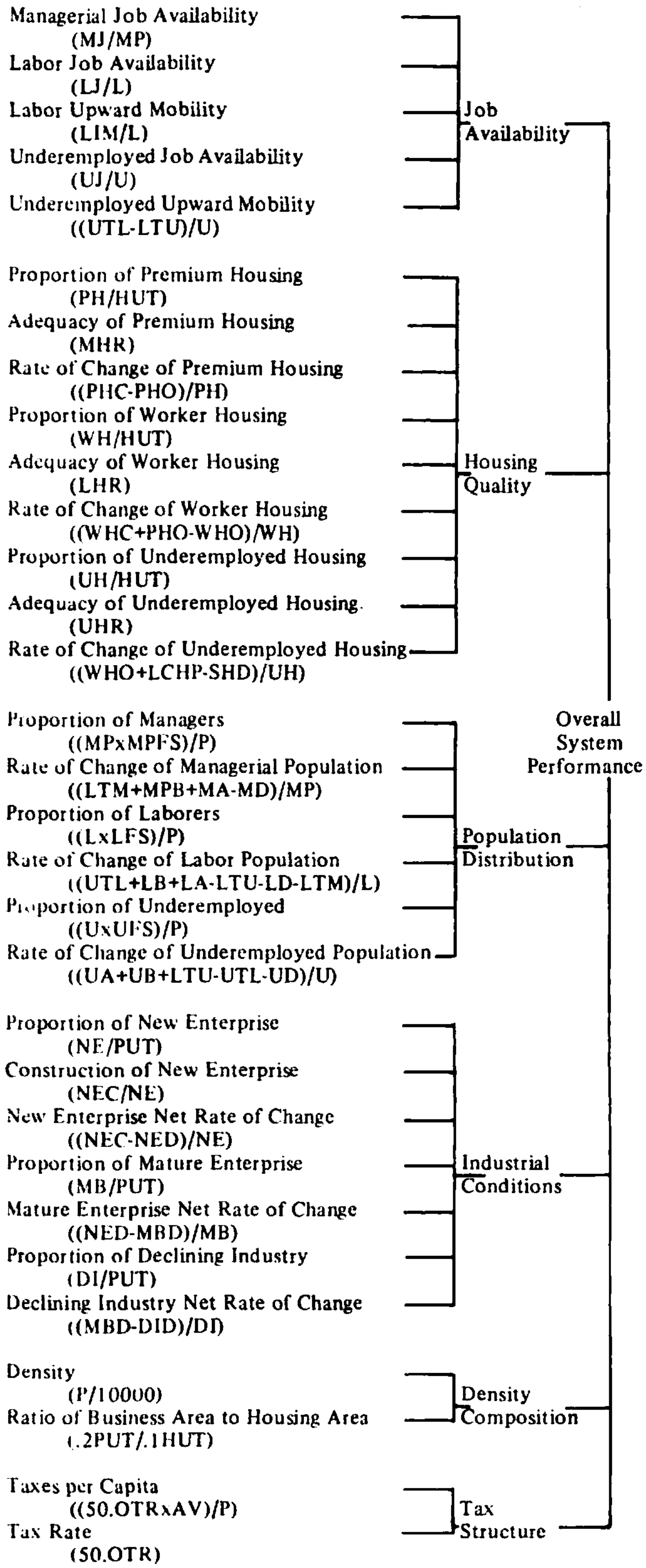

Figure 1: Hierarchical Judgement Model of the Urban System.

For many engineering problems and some economic problems, a priori specification appears to be justifiable. Most policy problems however, seem to demand a more complex assessment of the relative importance of a variety of competing social and political objectives. In the work reported below, social judgement analysis, a set of empirical techniques grounded in the theory of experimental social psychology, is proposed as a basis for the development of objective functions which could be used to assess the performance of a variety of social systems[1]. To illustrate the use of social judgement analysis in the development of workable and reliable objective functions, examples have been drawn from Forrester's **Urban Dynamics**[2], a complex, nonlinear, dynamic, feedback model designed to capture many of the interactions in a generic urban area.

The focus of the present research is on the construction of dynamic objective functions that would allow an individual decision maker to rigorously evaluate various policy tests conducted on the **Urban Dynamics** model. The two research questions (with implications extending well beyond the model being used) which guide the current investigation are: a) Can an objective function be constructed that accurately reflects a decision maker's preferences for various states of the urban system? and b) Can the overall dynamic trajectory be evaluated as the cross-sectional objective function varies over time? Although we do not expect to be able to answer fully either of these research questions, the present paper should provide a beginning to the solution of the complex problems surrounding the systematic investigation of workable and reliable dynamic objective functions.

4. DEFINING CRITERIA UPON WHICH TO BASE AN OBJECTIVE FUNCTION

Of the 124 system variables which Forrester tabled in **Urban Dynamics**, 36 were selected as potentially useful to the evaluation of the performance of the urban system. These 36 key system variables were combined and organized to form the hierarchical judgement model shown in Figure 1. More specifically, the 36 key system variables were variously combined to derive 31 explicit criteria at the bottom of the hierarchy which were subsequently clustered to form 6 separate system goals: job availability, housing quality, population distribution, industrial conditions, density composition, and tax structure. These 6 system goals, when integrated by a decision maker, provide the basis for constructing an objective function of overall system performance.

In order to develop a data base for the current research, the effects of 11 different urban policies on the 36 key system variables were extracted from the tabled values in **Urban Dynamics** for the two time periods cited: 10 years following implementation of a policy and 50 years following implementation. The 10-year cross-section captured the short-term effects of the policy being implemented and the 50-year cross-section reflected the long-run, equilibrium effects of the policy. By including one more set of conditions defined at equilibrium, a total of 23 alternative observations of the 36 system variables and, therefore, 23 alternative profiles of the 31 derived criteria, were constructed. Thus, the 23 profiles included one base equilibrium run plus one short-run (10 years) and one long-run (50 years) set of effects for each of the 11 policies investigated by Forrester (for a list

PROFILES OF JOB AVAILABILITY	
Managerial Job Availability (managerial jobs/manager)	.72
Labor Job Availability (labor jobs/laborer)	1.03
Labor Upward Mobility (% laborers to managers/year)	.9%
Underemployed Job Availability (underemployed jobs/underemployed)	.55
Underemployed Upward Mobility (net % underemployed to laborers/year)	1.5%
Managerial Job Availability (managerial jobs/manager)	.78
Labor Job Availability (labor jobs/laborer)	1.10
Labor Upward Mobility (% laborers to managers/year)	1.1%
Underemployed Job Availability (underemployed jobs/underemployed)	.75
Underemployed Upward Mobility (net % underemployed to laborers/year)	3.0%

PROFILES OF POPULATION DISTRIBUTION	
Proportion of Managers (managerial persons/total population)	.06
Rate of Change of Managerial Population (% growth of managerial population/year)	0.0%
Proportion of Laborers (labor persons/total population)	.41
Rate of Change of Labor Population (% growth of labor population/year)	0.0%
Proportion of Underemployed (underemployed persons/total population)	.53
Rate of Change of Underemployed Population (% growth of underemployed population/year)	0.0%
Proportion of Managers (managerial persons/total population)	.07
Rate of Change of Managerial Population (% growth of managerial population/year)	2.2%
Proportion of Laborers (labor persons/total population)	.44
Rate of Change of Labor Population (% growth of labor population/year)	1.6%
Proportion of Underemployed (underemployed persons/total population)	.49
Rate of Change of Underemployed Population (% growth of underemployed population/year)	-1.3%

Figure 2: Examples of Alternative Profile Segments.

of the 11 policies, see Table 2). Each complete profile was segmented according to the divisions specified by the 6 separate system goals. For example, Figure 2 illustrates 2 of the 23 profile segments for job availability and 2 of the 23 profile segments for population distribution. The data presented in Figure 2 were drawn from simulations of **Urban Dynamics**.

5. EVALUATING SYSTEM STATES ON THE BASIS OF THE CRITERIA

The evaluation of system states on the basis of 31 criteria is a complex problem which requires individual judgement. According to social judgement theory[3,4] such an evaluation process demands the integration of information related to any or all of the 31 criteria. Social judgement theory proposes that the integration of such information in the judgement process can be represented by a) the particular degree of importance placed on each criterion -- referred to as **weight**: b) the specific form of the functional relation between each criterion and the final judgement -- referred to as **function form**; and c) the particular method for integrating all of the criteria – referred to as the **organizing principle**. For example, judgement concerning the quality of various mass transportation system proposals might be described by a) placing twice as much weight upon the extent of air pollution and the amount of energy conserved as that placed upon the speed of travel; b) employing a negative linear function form for air pollution (the more pollution, the worse the judged quality) and a positive linear function form for energy conservation and speed of travel (the more conservation or speed, the better the judged quality); and c) additively integrating the information regarding air pollution, energy conservation, and speed of travel.

If repeated judgements are made about a variety of system states, the entire covert cognitive process of an individual's judgement can be mathematically modeled by the multiple regression equation[5,6,7,8,9,10,11].

$$\hat{Y} = \sum b_i X_i + c$$

where $\hat{Y}$ is the predicted judgement, b_i is the weight and functional direction of each criterion, X_i is the datum for each criterion, and c is a constant value. The equation can be extended to include quadratic and nonmetric relations[12]. The algebraic description of an individual's judgement can also be converted to a pictorial representation by means of interactive computer graphics[13].

Four students of system dynamics in the Graduate School of Public Affairs (SUNY-Albany) evaluated the 23 alternative profiles of system states (segmented into the 6 separate system goals) described above. Each of the 6 sets of judgements for each individual was made on a 20-point rating scale from 1 a) completely unacceptable system state) to 20 (a very acceptable system state). The six sets of judgements for each individual then become a final set of profiles themselves, about which a last set of judgements was made concerning the overall acceptability of the 23 system states.

Stepwise multiple regression analyses were used to develop models of the judgement processes of the four participants. Criteria were entered into the regression equations only if they were found to be statistically significant predictors of the participants' judgements ($p<.05$). The resulting multiple Rs ranging from .74 to .99 (an average of .94) indicated that a major proportion of the variation in judgements could be reliably predicted by the linear, additive models; nonlinear and nonmetric models requiring additional predictive terms were not tested due to the limited number of profiles available for the data base.

Table 1 presents the standardized regression coefficients in relativized form (i.e., constrained to sum to 100) for each

	Participants I	II	III	IV
Managerial Job Availability	44	46	32	30
Labor Job Availability	–	–	–	–
Labor Upward Mobility	–	–	–	–
Underemployed Job Availability	–	23	39	70
Underemployed Upward Mobility	56	31	29	–
Multiple R	.94	.89	.93	.93
Proportion of Premium Housing	-25	–	–	–
Adequacy of Premium Housing	–	–	-27	–
Rate of Change of Premium Housing	–	–	-26	–
Proportion of Worker Housing	–	-30	–	–
Adequacy of Worker Housing	14	–	–	–
Rate of Change of Worker Housing	–	–	–	–
Proportion of Underemployed Housing	–	–	–	–
Adequacy of Underemployed Housing	60	70	-47	100
Rate of Change of Underemployed Housing	–	–	–	–
Multiple R	.95	.79	.93	.87
Proportion of Managers	46	–	72	–
Rate of Change of Managerial Population	–	–	28	–
Proportion of Laborers	–	–	–	–
Rate of Change of Labor Population	54	44	–	54
Proportion of Underemployed	–	-27	–	-46
Rate of Change of Underemployed Population	–	-29	–	–
Multiple R	.86	.77	.93	.92

	Participants I	II	III	IV
Proportion of New Enterprise	79	–	69	69
Construction of New Enterprise	–	–	–	–
Mew Enterprise Rate of Change	–	100	–	31
Proportion of Mature Enterprise	-21	–	-31	–
Mature Enterprise Rate of Change	–	–	–	–
Proportion of Declining Industry	–	–	–	–
Declining Industry Rate of Change	–	–	–	–
Multiple R	.92	.92	.97	.95
Density	-12	-65	18	–
Ratio of Business to Housing Area	88	35	82	100
Multiple R	.99	.92	.95	.89
Taxes per Capita	23	32	–	–
Tax Rate	-77	-68	-100	-100
Multiple R	.97	.92	.83	.82
Job Availability	61	22	80	–
Housing Quality	–	12	–	43
Population Distribution	14	–	20	14
Industrial Conditions	–	31	–	43
Density Composition	25	–	–	–
Tax Structure	–	35	–	–
Multiple R	.95	.94	.97	.96

Table 1: Relative Weights Comprising Judgement Models.

participant's judgements on each of the 7 sets of predictors. Due to the tendency for the criteria to be highly interdependent, the regression coefficients cannot be considered to be extremely stable estimates. These interdependencies could have been eliminated by generating synthetic profiles with orthogonal criteria. However, this technique creates another set of problems. Correlations occur in the criteria because many of the system variables do, in fact, move together. Thus, to create synthetic data would require decision makers to make judgements about cities that probably could not exist. In effect, creating synthetic profiles forces judgement concerning "ideal" urban systems by ignoring real constraints.

6. USING JUDGEMENT MODELS TO EVALUATE POLICY OUTCOMES

Given the high multiple Rs associated with the judgement models derived above, it seems clear that social judgement analysis can provide an adequate method for predicting individual preferences. But does the technique provide consistent patterns of agreement or disagreement between individuals in their ranking of policy outcomes? Table 2 presents some suggestive results indicating how social judgement analysis may help in isolating both differences and similarities in individual assessments of policy alternatives. For each of the 11 policies tested by Forrester, a determination was made for each participant whether the policy resulted in

Urban Policy	Time Frame	Participants I	II	III	IV	Summary
Construction of low-cost housing	10 years	– (–)	– (–)	– (–)	– (–)	worsened
	50 years	– (–)	– (0)	(–)	– (–)	
Worker housing construction program	10 years	– (–)	– (0)	– (–)	– (–)	worsened or no change
	50 years	– (0)	0 (0)	– (0)	– (–)	
Tax subsidy of $100 per capita per year	10 years	– (0)	+ (0)	0 (0)	0 (–)	divided reaction
	50 years	– (0)	– (0)	0 (0)	– (–)	
Premium housing construction program	10 years	0 (0)	0 (0)	0 (0)	0 (–)	divided reaction
	50 years	0 (0)	+ (0)	– (0)	0 (0)	
Jobs created for 10% of the underemployed	10 years	0 (0)	0 (0)	0 (+)	+ (+)	divided recreation
	50 years	0 (0)	– (0)	0 (0)	0 (0)	
Declining industry demolition program	10 years	– (-)	+ (0)	– (0)	+ (0)	divided reaction
	50 years	0 (0)	+ (+)	0 (0)	+ (+)	
Underemployed training program	10 years	+ (+)	+ (+)	+ (+)	0 (0)	improved or no change
	50 years	+ (+)	+ (+)	0 (+)	0 (0)	
New enterprise construction program	10 years	+ (+)	+ (+)	+ (+)	+ (+)	improved or no change
	50 years	+ (+)	0 (0)	+ (+)	+ (0)	
Slum housing demolition program	10 years	0 (+)	+ (+)	+ (+)	+ (+)	improved or no change
	50 years	0 (0)	+ (+)	+ (+)	+ (+)	
Slum housing demolition and restrictions on worker housing	10 years	0 (+)	+ (+)	+ (+)	+ (+)	improved or no change
	50 years	0 (+)	+ (+)	+ (+)	+ (+)	
Slum housing demolition and encouraging new enterprise	10 years	+ (+)	+ (+)	+ (+)	+ (+)	improved
	50 years	+ (+)	+ (+)	+ (+)	+ (+)	

Note: "–" indicates worsened conditions; "+" indicates improved conditions; "0" indicates no change from equilibrium conditions. Symbols in parentheses are based on judgements predicted by the models of the participants' judgement policies.

Table 2: Comparison of Participants' Evaluations of 11 Urban Policies.

overall outcomes that were explicitly judged to be "better" (shown as a "+" within Table 2), "worse" (shown as a "-" within Table 2), or "the same" (shown as a "O" within Table 2) as the overall outcomes of the initial equilibrium condition. A rating of "the same" resulted if the evaluation of the system's state at the 10th year or the 50th year following policy implementation was within a 95% confidence interval constructed around the equilibrium judgement; judgements of "better" or "worse" fell outside of the interval. Shown in parentheses in Table 2 are parallel results based on judgements predicted from the judgement models derived above.

A comparison of the results from the actual judgements with results from the predicted judgements shows that the judgement models performed quite well in predicting individuals' actual reactions to the policy experiments. In no case did the models contradict the actual evaluations. Approximately 4% of the time, the judgement models predicted either "better" or "worse" when the actual judgements showed "the same". About 8% of the time, the judgement models predicted "the same" when the actual judgements showed slight support or rejection of the policy outcomes.

In one of the 11 policies investigated by Forrester, all four participants agreed that the outcome conditions represented a clear benefit for the city (i.e., slum housing demolition coupled with encouraging new enterprise construction). For one other policy (i.e., construction of low-cost housing), all participants agreed that the outcome conditions led to the city's deterioration. Four programs produced participants' conclusions of some betterment or no change (i.e., slum housing demolition and restrictions on worker housing, slum housing demolition alone, a new enterprise construction program, and an underemployed training program). Similarly, one program (i.e., the worker housing construction program) showed either no change or a clear worsening of conditions within the city. For the 4 remaining policies, the participants were divided. These programs produced a mixture of some better ratings, some worse ratings, and some ratings similar to equilibrium. These policies include job creation for the underemployed, a $ 100 per capita tax subsidy, a premium housing construction program, and a declining industry demolition program.

An investigation of Table 1 shows how these apparently contradictory ratings can result. For example, participants II and IV weighted industrial conditions rather heavily in their final evaluation of system states. Since the declining industry demolition program improved the city's industrial conditions at the expense of other system goals (most notably job availability), participants II and IV were inclined to favor declining industry demolition as beneficial to the city, whereas participants I and III found such a policy to be disadvantageous. For the other three split policy evaluations, the disagreement appeared to center on differences between short-run and long-run consequences. Although a complete examination of the systematic patterns of differences and similarities is beyond the scope of this paper, the foregoing discussion points to the potential usefulness of social judgement analysis as a means of analyzing individual attitudes towards policies tested in dynamic models.

7. USING JUDGEMENT MODELS TO EVALUATE TRAJECTORIES

The use of social judgement analysis may also provide an empirical approach to the problem of describing the characteristics of dynamic objective functions that make one trajectory

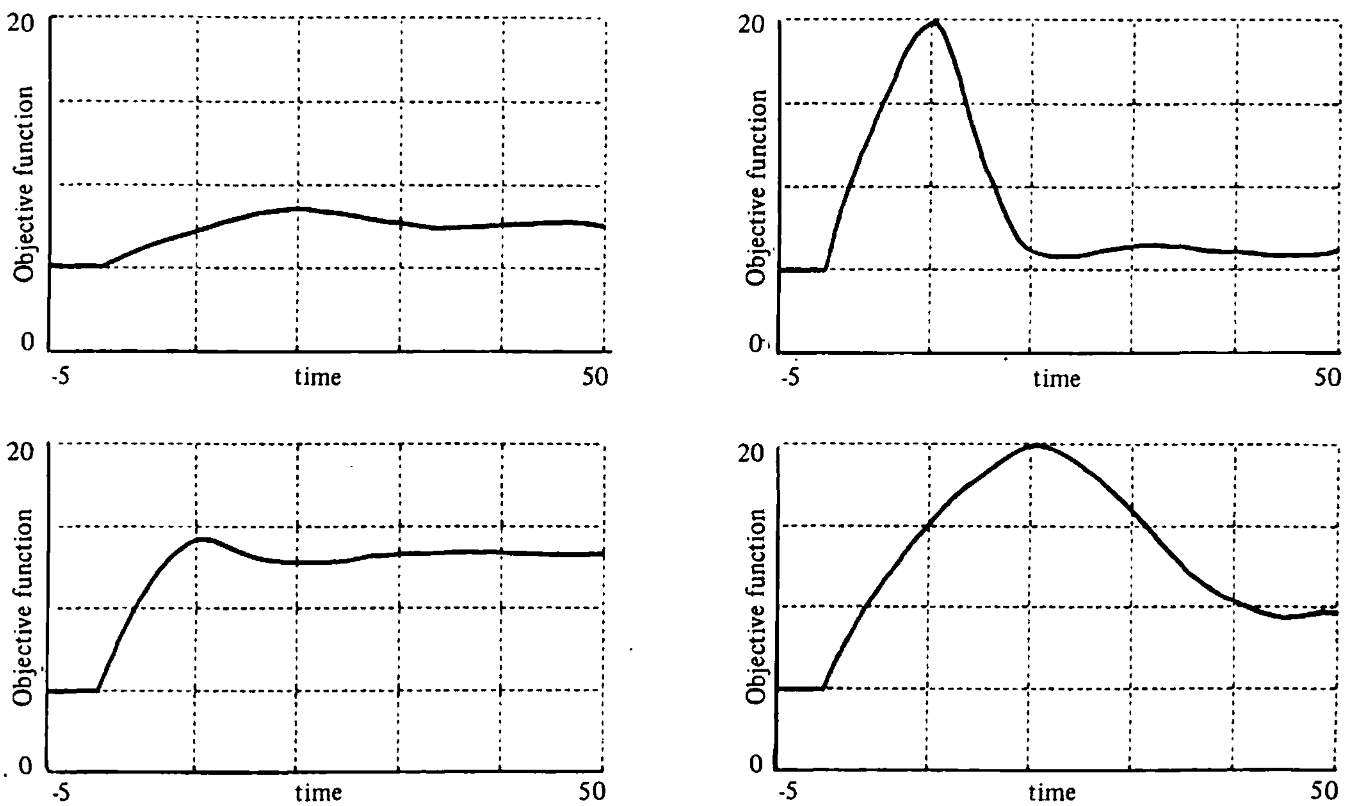

Figure 3: Examples of Alternative Trajectories.

preferable to another for a particular decision maker. Although a large number of characteristics for any given curve might be listed, four were selected as the basis for the present study: number of years from initial equilibrium to maximum, number of years from maximum to second minumum, maximum point, and final equilibrium point. By systematically varying these four characteristics at five levels (e.g., 5, 10, 15, 20 and 25 years from initial equilibrium to maximum), 25 curves were constructed that closely paralleled the trajectories generated by the Urban Dynamics model. These 25 curves had the special property that the four characteristics by which they were defined were uncorrelated across the full set of curves; examples of the 25 curves are shown in Figure 3.

The set of 25 curves were presented to four participants who were instructed that these curves represented hypothetical objective functions tracing the longitudinal acceptability of urban system states. The participants were asked to evaluate the curves on a scale of 1 (a completely undesirable trajectory) to 20 (a very desirable trajectory). Again, through the use of stepwise multiple regression analyses, judgement models were derived that could be used to predict consistently the desirability of a wide range of curves. Characteristics of the curves were entered into the regression equations only if they were found to be statistically significant predictors of the participants' judgements ($p<.05$). The resulting multiple Rs ranging from .89 to .96 indicated that a major proportion of the variation in judgements could be reliably predicted by the linear, additive models based on the four characteristics identified.

When the standardized regression coefficients were constructed in relativized form (i.e., constrained to sum to 100) for each participant's judgements, striking individual differences were found in the manner that the trajectories had been evaluated. One participant used only the characteristic of final equilibrium point (producing a relative weight of 100). Another participant used the final equilibrium point but also the number of years from maximum to second minimum (relative weights of 82 and 18, respectively). The third participant also used the final equilibrium point paired with the magnitude of the maximum point of the trajectory (relative weights of 75 and 25, respectively). The final participant used three characteristics: final equilibrium point, number of years from maximum to second minimum, and maximum point (relative weights of 40, 14, and 46, respectively). None of the participants appeared to rely on the number of years from initial equilibrium to maximum in evaluating the trajectories.

8. DIRECTIONS FOR FUTURE STUDY OF OBJECTIVE FUNCTIONS

Although the areas of research explored in this paper need further work and refinement, the following tentative conclusions can be drawn from the present research:

1. Social judgement analysis can be used to reliably predict individual preferences of a system's composite state at a single point in time.
2. The technique appears to be able to specify how and where individuals both agreed and disagree in their evaluations of policy alternatives.
3. There may be some clearly identifiable and consistent rules employed by individuals in evaluating dynamic trajectories of objective functions. Although this conclusion is the most tentative, it may also be the most promising.

Several conceptual and technical problems still exist, however, within the present approach. The set of feasible system trajectories are often highly correlated. This situation makes the application of social judgement analysis more technically difficult if the focus of research is on the specification of individual differences in judgement models. Further consideration must be given to alternative methods of structuring the hierarchy of criteria to insure that the reliable predictions do not shift if the judgement problem is slightly restructured. Finally, much more extensive work is needed in the evaluation of dynamic trajectories. In summary, the application of social judgement analysis to the objective function problems as explored in this paper is a promising first start toward the empirical derivation of truly workable and reliable objective functions. We conclude that the possibilities are sufficient to warrant a continued attack on the problems.

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