SYSTEMS MANAGEMENT: A GENERAL OVERVIEW
PART II - SYSTEM DYNAMICS MODELLING

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This is the second of a three part article concerning Systems Management. In Part I the concepts underlying the subject were developed. Part II is concerned with the scope of application of System Dynamics and is a forerunner to a case study concerning National Development in the Lebanon which will appear in Part III.

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1. THE MODELING PROCEDURE
The system dynamics approach begins with an effort to understand the circumstances that have created a problem and continue to sustain it. Relevant information is collected from literature and knowledgeable persons close to the problem such as managers, customers, workers, experts, etc. As soon as a rudimentary measure of understanding has been achieved, three complementary forms of a model of the situation is developed: verbal, visual and mathematical. The verbal description is a mental model of the system expressed in words. The verbal description is diagrammatic and shows cause-and-effect relationships between many variables in a simple, concise manner. The visual model, or “causal diagram”, is translated into a mathematical model, the system equations. All forms are equivalent, with any one form merely serving as an aid to understanding for someone who is not fluent in the other languages. However, the verbal description does not lend itself to formal analysis. The visual causal diagram can only be analyzed qualitatively. The mathematical model is by far the most precise and is the only representation of the system that permits quantitative analysis and the evaluation of alternative solutions to a problem.

2. CAUSAL DIAGRAMMING
An important tool for developing the model of a dynamic system is the causal diagram. The key variables which record the way the system works are identified from the verbal description and these are connected by arrows. The direction of the arrow shows the direction of causation for a pair of variables. The variable at the head of the arrow is the dependent variable; the variable at the tail of the arrow is the independent variable for the paired relationship defined by an arrow. The sign on the arrow identifies the polarity of the relationship between the two variables, plus or minus. A plus relationship means that the dependent variable changes in the same direction as the independent variable; a minus sign means that it changes in the opposite direction. Several examples of the development of causal diagrams from verbal descriptions, and the use of qualitative analysis, appear below.

Basketball Model
The entertainment value of the game is enhanced by teamwork, by long shots, and by dunk shots. However, many fans feel that the growing importance of tall players detracts from the game. These same purists contend that dunk shots place increased emphasis on individual play which, in turn, reduces teamwork. Shooting efficiency, the percentage of shots that are made, depends on the difficulty of shot and shooting accuracy. Players noted for dunking spend less practice on other shots which reduce their shooting accuracy. The lower the shooting efficiency, the greater the number of rebounds, and the greater the number of rebounds, the greater the importance of tall players to a team.

Eleven variables in the verbal model above have been identified and underlined. The most probable reason for someone, say the Commissioner of the National Basketball Association, wanting to model this problem would be to increase the entertainment value of the game. A “goal variable” such as this which can not be affected directly by the decision maker, is also referred to as an “uncontrolled variable”. The decision maker must use variables within his control, called “decision variables” or “controlled variables”, to achieve his goals.
Some controlled variables can be used to achieve the desired effect by altering system structure; others by changing the policies through which the system operates. Referring to the basketball model, two decision variables that might be used to improve the entertainment value through structural changes are the rim diameter and the basket height. Policy variables under control of the decision maker are the points per long shot, outlawing dunk shots and limiting maximum height of players.

Qualitative analysis of the causal diagram shown in Fig. 1(a) corresponding to the verbal model permits the decision-maker to evaluate the effectiveness of some of the decision variables and to enhance his understanding of the implications of the others.

**Narcotics Model (1)**

The poppy has been cultivated since antiquity as the source of opium. The principal alkaloid of opium is morphine, which is the raw material for the even more potent, addictive and illegal narcotic, heroin. The more opium poppy acreage (production), the more smuggling of heroin (distribution), the greater the supply of illegal heroin on the streets. The price of illegal heroin varies inversely with the supply. Also the higher the price, the greater the incentive to produce opium and traffic in heroin.

Secondly, the price affects the addiction rate. When the price is low, heroin is more accessible to the discretionary or non-addicted users and vice versa. In addition, low prices force pushers to seek buyers among non-addicts, thus increasing what might be called the “marketing-effort effect” which, in turn, increases the addiction rate. The addiction rate expressed in addicts per month also depends on the number of addicts on illegal heroin and a “normal addiction factor” which is the number hooked per month per addict. Lastly, the price of illegal heroin varies directly with the number of addicts on illegal heroin.

Narcotics is a business, one of the biggest in the world. It involves all the elements of legal industries — production, distribution, marketing, consumption, profit, government harassment, competition, risks, etc. Using the causal diagram in Fig. 1(b) which is taken from the verbal description, one can see the limited effectiveness of three very different strategies for coping with this industry:

1. Reduce production by paying farmers to grow other crops.
2. Reduce distribution through enforcement.
3. Legalize heroin.

**Financial Model of a Firm (2)**

The basic structure underlying processes in the firm and in society as a whole are causal feedback loops formed by two or more variables mutually affecting each other. Just as the individual relationships can be described as positive or negative, so can the loop containing the variables. The polarity of the feedback loop is obtained by counting the number of negative paired relationships around the loop: if even, the loop is positive; if odd, the loop is negative. Positive feedback is characterized by exponential growth or decay; negative feedback by a symptomatic growth or decay.

Pricing, credit, and investment decisions are imbedded in many separate but interrelated feedback loops within a firm. These loops connect the firm’s production sector to the product market.

An increase in product availability decreases delivery delay, which increases market share. The firm’s share of the market also increases with credit period and a lower price. Sales are determined by market share and market demand, the total sales by all firms in the market. Increased sales decrease inventory, which decrease product availability.

An increase in sales leads to an increase in the sales forecast, which causes an increase in investment. The increased investment eventually results in an increase in plant and equipment acquisitions, and therefore production capacity, a higher level of production capacity means increased production and, other things remaining constant, greater product availability.

An increase in acquisitions leads to an increase in cash expenditure. The increased cash expenditure causes the firm to increase debt financing, increased debt financing increases the level of debt outstanding. The firm’s unused debt capacity is the difference between the debt and the firm’s debt limit. The lower unused debt capacity means that a lower supply of funds is available for investment. The relationships between
investment and acquisitions already described completes the loop.

An increase in sales increases internal cash flow, which lowers the amount of debt financing required to meet cash expenditures for plant and equipment acquisitions. Moreover, increased internal cash flow provides a larger supply of funds for investment. An increase in sales also increases earnings. The increase in the firm’s earnings brings about an increase in equity, which brings a larger supply of unused debt capacity. However, a negative effect is also manifested with increases in earnings. Dividends increase, thereby lowering both internal cash flow and equity. Interest on the debt increases with the debt and this increase in interest expense will decrease the internal cash flow.

Desertification Model.

Everybody uses verbal models repeatedly in a variety of ways – whenever a complex phenomenon is perceived and described, whenever an opinion on politics is advanced, whenever a difficult decision or choice is to be made, etc. The following verbal model is taken from a cover story in TIME Magazine a couple of years ago:

More than a third of the earth’s land mass is desert or desert-like, and one out of seven people – some 600 million – dwell in these parched regions. Every continent is affected by the encroachment of deserts into usable lands, called desertification. It is generally agreed that this phenomenon is primarily the result of the misuse of the ecosystem by man. Africa’s Sahel, the band of impoverished land across the Sahara’s southern flank, is illustrative of this problem.

Various evidence suggests that much of north Africa formerly supported grazing cattle. Expanding human population over the years led to a increased cattle population; the increase in cattle population to a higher grazing density (cattle per hectare of grazing land); which caused over-grazing and this to land fertility degradation. The drop in land fertility means a lower land yield or yield per hectare, leading to the clearing of forests for additional grazing land. But at the same time this clearing of land or deforestation leaves the soil unprotected against winds thus accelerating erosion. Soon the thin layer of topsoil over the new grazing land area vanishes.

The supply of cattle feed depends on the amount of grazing land and the land yield. As the feed per head drops, genetic damage increases; this reduces the effective cattle per capita (of human population). The cattle acquisition rate depends on the difference between the effective cattle per capita and the normal supply of cattle per capita. In other words, genetic damage cuts the efficiency of a herd so that even larger herds are needed to supply a given amount of milk and meat.

The reduction of land available for agriculture leads to increased irrigation to reclaim land from the desert. However, the increased height of water table which results is counterproductive because of the high saline content of the subsurface water. The increased saline content of the affected areas further reduces land available for agriculture.

Transportation System Model.

The dynamics of a system as a whole raise a wide range of problems, for its parts may operate in conflict making it difficult to solve the individual problems separately. The urban transportation phenomenon affords an excellent example. The “parts” are highway transportation described in the first paragraph, and public transit, which is described in the second paragraph below.

It is not difficult to show, as the opponents of highway transportation have, that highway construction is an explosive phenomenon. The argument goes something like this: the more highways HWY (miles), the more travel, the more highway earnings HWYE (dollars per year). The Highway Trust Fund HWYF (dollars which is increased by highway earnings is used to maintain existing highways and build new ones. The amount of money spent on highway maintenance costs HWYM (dollars per year) depends on the miles of highways and the unit maintenance MC (dollars per mile per year). The number of miles of new highway construction HWYC (miles per year) depends on the balance in the Fund that is left after maintenance HWYC, the unit construction cost CC (dollars per mile) and the portion of the balance programmed for construction each year FPY (fraction per year). The land fraction occupied by highways LFO (dimensionless) depends on the average highway right of way ROW (acres/miles), the total area AREA (acres) served by the highways, and the miles of highways. As more and more land is consumed by highways, there is less land for other uses and therefore less trips generated leading to less automobile usage AU (miles/year per vehicle). Automobile usage, highway user taxes HUT (dollars per mile), automobile ownership AO (vehicles), and auto ownership fees AOF (dollars per year per vehicle) determine highway earnings per year.

Throughout the period of explosive growth of highway transportation, there was a steady decline in urban public transit operations in many places in the world. Many aspects of urbanization combined to form vicious circles which serve to suppress the variables of interest. Increases in per capita income PCI (dollars per year) lead to decreases in population density PD (per sq.mi.) and to increased automobile ownership AO (vehicles). Increased auto ownership means less transit ridership TR (trips/yr). Traffic congestion TC (veh./hr.) increases as auto ownership increases and transit ridership decreases. The more traffic congestion, the greater the travel time TT (minutes) for all traffic including buses. Higher travel times means higher turn-around-times for buses TAT (minutes) which reduce the revenues R (dollars/yr.) leading to increased fares F (dollars/trip). Increased fares reduce transit ridership. A decrease in ridership leads to less buses assigned per route BAPR causing larger headways H (minutes/bus). Reduced population densities lead to increased route lengths RL (miles/route) and longer route lengths cause increased headways.

Decision variables do not have to be affected by massive outlays of capital for new technologies as if often prescribed. Sometimes system structural changes or policy initiatives can do the job. Causal diagrams for the two component models are shown in Fig. 2. Three “cheap” strategies for alleviating the exponential growth of private highway transportation and the exponential decline of public transportation, which the interested reader should incorporate into the model by extending the causal diagrams are:
1. A land zoning policy which would reduce the land area in the highway component and increase the population density in the public transit component.

2. A financial allocation policy which would make budget transfers from the Highway Fund to a Transit Fund.

3. A capacity allocation policy which would convert existing freeways lanes to exclusive bus transit use.

3. **CLASSIFICATION OF MODEL VARIABLES**

There are two basic components of the structure of a system from the system dynamics perspective — flows and levels. Levels are state variables that represent the accumulations of resources in the system such as inventories of goods, persons, workers, jobs, housing and capital. Rates of flow represent the activities and decision functions in the system such as the movement of goods, births and deaths of persons, hiring and firing of workers, changes in employment opportunities, construction and demolition of housing, generation and use of capital, etc.

Our causal diagramming techniques is now ready to be refined so as to easily identify these two special types of variables — levels and rates — simply by noting that levels appear at the heads of solid arrows (physical flows) and rates at the tails of solid arrows. Similarly, information on the state of the system to be used as a basis for decision making must emanate from level variables and will be represented by dotted lines. A few verbal models will be converted to causal diagrams using this notation to illustrate this concept, which becomes important when it comes time to translate the causal diagram into the mathematical model — the system equations.

**Market Growth Model (3)**

Marketing couples the resources of a company to the desires of the customers. A causal-oriented approach to analyzing management problems and designing management policies finds numerous applications in the marketing distribution aspects of the firm. Consider the following verbal description of a model depicting sales growth and saturation.

Marketing effort is accomplished by the hiring of salesmen who provide the driving power for sales growth. The salesmen hiring rate \( SH \) which is the number hired per month adjusts the actual number of salesmen \( S \) to the desired sales force \( DS \) by comparing the actual to the desired number of salesmen. The time required to correct this discrepancy is the salesforce adjustment time \( SAT \) expressed in months. The number of orders booked \( OB \) (units per month) depends on the number of salesmen and on sales effectiveness \( SE \) which is defined as the units of product sold each month by each salesman. The budget \( B \) for salesmen’s monthly expenses is computed from orders filled (the delivery rate \( DR \) explained below) multiplied by the revenue to sales \( RS \) which is the dollars per unit that are allocated to selling cost. The desired sales force varies and is determined by dividing the monthly budget by the average salesman salary \( SS \). In other words the desired sales force is the number that can be justified.
been accepted. The simple model presented here describes some of the factors controlling the supply of a non-renewable resource such as fossil fuel.

Resource reserves are often classified as unproven reserves and proven reserves. The finite stock of virgin resources called unproven reserves UPR (tons) is discovered at a rate \( \frac{d}{dt} \) and after discovery these resources are classified as proven reserves PR (tons). The discovery rate depends on the amount of capital invested in exploration (dollars/year) and the cost of exploration (dollars/ton). As unproven reserves are depleted, the cost of exploration rises because less of the resource is found per exploration and because producers must look in less accessible places. The rising cost of exploration decreases the discovery rate slowing the depletion of unproven reserves. A rise in the cost of exploration also increases the total unit cost of production TC (dollars/ton), which decreases the return on investment in exploration ROI (dollars/dollar), causing a decrease in investment in exploration, and therefore in discovery rate.

On the demand side, an increase in proven reserves increases the reserve-to-usage ratio RUR (yrs), the measure of how long proven reserves will last at the current usage rate UR (tons/year). An increase in the reserve-to-usage ratio of the resource decreases its price P (dollars/ton). A price decrease results in an increase in demand D, which increases the usage rate and thus decreases proven reserves. Price depends on a second variable, total cost, as well as two constants, normal profit NPROF (dollars/ton) and normal coverage NC (years). The return on investment in exploration obviously depends on price as does sales revenues SR (dollars/year). Sales revenues also change with the usage rate. An increase in sales revenues will increase investment in exploration, for it is reasonable to assume that industry allocates its investment proportional to revenues. At the same time, investment in exploration is increased when the reserve-to-usage ratio drops below some specified critical coverage CC (yrs) because the need for further discoveries becomes more urgent.

Figure 3. Causal Diagram for Market Growth Model

by the present rate at which new orders are being booked and filled.

Sales effectiveness is a variable that depends on how long customers must wait for products — the delivery delay DD in months. The rationale is that when the product is available for immediate delivery a salesman has an easier job and can sell more per month than if the customer must wait for show delivery.

Orders booked are placed in a backlog of unfilled orders BL, which is reduced as orders are filled referred to as the delivery rate DR. The delivery delay DD is the ratio of the backlog to the delivery rate.

The delivery rate varies directly with production capacity PC. Production capacity in units per month is increased by the capacity acquisition rate CAR which is the difference between the desired production capacity DPC and the actual production capacity PC divided by the capacity acquisition delay CAD. The greater the backlog the greater the desired production capacity.

The causal diagram for this model which differentiates between physical and information flows is shown in Fig. 3.

Resource Development Model (4)

Only recently has the fundamental postulate that non-renewable resources are present on earth in finite supply

Figure 4. Causal Diagram for Resource Development Model
Iceberg – Water Supply Model

Of the world’s total water resources, only three percent is fresh water. Three-fourths of the fresh water total is held in the form of ice, and ninety percent of it is in Antarctica. Why not transport icebergs from Antarctica to arid regions and use this ice to satisfy the water requirements of agriculture, industry, and population in general?

The model described here uses Saudi Arabia to show how an iceberg delivery system might work. All that is needed is a few tugs and a small land-based reservoir located at the port of Jeddah on the Red Sea, near Mecca.

It is the great quantity of water that makes the iceberg delivery system economically attractive (less than one-tenth the cost of desalinated water, for example). Tugs would be despatched from Jeddah and travel to the South Pole, where they would “harvest” massive icebergs and hook them together. Each tug would pull a train of icebergs 150 meters wide, 100 meters deep, and at least 2 kilometers long. Part of the harvesting task would be wrapping the iceberg train in plastic insulation to retard melting in transit. A tug would spend three weeks traveling to Antarctica, twelve weeks collecting the icebergs, insulating them, and forming the train, and thirty-six weeks on the return trip. Towing the train back would be resisted by skin friction and the Coriolis force due to the Earth’s rotation. Upon reaching port, one week would be allowed for mooring the train in the harbor and performing maintenance on the tug before steaming south again.

Once delivered, the icebergs must be melted. This is best accomplished using waste heat from power plants. A 10,000 MW power plant would be located at Jeddah which would process 10% of the ice per week into fresh water which would be transported and stored in a nearby, relatively small land-based reservoir which has the advantages of not costing much to build, not taking much land, and not losing much water to evaporation. Because most of the water would be held in the continuing supply of icebergs in the “pipeline” from Antarctica, all of Saudi Arabia’s water requirements for its 7 million people would be met from a reservoir the size of those generally used for small cities where the source of the water supply is run off.

The causal diagram representation of this model appears in Fig. 5.

Commodity Supply and Demand Model (5)

In modern times the vulnerability of developing countries is due to a large extent to their dependence on a few basic commodities for earning foreign exchange. Here a commodity is taken as any raw material that is undifferentiable and is relatively price inelastic with respect to demand. It is well-known that millions of dollars have been spent by national governments and international agencies to control the fluctuations in commodity prices that drag countries through successive periods of depression, recovery, prosperity, and crisis, and that these attempts, for the most part, have been unsuccessful.
This model which is shown in Fig. 6 is based on the interactions among three market sectors: production, distribution and consumption. Price (per unit) links the three sectors. Producers try to adjust their production capacity to the profit maximizing level for a given market price. Distributors try to adjust the market price to maintain an optimal inventory. Consumers respond to the market price as they attempt to maximize their own utility. Structural additions, appropriate to certain commodities, include recycling and synthetic substitutes.

With minor changes, this model may be applied to many different resources and commodities. Try this by circling the phrase that completes the following statements correctly:

(1) Assume that the commodity described in the I, P, CR, PR supply and demand loops is a construction material like steel and that the substitute material SS is a high strength plastic. We see from the model that as P the price of steel increases, SSR the scrap salvage rate (increases) (decreases) (is unaffected) (may increase or decrease).

(2) Assume that the commodity described in the I, P, DR, PR supply and demand loops is an energy resource like petroleum and that the substitute resource is coal. As more coal is used, petroleum production capacity (increase) (decreases) (its unaffected) (may increase or decrease).

(3) Assume that the commodity modeled is livestock and that the substitute and scrap considerations are irrelevant. As the price of livestock goes up, it may be desirable from the farmers' point of view to hold the livestock off the market until they are older heavier. Show how this strategy could be incorporated into the model by making the appropriate additions to the causal diagram.

National Development Model
Contemporary problems are so complex and ensnarled that the situation has been referred to as a "problematique" -- a super problem. Modeling provides us with greater understanding, and greater understanding gives us greater power to intervene. Consider an economic development model corresponding to the following verbal description and the causal diagram in Fig. 7 showing the interrelationships among the important elements underlined.

Each year the nation's population (P) is increased by the births and decreased by the number of deaths that have occurred during the year. The birth rate (BR) is a function of population and average fertility (AF). The death rate (DR) is determined by the population divided by life expectancy (LE).

Industial Capital (IC) grows in much the same manner as population. A given amount of industrial capital, operating at
a certain efficiency of capital (EOC) will be able to produce a certain amount of industrial output (IO) each year. Some of that industrial output is in the form of factories and machines, which represent the industrial capital investment rate (ICIR) that adds to existing industrial capital. The ratio of this investment to the total industrial output is the fraction of industrial output to industrial capital (FOIC). Similarly, industrial capital is reduced by the industrial capital depreciation rate (ICDR) which depends on the amount of industrial capital and the useful life of industrial capital (ULIC).

The fraction of industrial output to industrial capital will depend on the fraction of industrial output allocated to agriculture capital (FOAC) and the fraction of industrial output allocated to service capital (FOSC). The amount of agriculture capital (AC) — tractors, irrigation facilities and farm inputs such as fertilizers and pesticides — and the arable land per capita (ALPC) determines the food per capita ratio (FPFR) — the amount of food per capita compared to the subsistence level food per capita. A low food per capita ratio increases the death rate. The amount of arable land available depends on the total land area (LA) and the amount of potential arable land consumed by infrastructure — infrastructure land per capita (ILPC).

Service capital per capita (SCPC) is allocated to health services per capita (HSPC) and educational services per capita (ESPC) based on the fraction of service capital to health (FSCH) and the fraction of service capital to education (FSCE). Accordingly, emphasis on health reduces deaths per year; emphasis on education reduces births per year. The crowding ratio (CR) which is the population divided by the land area (LA) compared to some normal population density (NPD), also affects the birth rate in an inverse manner.

Industrial output leads to the depletion of natural resources (NR) by increasing the natural resource usage rate (NRUR). The efficiency of capital drops as natural resources are depleted, and also depends on the initial supply of natural resources (NRI).

4. FEEDBACK SYSTEM ANALYSIS
Causal diagrams help to identify the feedback structure of systems as a prelude to analyzing the behavioral characteristics of the feedback systems. (6)

A feedback loop may be either positive or negative. In a positive feedback process, a variable continually feeds back upon itself to reinforce its own growth or collapse. Such
terms as "band-wagon effect", "vicious circles" and "virtuous circles" characterize positive feedback. Negative feedback is characterized by goal-directed or goal-oriented behavior. Such terms as self-regulating, homeostatic, and adaptive are synonyms for negative feedback.

When two or more feedback loops are interconnected, the variables in each loop can propagate through the connections to affect variables in other loops within the feedback system. Coupled feedback loops may contain a dominant positive loop, a dominant negative feedback loop, or shifts in dominance. Some of these cases will be illustrated.

Feedback Systems with Dominance

Consider the Industrial Capital subsystem of the National Development Model described in the causal diagram in Fig. 7. The system equations for this portion of the model are as follows:

- \( I.C = I.C.I \)
- \( F.O.I.C = 1 - F.O.I.C.A - F.O.I.C.S \)

- \( I.C = \text{INDUSTRIAL CAPITAL (S)} \)
- \( I.C.I = \text{INDUSTRIAL CAPITAL INITIAL (S)} \)
- \( I.C.R = \text{INDUSTRIAL CAPITAL INVESTMENT RATE (S/yr)} \)
- \( I.O = \text{INDUSTRIAL OUTPUT (S/yr)} \)
- \( F.O.I.C = \text{FRAC. OUTPUT TO INDUS. CAPITAL (DIMENSIONLESS)} \)
- \( F.O.I.C.S = \text{FRAC. OUTPUT TO SERVICE CAPITAL (DIMENSIONLESS)} \)
EOC  —  EFFICIENCY OF CAPITAL (1/YR)
ICDR — INDUSTRIAL CAPITAL DEPRECIATION
       RATE ($/YR)
ULIC — USEFUL LIFE OF INDUSTRIAL CAPITAL
       (YR)

The analytical solution is

$$IC = IC\times e^{(1 - FOAC - FOSC) (EOC) - (1/ULIC)}$$  \hspace{1cm} (1)

Referring to the exponent in Eqn. (1), three possibilities exist:

1. if \((1 - FOAC - FOSC) (EOC) > (1/ULIC)\), IC grows exponentially
2. if \((1 - FOAC - FOSC) (EOC) = (1/ULIC)\), IC = ICI
3. if \((1 - FOAC - FOSC) (EOC) > (1/ULIC)\), IC decays to zero

These three cases are shown in Fig. 8. In case 1, the country's industrial capital and hence its industrial output and gross national product GNP is growing exponentially. In case 2, there is no growth. In case 3, not enough is being reinvested to maintain existing industrial capital and the GNP is decaying exponentially. In case 1, the positive feedback loop dominates; in case 3, the negative feedback loop dominates; and in case 2, the two opposing forces are in equilibrium.

**Shifts in Dominance**

Consider the population subsystem of the National Development Model described in Fig. 7. The system equations for population controlled by crowding are as follows:

- \(P.K = P.J + (DT) (BR.JK - DR.JK)\)
- \(P = PI\)
- \(BR.KL = P.K \times AF/CR.K\)
- \(CR.K = P.K/(LA \times PDN)\)
- \(DR.KL = P.K/LE\)
As is invariably the case at the present time throughout the nations of the world, average fertility is greater than the reciprocal of life expectancy and so positive feedback dominates. However, population growth cannot continue indefinitely within a finite land area. Gradually crowding drives up prices, for example, making for smaller families. Eventually the number of births per year equals the number of deaths and population is in equilibrium. This form of dynamic behavior is referred to as S-shaped growth and it is depicted in Fig. 9. The steady state population \( P \) is obtained by equating \( BR \) to \( DR \) and is found to be

\[
P_e = AF \times LE \times LA \times PDN
\]

Second-Order Negative Feedback
A second-order system has two level variables. In the previous examples dealing with first-order negative feedback, the level is seen to have a time response that simply approaches its final value without overshoot or oscillation. Now, a second level variable will be introduced into the feedback loop to show how the mode of behavior can change to one of oscillation. These oscillations can be troublesome to countries trying to avoid economic cycles and to corporations trying to avoid business cycles.

Four general cases of second-order negative feedback systems with first-order loops are identified in Fig. 10. In Fig. 11 the system equations and response curves are given.

The merits of System Dynamics will be explored in a specific, detailed case study to be presented in Part III of this paper.

REFERENCES


