Calculating Total Capital Expenditures
On Existing Projects up to the Forecast Horizon*

By

Ali N. Moslehirazi
University of Bradford
System Dynamics Research Group

Abstract

One of the factors influencing decisions on any new Capital Expenditure to be incurred in the future is the availability of financial resources up to the Planning Horizon to finance such investment. This is determined after setting aside the resources needed for projects already commissioned but not fully paid for.

This paper presents a method of calculating expenditures on already commissioned projects up to the Planning Horizon through calculating coefficients representing the amount of progress that will be made on those projects. The method assumes progress stage payments which is customary both in industry and practical System Dynamics modelling.

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In preparing a sound Capital Expenditure Budget within a well developed strategic planning environment, it is often extremely useful to know just how much is to be spent on existing projects each of which may be at a different stage of progress and, of course, have different financial requirements. (1) The virtue of knowing this lies in the fact that given the estimates of the funds to become available and the financial requirements of those projects likely to be chosen over the forecast horizon, the investment situation (the Investment Gap) (2) becomes clear. This financial situation, once known by the management usually encourages more serious search for profitable uses of the company's financial resources and often leads to diversification.

In practice, there are a number of methods being employed in finding this gap, ranging from simply considering the cash situation at the present to choose or reject a project, to more sophisticated ones like taking into account the total capital expenditures committed and authorized on projects that will become completed up to the forecast horizon. These methods often fail to show the potential capability of a firm to invest and perhaps finance it successfully. The reason is that the planners usually regard the authorized projects and the ones the firm is likely to commit itself to in the near future as if they were going to be completed and totally paid for when the forecast horizon is reached. So they set aside financial resources for them, whereas this may not be the case.

The problem of defining the Investment Gap can be divided into three parts.

(a) Estimating total funds that will be available for investment up to the forecast horizon.

(b) Calculating total expenditures up to the forecast horizon incurred on capital projects already authorized.

(c) Estimating total expenditure up to the forecast horizon on projects likely to be authorized between the current time and the forecast horizon.

This paper presents a method for modelling part (b) of this problem. Such a method needs to take into account that most capital expenditure projects involve stage payments so that the total cost of the project is spread over its construction period. For instance, many chemical projects are divided into three stages with roughly 33% of the project cost being paid at the beginning of each stage. Furthermore, at any particular time a large company will have a large number of capital projects at various stages of completion some of which will be completed by the forecast horizon whereas others will still not be complete when the forecast horizon is reached.
The payments made for new additions to existing capacity depends on the nature of capital expenditure projects. A company may purchase an already existing firm where the total amount or a large part of it is paid at the time of purchase. Or it may undertake a project which requires contracting the whole work to outside contractors and payments are made in accordance with its progress. The company may also start another project which most of the development is made by the company itself and only a few equipment and machines have to be purchased. These different possibilities and many more, require different types of payments spread over the project's completion time. Four basic cases are illustrated in Fig. 1. Whichever of these possibilities or a combination of them be the case, the actual payment is not continuous as it might look, but is made in different time intervals, perhaps, geared with the progress stages of the project.

Having said that, for all practical purposes, one can assume that many projects go through three stages of completion represented by X, Y and Z in Fig. 2. below:

![Diagram](image)

where:  
\[ \text{CASR} = \frac{(\ell/m)}{m} \text{ capacity start rate} \]  
\[ \text{ACASR} = \frac{(\ell/m)}{m} \text{ average capacity start rate} \]  
\[ \text{CACR} = \frac{(\ell/m)}{m} \text{ capacity completion rate} \]  
\[ \text{ACACR} = \frac{(\ell/m)}{m} \text{ average capacity completion rate} \]

And, if the total completion time is CIDEL months, then the duration of each stage is \( \tau = \text{CIDEL}/3 \) months.

It is clear that the above arrangement actually describes the internal structure of a third order delay function in DYSMAP which is linear. However, it is possible to assign different time durations for each stage if there is a need to do so. For the first of the possibilities we have:

\[ \text{CACR}_{KL} = C = \text{DELAY3(CASR}_{JK}, \text{CIDEL)} \]

(where we ignore the smoothing stages at the beginning and end of the above structure for a moment)
which is equivalent to the differential equations:

\[
\begin{align*}
\dot{X} &= \frac{S - X}{t} \\
\dot{Y} &= \frac{X - Y}{t} \\
\dot{Z} &= \frac{Y - Z}{t} \\
C &= \frac{Z}{t}
\end{align*}
\]

where \( S = \text{CASR} \).

Now, at time \( t \), if no further orders are placed, then the total capital expenditures on projects started before \( t \), to Forecast Horizon, \( \text{FHOR} \), will be given by:

\[
\text{IF} = \int_t^{t+\text{FHOR}} I'(t')dt'
\]

the integration of all expenditures spent on projects at different stages up to \( t+\text{FHOR} \).

Clearly, depending on the relative lengths of \( \text{CID} \) and \( \text{FHOR} \), at time \( t+\text{FHOR} \), there may remain unfinished projects in the stages.

There are several ways to calculate the amount spent to \( \text{FHOR} \), e.g. solving the integral or differential equations which follow from the foregoing logic analytically. A neater way – having a DYSMAP Compiler available – perhaps is to write a small DYSMAP program to do the job. This is done by assuming that at time \( t \) there is only £1.00 worth of projects in stage \( X \) and nothing else in the rest of the system. Then we let this work its way through the system over \( \text{FHOR} \); the \text{LENGTH} of the program. At the end, one can see what proportion of this unit has passed through all other stages and from preceding stages to the others. These proportions then can be treated as coefficients for corresponding stages; \( \text{Bi}, \text{Ci}, \) and \( \text{Di} \) as shown on the above figure. For coefficients \( \text{Al} \) and \( \text{El} \) corresponding to smoothing stages with equal averaging time, \( \text{PLAT} \), we have:

\[
\text{Al} = \text{El} + (1 - \exp(-\text{FHOR}/\text{PLAT}))
\]

where: \( \text{PLAT} = (m) \) planning and review averaging time.
The DYSMAP Program for calculating the other coefficients is attached to this paper. The first three level equations carry the one unit investment at the beginning through the system. The level equation for Bl is to calculate the proportion of the unit at X, gone through Y and Z. For proportion that initialized at Y and should go through Z, the program takes it as it was initialized at X and gone through Y and then came out there instead. This trick prevents writing out a longer program and is correct since \( \tau \) is constant in this case. This is done by the level equation for Cl. The same applies to the level equation for Dl.

The program also contains a few RERUNS of some possible combinations of CIDEL and FHOR. The LENGTH represents the Forecast Horizon and from the output prints the final values of coefficients Bl, Cl and Dl can be read off.

For example:

(1) If:

\[
\begin{align*}
\text{FHOR} & = 60 \text{ months} \\
\text{CIDEL} & = 36 \text{ months} \\
\text{PLAT} & = 3 \text{ months}
\end{align*}
\]

then the coefficients are:

\[
\begin{align*}
A1 = E1 = (1 - \exp(-60/3)) & \approx 1.0 \\
Bl & = 0.85201 \\
Cl & = 0.94974 \\
Dl & = 0.99113
\end{align*}
\]

(2)

\[
\begin{align*}
\text{FHOR} & = 60 \text{ months} \\
\text{CIDEL} & = 60 \text{ months} \\
\text{PLAT} & = 3 \text{ months}
\end{align*}
\]

then the coefficients A1 and E1 are the same as for Example (1) and:

\[
\begin{align*}
Bl & = 0.57716 \\
Cl & = 0.80132 \\
Dl & = 0.95045
\end{align*}
\]

Other combinations of CIDEL and FHOR are readily evaluated.
One can make a series of tables of these coefficients by RERUNS of a range of
CIDE1 with a reasonable FHOR. Since the LENGTH of the program is FHOR, the entries
to those tables are actually the printed values of the coefficients against the
column headed TIME in the print-out table of the program with an accuracy of ± 0.001.

There are a few other points to mention:
First, from the second example, we have the value of B1 = 0.57716. This suggests that
from a unit pulse to a DELAY 3 with a Delay Time of 60 months, only 57% of that unit
has actually come out at the end of that period. The remaining 43% is still in the
DELAY. This dynamic characteristic of the DELAY3 Macro should be kept in mind by a
modeller using this function in his model. For instance, if the actual Average Delay
Time in the industry is 60 months, and the modeller puts it as 36 months in his model,
the first example shows that only about 85% will come out of the delay after 60 months.
Secondly, these coefficients can be evaluated using Laplace Transform Techniques as
indeed very much the same way we evaluated A1 and E1.

However, these coefficients are obtained, they can then be used to calculate the
Total Investment to Forecast Horizon; IF. Assuming payments are made in proportions
A, B and C of the total capital cost corresponding to stages X, Y and Z, where:

\[ A + B + C = 1 \]

We have:

\[ A \text{ IF.} K = B1 \times X.K \times (A + B + C) + C1 \times Y.K \times (B + C) + D1 \times Z.K \times C \]

or simply:

\[ A \text{ IF.} K = B1 \times X.K + C1 \times Y.K \times (1 - A) + D1 \times Z.K \times C \]

thus expenditure on existing projects from the current time to the forecast horizon is
easily computed from the values of the projects in each of the stages.

Finally it is worth noting that though the method outlined deals with projects
that can be divided into stages it can be readily extended to projects composed of any
number of stages, each of arbitrary length provided the stage lengths are constant.
DYSMAP PROGRAM

0 DIM
1 * COEFFICIENTS FOR INVESTMENT T U FORECAST HORIZON
2 L X,K=X,J+(DT/TAU)*(-X,J)
3 N X=0
4 N TAU=C1DEL/3
5 C C1DEL=36
6 L Y,K=Y,J+(DT/TAU)*(X,J=Y,J)
7 N Y=0
8 L Z,K=Z,J+(DT/TAU)*(Y,J=Z,J)
9 N Z=0
10 L B1,K=B1,J+(DT/TAU)=Z,J
11 N B1=0
12 L C1,K=C1,J+(DT/TAU)=Y,J
13 N C1=0
14 L D1,K=D1,J+(DT/TAU)=X,J
15 N D1=0
16 C LENGTH=60
17 C DT=0.0625
18 C PLTPEM=0.25
19 C PLOPER=0.25
20 PRINT B1,C1,D1
21 PLOT B1=C,D1=0
22 RUN 60 MONTH FHOR
23 C LENGTH=48
24 RUN 48 MONTH FHOR
25 C C1DEL=48
26 C LENGTH=60
27 RUN C1DEL=48 II, FHOR=60
28 C C1DEL=60
29 RUN C1DEL=60 II, FHOR=60
30 D B1=(E/H) COEFFICIENT CORRESPONDING STAGE X
31 D C1=(E/H) " " " STAGE Y
32 D C1DEL=(H) PROJECT COMPLETION TOTAL TIME
33 D D1=(E/H) COEFFICIENT CORRESPONDING STAGE Z
34 D DT=(H) SOLUTION INTERVAL
35 D LENGTH=(H) FORECAST HORIZON
36 D TAU=(H) TIME DURATION EACH STAGE
37 D TIME=(H) MONTH SIMULATION TIME
38 D X=(E/H) 1ST STAGE CAPACITY IN PROGRESS
39 D Y=(E/H) 2ND STAGE CAPACITY IN PROGRESS
40 D Z=(E/H) 3RD STAGE CAPACITY IN PROGRESS
41 +
42 ****
Notes and References

For a full treatment on Capital Budgeting see:

(1) Weston, J. F. & Brigham, E. F., Managerial Finance

(2) Malmlow, E. G. "A Systematic Approach to Diversification"

(3) For more details on Dynamic Characteristics of Delay see:

    a) Forrester, Jay W. Industrial Dynamics, the MIT Press 1961, Chapter 9.
    b) Coyle, R. G. Management System Dynamics