What Counts As An Explanation For System Behavior? A Brief History of SD from a Dominance Perspective

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Abstract

A fundamental pursuit in system dynamics is explaining how structure drives behavior. A diverse set of practices and tools have emerged for identifying which parts of structure *dominate* behavior, and when. Over time, various definitions and criteria have been offered for *dominance*, however today there still lacks a single formal and rigorous definition agreed upon by the field. The aim of this paper is to conduct a systematic review of SD literature from the perspective of "What constitutes an explanation of behavior?"

The review is organized around four time periods, from the origins of the field to current day, tracing the evolution of definitions for structure, behavior, and dominance. The review begins with examining how Jay Forrester and his contemporaries provided explanations of behavior, then examines how methods of explanation evolved over time, making particular note of the context, definitions, and descriptive phrases surrounding the term *dominance*.

The review identified 18 distinct definitions of the term *dominance*, many of which lack formal criteria. Of the 44 articles reviewed which are focused on dominance analysis methods, only 16 offer definitions for the term. Methods also vary in their definitions of *structure* and *behavior*.

The review identifies several research gaps in dominance analysis which could benefit from a formal, rigorous definition. The insights and conclusions from the review serve as a foundation for arriving at a rigorous and formal definition of *dominance* as it is pertains to structure-behavior relationships in dynamic systems.

1 Introduction

A central premise in System Dynamics is that the behavior of a system arises from its own structure, where structure consists of the nonlinear interactions between variables and feedback loops [89, p. 107]. This premise is also referred to as the *endogenous perspective*, in which explanations for behavior look within the system [80].

SD practitioners formulate a dynamic hypothesis (expressed formally as a dynamic model) about how causal mechanisms lead to observed behavior over time [89, pp. 94-95]. Questions then aim at uncovering and explaining the endogenous sources of behavior, such as, "What are the causal structures in the system that are primarily responsible for generating the observed behavior? Under what conditions are these causal structures influential?" Such questions aim to identify potential places to intervene in the system, or leverage points, in which small changes can have significant impact and where policies can be designed to achieve an objective. Since models may have hundreds or thousands of feedback loops, answering this question is non-trivial, especially considering that in practice, typically only a few loops have significant influence on the behavior at any given time [38]. Methods have been developed within the field of system dynamics to help address questions such as, "What is the dominant feedback loop?" According to some, "to identify these (dominant) loops...is to identify the fundamental causes of the system's behavior."[37].

Early contributions in the field of system dynamics by Michael Goodman [35] and Alan Graham [38] applied basic principles from linear systems theory and feedback control to make general statements about the relationship between structure and behavior in certain classes of oscillatory nonlinear systems. However, the extent to which these principles could be applied to most social systems of interest to SD practitioners was limited due to their dynamic complexity [54][1]. Nathan Forrester applied techniques from engineering to linearize a system around an operating point and applied methods from linear systems theory [28]. Several other methods have been developed based on these analytical foundations and fall in the category of *dominance analysis* [8]. Today, however, analysis is still mostly performed experimentally where models are investigated through the removal or deactivation of partial structures and simulated to examine the impact, or through sensitivity analysis in which parameters are varied and results observed. The effectiveness of these methods varies depending on the size or dimension of the model and the nature of nonlinear relationships.

Dominance analysis became a focused area of research shortly after George Richardson proposed the research problem of detecting dominant structure in nonlinear systems [78]. In the years to follow, new methods had emerged for detecting *dominant* parts of system structure. Kampmann and Oliva summarized many of these methods and also described the *structure-behavior problem* as a fundamental pursuit in the field of system dynamics [54], but the authors also raised the more fundamental question of what constitutes an explanation of the link between structure and behavior. Since then, while many methods have been developed in an attempt to identify dominant structure [8], there has been a lack of debate regarding what should actually count as an explanation for structure-behavior relationships. But the main questioned posed here is, what is the importance of this anyway to everyday SD practitioners and researchers?

Today, there still lacks a formal, rigorous definition of and criteria for *dominance* that is universally accepted within the field. Researchers have used the term *dominance* to represent different concepts, from *most influential* to *solely determines the behavior*. If, as SD practitioners, we seek to improve the scientific rigor of our work and grow its influence in other domains, we would benefit from greater clarity on what constitutes an acceptable explanation for structure-behavior relationships, given that this is often central to our work. How we use terms such as *dominance* matters when it comes to making testable and refutable claims, as well as the repeatability of our simulation studies and analysis.

This review examines, within the field of SD, what has constituted an explanation for

[1]Dynamic complexity is a result of the types of nonlinear interactions between the state variables and feedback loops and the number of state variables or stocks in the systems.

how structure determines behavior, towards the goal of laying a foundation for future work in defining *dominance*.

2 Literature Review Method

The search set includes all literature commonly accepted as belonging to the field of SD, including the primary peer-reviewed journal *System Dynamics Review*, proceedings from the annual International System Dynamics Conference, and the MIT SD literature collection (including over 5,000 memos, course material, research proposals, masters and PhD theses, historical documents, and miscellaneous publications). The search set also included articles in other journals in which SD was used as a primary method. Finally, the review includes classic and modern texts in SD.

Within the search set, the primary criteria for inclusion was the appearance of one or more of the following words: *dominant*, *dominance*, *dominate*, *dominates*, *dominated*, and *dominating*. Earlier literature also revealed that the words *predominant* and *predominate* were used in a similar fashion to *dominant* and *dominate*, therefore the various forms of these words were also included.

All literature considered foundational to the field (from 1956 to 1963) were included and reviewed, regardless of the presence of the term *dominant*, in order to observe the components of explanations for behavior and the words and methods used to describe structure-behavior relationships.

Additional criteria for the inclusion set:

- Excluded instances in which the term *dominant* (and variations) were not used to explain behavior or structure-behavior relationships. Specifically, if the term did not refer to model structure, model behavior, or the relationship between structure and behavior, it was not included. For example, the following search result was excluded from the review: "The so-called Phillips Curve has dominated much of the debate about inflation." (D-3606).

- Excluded instances in which the word *dominant* was used without any context from which its meaning could be inferred.

- The review noted cases in which authors cited and re-used previous uses and definitions of the term *dominant*, however not every re-use or repetition was included, especially when it did not offer any new context or information about its meaning.

3 Limitations

There are several limitations of the systematic review. First, while attempts were made to include all literature in the field of SD addressing the concept of dominance, the search may have missed some literature which discussed the concept of dominance without using the word or any of its derivatives. The greatest risk of missed literature occurs during the foundational time period (pre-1964), prior to when the term *dominance* was in wide-spread use. Attempts were made to mitigate this risk by including nearly every literature found during this time period, regardless of whether the word *dominant* or its derivatives appeared. This strategy was effective in identifying early thoughts on structure-behavior explanations. Post 1964, however, the volume of SD literature grew rapidly and therefore it was no longer practical to include every piece of literature. Instead, the keyword search criteria was expanded to include *cause*, *explain*, *explanation*, and *influence*, however this also greatly expanded the inclusion set to the point where the majority of search hits were not relevant to dominance, and so many were excluded. It is possible some of these exclusions may have resulted in missing important literature which, while not using the term *dominance*, would have added value to understanding the nature of explanations.

Second, there is a degree of subjectivity in judging whether or not each use of the term *dominance* applies to behavior-structure relationships.

Third, there may have been misses in the older literature due to quality of scans and text recognition software. For example, at least one instance of this was caught in *Dynamics of Product Growth in a Competitive Market*, Miller, Jr., where the term *dominate* was missed by text recognition software, but discovered through manual inspection.

With the above limitations in mind, the review is considered to be both systematic and exhaustive in that for the foundation documents, nearly all literature was manually inspected for relevance. Furthermore, the search criteria included all possible derivations of the word *dominance*. Precise documentation was kept for all literature included in the review, including the reason for inclusion. The search included textbooks, educational and training literature, journal articles, conference proceedings, memos, research proposals, and masters and PhD theses.

4 Organization of Results

From the review, four time periods emerged which provide a helpful way to understand the development of *dominance analysis*.

- Period 1: Foundational system dynamics literature (1956-1963)
- Period 2: Early uses of the term *dominance* (1964-74)
- Period 3: Analytical foundations for *dominance analysis* (1975-1986)
- Period 4: The development of dominance analysis methods (1989 to present)

The following sections present the findings of the systematic review, including the trends and evolution of the use, definitions, and methods of *dominance* which are observed across the four time periods.

5 Period 1: Foundational System Dynamics Literature (1956-1963)

The foundation literature begins with Jay Forrester's letter to the MIT faculty research seminar in 1956 which proposed a new approach to modeling industrial systems that later become known as *industrial dynamics* [15], and more generally, *system dynamics*. The inaugural publication of the field came two years later in the *Harvard Business Review* [16] and the first text, *Industrial Dynamics*, was introduced three years after [17]. This period also includes the first documents, called *D-memos*, from 1958-1963 in which the first instances of *predominance* and *dominance* appear in SD literature.

In his earliest writings on system dynamics, Forrester does not use the word *dominant* (or any of its forms) [15, 16, 17][2]. However, system behavior is inextricably linked to structure since the beginning of the field. The first D-memo traces behavior to accumulations, delays and decision making criteria [15].

In *Industrial Dynamics* [17, p. 66], several statements refer to structure-behavior relationships and the role of nonlinearity in determining behavior.

> Our social systems are highly nonlinear. It seems likely that such nonlinearities, coupled with the unstable tendencies caused by amplifications and time delays, create the characteristic modes of behavior...

In the written history of SD, the term *dominance* is actually first used by Forrester's contemporaries[3]. The term is used to describe behavior characteristics, and specifically, the amplification or dampening of oscillations [50], the periods in oscillatory systems [4, 99], and frequencies or modes of behavior [12]. For example:

> ...the second peak in the orders to the manufacturing sector is thirty-eight weeks removed from the initial surge, after the shut-down. This may be a reflection of the natural period, but it is not a dominant characteristic in this case [4, p. 36].

The systems being described were typical of those studied by Forrester in *Industrial Dynamics*, characterized by steady-state oscillations. This use of the term *dominant* is also consistent with how feedback control engineers describe dominant frequencies and dominant modes in classical control theory [70, p. 304]. This language, therefore, is not surprising given Forrester and his colleagues' background in servomechanisms. The term *dominant* or *dominate* is also used to describe structural elements, such as system parameters [7, p. 2].

Interestingly, the term *feedback loop dominance*, of common use in the field today, is never used to describe the industrial systems exhibiting oscillatory modes which were studied in the earliest literature. Rather, the term is first used to describe growth processes which exhibit transient modes, as in Nord and Swanson, 1962, in "Growth of a New Product" [67, p. 37]

> Because the forecast uses information which is an integral part of the feedback loop, the loop itself dominates the forecast.

Then, in his thesis "Dynamics of Product Growth in a Competitive Market," Miller describes the activity of examining feedback loops with respect to explaining behavior [61, p. 7].

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[2]In *Industrial Dynamics*, Forrester uses the word *predominate* several times, but without definition.

[3]The terms *predominance*, *predominate*, *predominant* have definitions similar to that of *dominance*, *dominate*, *dominant* and are used in a similar fashion, and therefore usage of these words are included in the literature review.

> This will be done first by tracing through a time history which might be typical of many products, and later by examining the most important feedback loops in the system to determine how the characteristics of these loops influence the behavior.

These are the earliest references for what today is termed *feedback loop dominance*. In the following years, the term *dominant* is used with significantly increased frequency. Also, during the next period the concept of *shifts in dominance* is first introduced.

6 Period 2: Early Uses of the Term *Dominance* (1964-74)

The next period examines how the term *dominance* is used as the field of SD begins to take shape. The review examines the main contributions of Forrester and his contemporaries such as Carl Swanson, Dennis Meadows, Donella Meadows, and Michael Goodman, and how the term *dominant* takes form. New phrases emerge, such as *shift in dominance*. This period includes classic publications of the field such as *Market Growth* [22], *Principles of Systems* [23], *Urban Dynamics* [24], *The Limits to Growth* [60], and *Study Notes in System Dynamics* [35]. During this time, while many descriptive phrases and synonyms are used for *dominance* and *shifts in dominance*, a formal definition has yet to be offered.

One of the first uses of the word *dominance* by Jay Forrester is to describe growth processes and also indicates a distinction between engineering and social systems [18].

> Most of the literature on feedback systems deals with the negative feedback loop. Negative feedback is the form normally encountered in the control of physical systems. Yet, positive feedback dominates in the growth and decline patterns of social systems.

Forrester also notes the transition from growth processes of positive loops to stagnation of negative loops, the first indication of what is now referred to as *shifts in loop dominance* [18]. Then, Swanson discuss how shifts in dominance occur [92].

> These loops that dominate the behavior of a variable shift and usually produce different characteristic behavior due to the shift. The mechanisms that shift the dominance are the nonlinearities in the system which change the gain and delay of feedback loops.

Meanwhile, the phrases *feedback loop dominance*, *loop dominance*, and *shift in dominance* do not appear in key word searches outside of SD, during any time period. It appears the concept originates entirely from within SD, and furthermore, it does not appear until the study of growth processes involving the coupling of reinforcing (positive) feedback with balancing (negative) feedback which limits growth[4]. Another researcher observes, "As positive feedback is not of interest for engineering design purposes there is little if any literature with which to compare this approach" [71].

[4] This is consistent with recent conversations with George Richardson (2016) on the history of feedback loop dominance, who suspects that the concept originated from within the field.

Post-1964, references to *loop dominance* and *shifts in dominance* become prolific in the field. Many references to *shifts in dominance* refer specifically to the shift that occurs in growth processes, as mentioned above, in which there is a shift from a reinforcing to balancing loop [91, 19, 3, 20, 21, 46, 34, 35]. In the vast majority of references to *shifts in loop dominance* during this period, it is the nonlinear relationships which are implicated as the cause of the shifts [19, 92, 93, 100, 20, 21, 34, 35]. In fact, even before the phrase *shifts in dominance* was in use, nonlinearities were implicated as the source for interesting and counter-intuitive behavior of systems, tracing back to the first D-memo by Forrester [15] and the text *Industrial Dynamics* [17][5].

As the term *dominance* gained traction, assertions and general principles about *dominance* were proposed based on experience and anecdotal evidence, but without formal proof or rigorous definition. For example:

> Even in a complex system only one or a few loops dominate the behavior of a variable of interest over an interval of time... These loops that dominate the behavior of a variable shift and usually produce different characteristic behavior due to the shift [93].

The assertion is also made that linear systems cannot shift in loop dominance and that only nonlinear systems are capable of shifts in dominance [34, 35].

Michael Goodman is the first to offer precise statements about how shifts in loop dominance result in S-shape growth [33, p. 3].

> Previous knowledge of positive and negative feedback would indicate that the S-shape phenomenon is a two-stage process beginning with positive feedback and after some time becoming dominated by negative feedback.

Goodman also makes several generalizations about system structure and behavior while employing the term *dominance* in his masters thesis "Elementary SD Structures," directly linking dominance to sigmoid growth which produces an S-shape behavior [34].

In *Study Notes in System Dynamics* [35], Goodman introduces generic feedback structures such as positive and negative feedback loops, relating them to behavior modes such as exponential growth, goal-seeking behavior, and S-shape growth. He also mentions *shifts in loop dominance*, most often in the context of shifts between positive and negative loops and uses this notion as a key concept for explaining how S-shape growth occurs.

While *dominance* is not defined by any of the authors, many phrases are used to describe the term, listed alphabetically in Table 1.

Some phrases indicate a relative or subjective measure of influence such as *affects*, *encouraging*, *important*, *influence*, *powerful*, and *role in determining*. For example:

> As the periods increase, the third order delay plays less and less of a dominant role in determining the behavior of the system [5].

While other phrases indicate an objective causal relationship that is linked to a direct behavioral outcome, such as *cause*, *determine*, *governs*, *necessary to produce*, and *responsible for*. For example:

[5] Email exchanges with George Richardson (2016) on the history of feedback loop dominance also indicate a strong, historical association between nonlinearity and shifts in loop dominance.

achieving	influence
affect	leading to
cause to occur	most important factor
causing	necessary to produce
control	overtake
determinant of	powerful
determine	predominate
driven by	primary determinant
encouraging	produce
governs	responsible for
important	role in determining

Table 1: Words and phrases used to describe *dominate* or *dominant* in literature from 1964-74.

> Loop 3, an additional negative feedback loop based on Calhoun's observations, must then be responsible for transferring dominance from Loop 1 to Loop 2, necessary to produce equilibrium [34].

Lastly, other phrases are used in both a relative and objective sense. For example, *determine* vs. *role in determining.* The former seems to be an objective statement of causality between structure and behavior, whereas the latter indicates a relative relationship.

During this time period, the word *dominant* is not only used to describe behavior patterns, feedback loops, and parameters (as in the previous period), but it is now expanded to describe a wider variety of concepts such as dominant pressures, forces, time constants, variables, nonlinear interactions, subsystems, causal mechanisms, inputs, and flows. The word is used to identify whatever aspect of a system is considered important or critical in affecting behavior, but it is not accompanied by formal criteria.

The next time period sees the first proposed definition for *dominance* and the development of analytical and experimental methods for identifying *dominant* structure. This period establishes the mathematical foundations of future work, culminating with the emergence of *dominance analysis* as a sub-discipline within SD.

7 Period 3: Analytical Foundations for *Dominance Analysis* (1975-1986)

This time period includes the work of Alan Graham, Nathan Forrester, George Richardson, Alexander Pugh III, Robert Eberlein, and many others who laid the analytical foundation for future model analysis research. The term *dominance* had been used for nearly twenty years without formal definition, which was offered for the first time by Alan Graham in 1977 [38].

> Here, a loop dominates the behavior in the sense that if the loop is disconnected or substantially altered, the behavior mode also changes substantially.

Graham's definition influenced future work by Ford, who used a similar definition and test of *dominance*, and Rahmandad, who stressed the importance of counterfactual testing. In Graham's definition, the counterfactual is the objective criteria for dominance in which the *feedback loop* is the structural unit of interest, and behavior is defined by *behavior modes*. Other definitions of dominance during this time period focus on causal links as well as feedback loops as the important elements of structure [26]. Other definitions also appeal to different quantifications of behavior, such as eigenvalues [26], while other definitions do not precisely define behavior [25].

In contrast to Graham's definition for *dominant*, an alternate one using a more relative criteria, is:

> ...In a feedback structure, a loop that is primarily responsible for model behavior over some time interval is known as a dominant loop [81, p. 285].

Shortly after, Richardson formalizes what is meant by behavior and introduces the concept of *dominant polarity* for first order systems ($\dot{x} = f(x)$) and used the concept to classify behavior as either goal-seeking/convergent if $dominant\,polarity = -1$, or goal-divergent if $dominant\,polarity = +1$, where:

$$dominant\,polarity = \operatorname{sgn} \frac{\partial \dot{x}}{\partial x} \tag{1}$$

Furthermore, he uses the concept of *dominant polarity* to define *shifts in loop dominance*:

> ...a shift in loop dominance is said to occur if and when $\partial \dot{x}/\partial x$ changes sign, that is, when the dominant polarity of the system changes [79].

However, Richardson acknowledged that not all shifts in dominance may be associated with changes in dominant polarity, and many of the definitions and results he proposed apply only to first-order systems.

During this time period, many of the previous words are still used to describe dominance, however new phrases are also introduced (Table 2). Some phrases indicate a subjective or relative sense of dominance: *accounts for the majority of*, *active*, *come into operation*, *exerting greater pressure*, *high elasticity*, and *significant*. Other phrases indicate an objective sense of dominance: *accounts for*, *essential to behavior*, and *generates*. Some words are used in both a subjective and objective manner, for example: *accounts for* and *accounts for the majority of*.

During this time period, the term *dominance* is also applied to several new structures such as dominant links, effects, factors, components, mechanisms, and constraints. It is also applied to a variety of behavioral concepts such as dominant behavior modes, patterns, trends, and eigenvalues. This usage seemed to both expand and confuse what was meant by the term dominant. The phrase *shifts in dominance* was used by an increasing number of authors to explain sources of system behavior, often without formal explanation or definition. For example:

accounts for	essential to behavior	overpower
accounts for the majority of	exerting greater pressure	overtake
active	exerts increasing control	powerful
becomes active	fundamental cause of behavior	predominate
cause	generates	primarily responsible
cause the behavior	greatest effects on behavior	principally responsible
come into operation	high elasticity	produce
control	important	produce behavior
critical in producing	influential	relative importance
determine behavior	largely responsible for	significant
domination	lead to	significant influence
drives	most important	takes full control

Table 2: Words and phrases used to describe *dominate* or *dominant* in literature from 1975-1986.

> A nonlinear relationship causes the feedback loop of which it is a part to vary in strength, depending on the state of the system. Linked nonlinear feedback loops thus form patterns of shifting loop dominance- under some conditions one part of the system is very active, and under other conditions another set of relationships takes control and shifts the entire system behavior. A model composed of several feedback loops linked nonlinearly can produce a wide variety of complex behavior patterns [77, p. 33].

In this text, the words *strength*, *active*, *takes control* indicate something occurring mathematically but are not formally defined.

By this point, the sigmoid growth process, or logistics growth (Verhulst equation 2 [97]) emerged as the canonical example of *shifts in loop dominance* [34, 35, 57, 96, 82, 79, 25]. Logistic growth corresponds to the simplest of growth processes consisting of one reinforcing and one balancing loop, and nearly all dominance claims, examples, and methods were illustrated using this model.

$$\frac{dP}{dt} = aP - bP^2$$

$$\begin{aligned} \text{where :} & \\ a &\geq 0 \\ b &\geq 0 \\ P(t) &= \text{population at time } t \end{aligned} \tag{2}$$

This trend continued into the next time period as well, especially with the development of SD training and instructional material [56, 98, 30, 103, 59, 2].

Methodological developments during this time period were derived primarily from linear feedback control theory. In his thesis, Alan Graham offered the first focused attempt to generalize principles for inferring relationships between feedback loops and oscillatory behavior, drawing insights from classical control on how signals propagate through a system [38]. Similarly, Nathan Forrester conducted research on the sensitivity and elasticity of the eigenvalues of a linearized system, laying the foundation for future eigenvalue and eigenvector analysis in the field [26, 27]. Nathan Forrester's work was based on prior eigenvalue sensitivity research in modal control theory with applications to power systems by Perez-Arriaga in 1981 [72], tracing back to methods developed by Porter and Crossley in 1972 [75, p. 53]. Also motivated by linear analysis, Robert Eberlein researched how to simplify and reduce linear dynamic models by retaining specific behavior modes using selective modal analysis, drawing from dominant mode techniques in engineering feedback control [10]. This laid the foundation for future work on model simplification techniques for the purposes of identifying the minimum structure needed to explain behavior.

Richardson and Pugh were the first to describe an analytical procedure specifically for nonlinear systems that did not leverage linear systems theory [81, pp. 268-272]. They describe the *loop knock-out* experimental procedure in which a feedback loop is deactivated (for example, by making the associated time constant sufficiently large) and the results observed through simulation. If the simulated results are significantly different, the feedback loop may be dominant.

With all the methodological advances and the broad use of the term dominance, Richardson observed that the field was in need of more rigorous definitions. He helped establish the line of research called *dominance analysis* which emerged as one of the field's top priorities [78]. In establishing *dominance analysis* as a research focus, Richardson states:

> There is a conspicuous gap in our literature between intuitive statements about loop dominance and precise statements about how to define and detect dominant structure.

This time period concludes with Richardson proposing specific research questions aimed at advancing the field of dominance analysis. Among his questions are how to precisely define *dominance*, *structure*, and *behavior*.

> To move beyond intuition, two questions need to be answered: What do we mean by a particular pattern of behavior? and What do we mean by principally responsible? Precise answers would raise exciting possibilities. If we can define these terms formally and unambiguously, we might then be able to devise means of detecting, rapidly and with certainty, the dominant structures underlying the patterns of behavior exhibited by a model [78].

With dominance analysis identified as an important research topic for the next decade, attention now shifts to literature in which specific methods are developed and matured. The literature review no longer includes every instance of the term *dominant*, which by this point seems to have reached widespread use, but rather focuses on the *dominance analysis* literature which aims to develop methods for identifying dominant structures.

8 Period 4: Dominance Analysis Literature (1989 to Present)

Following Richardson's challenge, many new methods were developed to detect *dominance*, building upon the foundational research from the 70s and 80s, but each offering their own interpretation of *dominance*. At least 14 new definitions for dominance were proposed, making a total of 18 documented definitions (see Appendix A for a list of all definitions of dominance from the literature review). Although, of the 44 articles on dominance methodology during this time period, only a third explicitly define the term. Some new descriptive phrases for dominance also emerged. New phrases which describe *dominance* in a relative/subjective manner were used by researchers who proposed methods using normalized metrics to determine relative dominance. These terms include: *contribute most to*, *contribute positively in the same direction*, *contribute significantly to*, *having larger magnitudes*, and *mainly contributes*. Other new terms which indicate relative influence include: *largest gain*, *mainly influences*, *mainly responsible for*, *more important*, *outweighs*, and *stronger*. New phrases describing dominance in an objective manner also emerged such as *explains* and *power of changing*, indicating that the structure generates or determines the behavior.

As more models were evaluated, new phenomena were also observed which also led to new terms. For example, *shadow loop dominance* was introduced to describe situations in which two or more loops are required to be deactivated in order to change behavior [14]. The term *multiple loop dominance* was used to describe situations in which there exists more than one loop which independently affects behavior. A common theme throughout this period, however, is the lack of reference to a single formal definition or criteria for dominance. Each method introduces its own criteria, and new seemingly scientific terms are introduced but without formal and rigorous definition.

8.1 Dominance Analysis Methods

Dominance analysis methods can be categorized as either *exploratory methods* or *formal methods*, as recently summarized by Duggan and Oliva [8]. A similar dichotomy was offered earlier by Ford, who distinguished between *behavioral* and *structural* methods [14]. This paper refers to *exploratory/behavioral* methods as those which use changes in simulated model behavioral as the criteria for dominance. The term *exploratory* refers to the exploration of model behavior by changing different elements of model structure and then simulating. In contrast, *formal/structural* methods refer to those which analyze the structure (equations) of the model mathematically.

8.1.1 Exploratory/Behavioral Methods

With exploratory or behavioral methods, a structural element is dominant if it determines behavior, which is tested by deactivating the structure and examining how the system is affected. This is a counterfactual method of explanation, and the strength of this approach is the explicit connection between structure and behavior.

Ford's behavioral analysis method. Researchers discovered challenges applying concepts such as *loop polarity* and *dominant polarity* to develop intuition for large and complex models [66], and there was a clear need to establish methods which would scale. Ford introduced a behavioral approach to feedback loop dominance which formalized a routine for performing *loop knock-outs*, as described earlier by Richardson and Pugh. That is, systematically testing (through simulation) the deactivation of each loop in order to evaluate its impact on behavior [14, 74]. In doing so, Ford extended the concept of *dominant polarity* and defined three *atomic behavior patterns (ABP)* to classify behavior as linear ($ABP = 0$), logarithmic ($ABP < 0$), or exponential ($ABP > 0$), where:

$$\textit{atomic behavior pattern } (ABP) = \frac{\partial|\dot{x}|}{\partial t} \tag{3}$$

Similarly, Saleh and Davidsen [85] described Ford's atomic behavior patterns as convergent ($ABP < 0$), and divergent ($ABP > 0$), and defined a normalized proxy measure for ABP to indicate convergence/divergence of a variable, calling it *Behavior Pattern Index (BPI)*:

$$\textit{behavior pattern index } (BPI) = \frac{\ddot{x}}{\dot{x}} \tag{4}$$

Ford's definition of dominance built upon the earlier work of Graham, Richardson, and Pugh who defined dominance from a behavioral perspective. According to Ford, "A feedback loop dominates the behavior of a variable during a time interval in a given structure and set of conditions when the loop determines the atomic pattern of that variable's behavior" [14]. One limitation of Ford's method was that it did not specify exactly how to deactivate a feedback loop, which presents challenges when a loop does not contain a unique variable. Methodological extensions, such as the Generalised Loop Deactivation Method (GLDM), provide heuristics for deactivating loops which do not contain a unique control variable [74, 48], however, these extensions may not work in every situation.

Statistical screening. Statistical screening methods are also considered *exploratory/behavioral* in that the criteria for dominance is based on the sensitivity of simulated responses based on changes in model structure [13, 94, 95].

8.1.2 Formal/Structural Methods

A discussion then emerged regarding whether or not methods should identify a single or multiple dominant loops [65, 73, 52]. There was also a desire to understand not just which loop(s) were dominant, but which parts of the system were causing the loop(s) to be dominant. Ford indicated that perhaps both behavioral and structural approaches could be used together.

> Future research can further validate our procedure, expand our initial investigations of simultaneous dominance and shadow feedback structures and integrate our behavioral perspective with structural approaches to feedback loop dominance analysis [14].

Structural or formal methods (i.e. PPM, LEEA, DDWA, Loop Impact Method) were developed which assess the relative strengths of loops through normalized metrics derived directly from the equations in order to determine which loops and links are most influential. Formal/structural methods identify which structural elements are the most influential in determining behavior and rank orders the elements based on their relative sensitivity.

Loop eigenvalue elasticity analysis (LEEA). [26, 51, 85, 69, 84, 1, 42, 73, 52, 86, 53, 54, 62, 101, 32, 63] In this method, systems are linearized at each point along a variable's trajectory, and the eigenvalues are computed in order to identify the dominant behavior modes of the system. The feedback loops and corresponding links of each loop are identified. The sensitivity of the eigenvalues to each link gain is calculated using partial derivatives. From this, feedback loop sensitivities are calculated and then normalized to produce feedback loop elasticity values. This procedure is conducted for each feedback loop, and the loop with the largest elasticity is determined to be dominant at that local point in the trajectory. This procedure is then iterated along the entire state trajectory.

An important finding is that the number of loops in a maximally connected system with n state variables and p auxiliary variables grows more than factorially ($2^{np} \cdot (n-1)!$) while the number of independent loops with N links only grows linearly: $N - n + 1$. The challenge, therefore, is how to identify and choose a suitable independent loop set (ILS), which may not be unique [51]. Different methods using graph theory have been developed to identify a minimum set of loops such as the shortest independent loop set (SILS) and minimal SILS (MSILS) [51, 85, 68, 69, 47, 49]. This remains one of the research challenges today for models in which a minimal SILS does not exist.

Eigenvector or dynamic decomposition weight analysis (DDWA). The dynamic decomposition weights analysis (DDWA) method improves upon LEEA to consider not only the entire system behavior, but that of specific variables of interest [87]. This method extends LEEA and examines sensitivity and elasticity of loops with respect to the eigenvectors of the linearized system [31, 32, 47]. While the eigenvalue and eigenvector based methods have been implemented in software such as *Mathematica* and have been tested against relatively simple models, there are no documented cases of applying these methods to realistic models [76].

One of the challenges for both eigenvalue and eigenvector methods is developing intuitive interpretations for loop elasticities [54, 87]. Additionally, it has been found that eigenvalue elasticities can be misleading and result in missed detections of high-leverage intervention points as well as false positives due to phantom loops in some highly nonlinear models [101, 87, 63].

Pathway participation method (PPM). In this method, local behavior is determined by the variable's first and second time derivative, as shown in Figure 1, resulting in nine possible local behavior patterns [65].

Mojtahedzadeh and colleagues use the BPI metric for a variable of interest, calling it the *Total Pathway Participation Metric (TPPM).*

$$total\, pathway\, participation\, metric\, (TPPM) = \frac{\partial \dot{x}}{\partial x} = \frac{\ddot{x}}{\dot{x}} \tag{5}$$

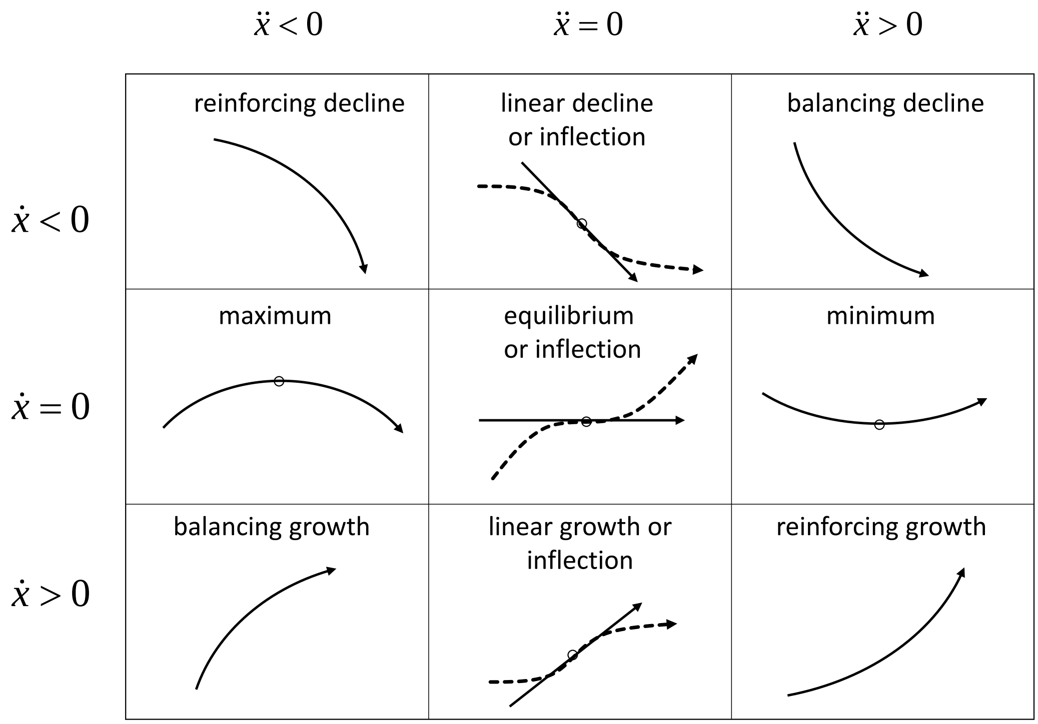


Figure 1: Nine possible local patterns based on the signs of the first and second derivatives.

The PPM algorithm evaluates the contribution of each causal pathway to the TPPM and identifies the dominant pathway as that which has the largest contribution. The procedure then iterates on the next most influential variable and continues until either a closed loop or exogenous variable is identified. The procedure identifies either a dominant feedback loop containing the variable of interest, a dominant pathway from the variable of interest to a feedback loop elsewhere in the system, or a pathway to a dominant exogenous variable.

Loop impact method. Similar to PPM, this method looks at the strength or impact of causal links as opposed to feedback loops and uses a metric similar to the PPM. It addresses one of the critiques of PPM and performs a breadth-first (as opposed to depth-first) search of the set of feedback loops that together have the largest impact and dominate the dynamics [43, 45, 44]. The Loop Impact method considers the cases of $\dot{x} = 0$ and $\ddot{x} = 0$ as transition cases, and simplifies the scheme in Figure 1 to the four local modes of behavior in the corners. Dominance is defined as the loop, or minimum combination of loops of like polarity, whose (combined) impact is greater than the sum of all loops of opposite polarity.

A summary of the strengths and limitations of each dominance method discussed, along with their criteria for dominance, is included in Appendix B.

8.2 Definitions for System *Structure*

Returning to Richardson's question of how to define *structure*, there have been primarily three ways to define sub-structure for the purposes of explaining model behavior: causal links, causal pathways, and feedback loops. Causal pathways consist of one or more causal links connecting a state variable (stock) to a derivative of a state variable (net flow). Feedback loops consist of one or more causal pathways which form a closed causal loop, as shown in Figure 2.

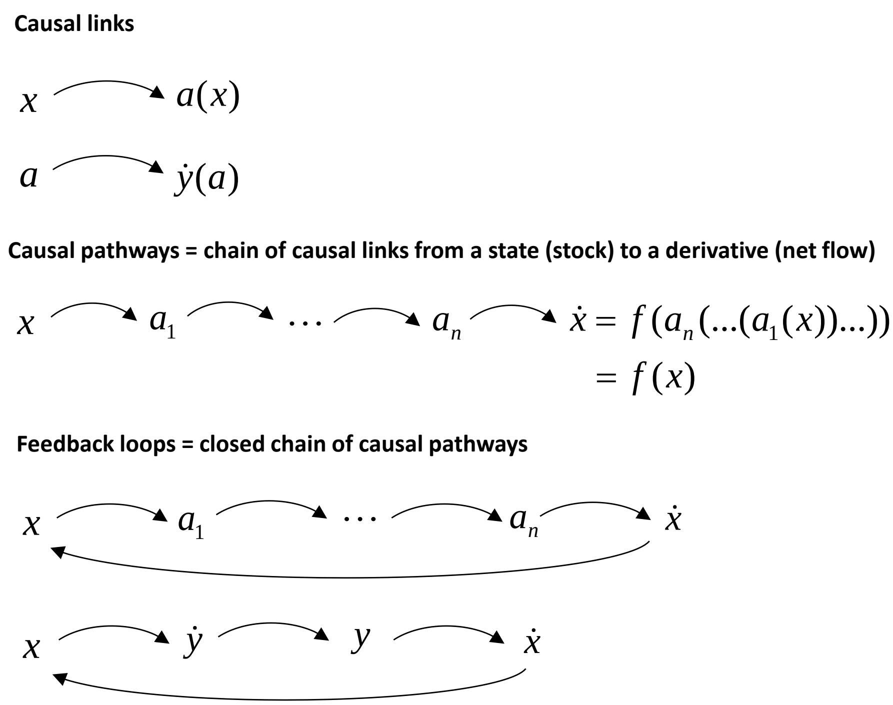
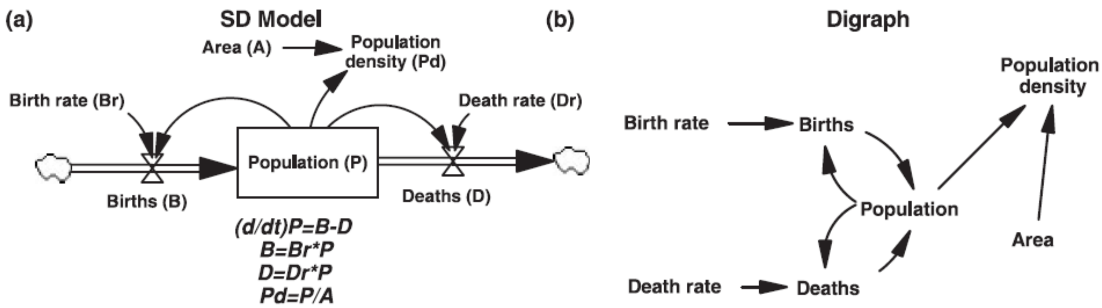


Figure 2: Causal links, causal pathways, and feedback loops.

The causal links and feedback loops in an SD model can be succinctly defined as a directed graph in which the nodes represent the variables, and the links or edges represent causal relationships between the variables [51, 68]. Figure 3 illustrates how a simple birth and death model can be represented by a graph.

The question of whether causal links, causal pathways, or feedback loops should be used to explain behavior has not been entirely resolved. However, the vast majority of methods focus on feedback loops as the explanatory element of system structure. Even pathway approaches, such as PPM, are constructed such that they are able to identify feedback loops (as a closed chain of causal pathways) as being dominant. Only a few papers focus on variables, parameters, or links as the influential elements of structure [40, 13, 102, 95, 87]. These approaches mainly look at the sensitivity of individual variables and parameters on the system behavior (or proxy measure of behavior).

There are notable challenges with identifying feedback loops as the explanatory element of system structure. As noted above, the number of possible feedback loops in a maximally con-



(c) **Adjacency matrix**

	Br	*B*	*P*	*Dr*	*D*	*Pd*	*A*
Br	0	1	0	0	0	0	0
B	0	0	1	0	0	0	0
P	0	1	0	0	1	1	0
Dr	0	0	0	0	1	0	0
D	0	0	1	0	0	0	0
Pd	0	0	0	0	0	0	0
A	0	0	0	0	0	1	0

(d) **Reachability matrix**

	Br	*B*	*P*	*Dr*	*D*	*Pd*	*A*
Br	1	1	1	0	1	1	0
B	0	1	1	0	1	1	0
P	0	1	1	0	1	1	0
Dr	0	1	1	1	1	1	0
D	0	1	1	0	1	1	0
Pd	0	0	0	0	0	1	0
A	0	0	0	0	0	1	1

(Oliva, 2004)

Figure 3: Graph representation of system structure [68].

nected system, that is, where every state derivative depends on every variable (represented as a directed cycle within a directed graph), assuming the graph is strongly connected[6] with N links and n state variables and p auxiliary variables grows by $(2^{np} \cdot (n-1)!)$ [51]. While it has been shown that the number of independent loops in such a set only grows linearly $(N - n + 1)$, it has also been shown that for some models, a shortest independent set (SILS) cannot be generated [49], and in other cases it is not unique, therefore introducing subjectivity into how the loops are defined [64]. Another research challenge has been how to test a loop, independent from all other loops, once it has been identified [14]. While methods have been developed to test loops independent from one another, they are not adequate for all systems [74, 48].

Finally, it is interesting to observe that a few researchers have raised the question to what extent the very notion of feedback loops makes sense as explanatory structure [54]. The literature review did not reveal any papers that have specifically dealt with this question, although observations have been made about the limitations of being able to distinguish between the effects of feedback loops which share common links [45].

While progress has been made in defining structure and behavior independent from one another [14], it is clear there still exists multiple competing definitions in use today.

[6]Strongly connected graph is one in which for any pair of nodes, there exists is a directed path in both directions.

8.3 Definitions for *Dominance*

There have been two approaches to defining *dominance*. Exploratory/behavioral methods define dominance with respect to behavior (e.g. a feedback loop is dominant if its deactivation causes a significant change in behavior), whereas formal/structural methods define dominance relative to structure (e.g. a feedback loop dominates if its relative influence is greater than that of other loops). The exploratory/behavioral methods detect dominance based on an objective criteria for how structure determines behavior, using phrases such as *determine*, *results in* and *cause*.

Structural methods, on the other hand, use subjective or relative terms for dominance, relating one piece of structure to another, such as *most influence*, *greater than*, and *larger*.

These two approaches to defining dominance lead to different questions and answers. Behavioral methods ask which loops determine behavior, and when. Structural methods ask which loops are more influential than others and how that influence changes over time. Both questions are important for explaining how structure produces behavior. Each captures a different dimension of dominance. The exploratory/behavioral methods employing objective criteria captures the structure-behavior dimension, whereas the formal/structural methods employing relative criteria capture the structure-to-structure dimension.

There is ample support for the position that both behavioral and structural approaches are needed to provide satisfactory explanations of system behavior. For instance, Nathan Forrester, who first introduced the eigenvalue methods to SD, states:

> ...the elasticities may not be clearly grouped by magnitude, in such cases the cutoff between "dominant" and "secondary" feedback loops is arbitrary [26].

Others using the eigenvalue approach have suggested that the methods could be used well in conjunction with behavioral approach.

> In this context primarily the most dominant eigenvalue would be considered, - i.e. the eigenvalue with the most significant contribution, yet one may test the effect of the second and/or the third and/or any higher order dominant eigenvalue on the behavior of a state [1].

8.4 Research Gaps in Dominance Analysis

While early feedback loop methods and definitions were being formalized and tested on small models, the question was raised as to the usefulness of the feedback loop concept for large-scale models [51]. The development of several dominance methods shows promise in this regard, however most have still only been tested on relatively small models. For all the methods, additional testing is required on large-scale nonlinear models [11, 6, 55, 51, 41, 1, 86, 48, 45].

Today, there remains significant limitations of not having a single, formal, and rigorous definition for *dominance*. The systematic review found multiple inconsistencies between methods which are based on different definitions and criteria for detecting *dominance*. These differences have been acknowledged and studied by several authors in the field [42, 73, 52, 54, 63].

It seems possible that in some of these instances, a formal definition of dominance would reveal that the methods being compared are asking different questions about the same model, explaining why they produce different answers. Additionally, the subjective nature of terms used to describe dominance in structure-behavior explanations, such as *highly influential* and *important* makes it challenging to infer precisely how and to what extent certain structures determine behavior.

The lack of a rigorous definition of *dominance* also makes it difficult to test, prove, or falsify assertions that have been made about dominance, such as, “Additive effects cannot dominate an expression; multiplicative effects can” [88]. It is also commonly asserted that linear systems are incapable of shifting dominance [34, 83, 82, 25, 89]. This statement seems like it could be formally proven or falsified. However, while there have been compelling arguments and examples offered in support of this claim, using the superposition principle in linear systems theory and appealing to metrics such as dominant polarity, the systematic review found no instance of a formal proof of this claim. Furthermore, a counter-example is offered by Guneralp in which a linear system appears to exhibit a shift in dominance [41]. Upon closer inspection, however, the counter-example employs a different criteria for dominance than that used by previous authors. This illustrates the challenge of not having a common, formal definition of dominance.

It is also commonly claimed that in large systems only a few loops dominate [36, 39, 58]. This claim also seems testable, but to do so requires a precise definition of dominance which allows for more than one loop to be dominant. For many methods, such as LEEA, there is not objective criteria for the number of loops that are dominant [42], while other methods only identify a single dominant loop.

Important questions are being asked in the field which are challenged by the fact that a single formal definition of the term *dominance* does not exist. The following lines of research in dominance analysis address important research gaps and would benefit from a precise definition:

- Identifying the proper set of loops to analyze when more than one unique set exists [51, 64, 68, 42, 53, 54, 49].

- The manner in which to deactivate loops in tests for dominance [14, 74, 48].

- The threshold or criteria for establishing whether one, two, or multiple loops are identified as dominant [65, 73, 52].

- Understanding why some methods, in some cases, result in missed detections or false positive (e.g. phantom loops) [101, 87, 63].

Lastly, each dominance method operates in the time-domain and is not designed to detect how dominance changes across the state-space, and thus does not evaluate sensitivity to initial conditions [74, 102]. In fact, often it is advised to set the initial conditions such that the model begins in equilibrium, prior to conducting analysis, which inherently confines the analysis to specific regions of the state-space [81, p. 286].

9 Summary and Conclusions

In response to the challenges proposed by Richardson in 1986, the field has made considerable progress in developing methods to detect dominance. Methods have been developed based on deactivation of entire elements of model structure, and others based on marginal changes in model structure. Each have advantages and limitations and provide some explanation for how and why structure drives behavior, especially when combined with the intuition afforded by linear systems theory. However, there remains significant challenges. There are instances when different methods produce different results and some methods are not suitable for addressing certain classes of systems. Some methods identify instances of no dominant structure and multiple dominant structures, while others always detect a single dominant structure. This is further complicated by the fact that for some models, there may not exist a unique set of loops, and so results may depend on the analyst's choice of loop set [49]. Some methods consider behavior at a single point in time, while others consider global behavior patterns observable over longer time intervals[7] [90]. Some methods consider the behavior of all variables while others focus on a specific variable. The lack of a formal and universally accepted definition of dominance has led to explanations of structure-behavior relationships in some cases that are ambiguous and untestable (and therefore not falsifiable), which can hinder progress in the field. Furthermore, some questions are not being asked, such as how *behavior*, *structure* and *dominant* should be defined in order to best explain structure-behavior relationships.

Lastly, widespread use of dominance tools has still not occurred and there is a lack of real-world examples showing the relative utility of dominance methods over other methods, such as sensitivity analysis [76].

Kampmann and Oliva, 20 years after Richardson's initial challenge, reiterate the need to clearly define terms for the sake of progress in the field.

> To the extent that we can both rigorously define and identify such dominant structures, we choose to say that we have found a 'theory' of the observed behavior [54].

The literature review concludes that, in fact, there still exists a need for a formal and rigorous definition of *dominance* to facilitate further advancements in the field.

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Appendix A: Definitions of *Dominance* From Literature Review

The literature review found the following definitions of *dominance*, listed in chronological order:

> Here, a loop dominates the behavior in the sense that if the loop is disconnected or substantially altered, the behavior mode also changes substantially [38].

> ...In a feedback structure, a loop that is primarily responsible for model behavior over some time interval is known as a dominant loop [81, p. 285].

> Links that have large-magnitude eigenvalue elasticities are particularly important. If a small number of elasticities have markedly greater magnitudes than others, then (they) define a dominant subset of model structure [26].

> In a first-order system with level x and net rate of change x, a shift in loop dominance is said to occur if and when $\partial\dot{x}/\partial x$ changes sign, that is, when the dominant polarity of the system changes [79].

> If we say that at a particular time one feedback loop is stronger or dominant over another we mean that the system is undergoing behavior associated with the dominant type of feedback at that time [29].

> The stronger loop is said to have loop dominance [56].

> When a positive and negative feedback loop are used together, as shown in Fig 9, the strongest loop is the dominant one. In Fig 9, for one loop to be dominant, it must have a greater effect on the population [9].

> We can trace several feedback loop gains simultaneously by simulating model equations, and then we can select a dominant feedback loop which has the largest gain in the specified periods [55].

> Loop dominance: A system in which one loop is stronger. In a system with multiple loops, magnitudes and algebraic signs of variables determine what kind of behavior, positive or negative feedback, is dominant at any given time. If the system exhibits exponential growth, then the positive loop is dominant. If asymptotic behavior is evidenced, the negative loop has dominance. S-shape growth is a common behavior of a system in which loop dominance shifts with time [103].

A feedback loop dominates the behavior of a variable during a time interval in a given structure and set of conditions when loop determines the atomic pattern of that variable's behavior [14].

Dominant loops can be seen as a reduced set of closed feedback paths that contribute most to the overall behavior mode of a model [65].

Contributes the most to $\partial \dot{x}/\partial x$ [65].

Mojtahedzadeh then considers each possible pathway and defines the dominant pathway as the one with the largest numerical value and the same sign as PPM_i [54].

By dominant structure we mean particular feedback loops, or possibly external drivers, that are in some sense "important" in shaping the behavior of interest [54].

...the one considered as a dominant loop should exert most significant influence to the behaviour, i.e., when the dominant loop is deactivated, the behaviour diverts most from its original trajectory [48].

...pathways with higher magnitude of frequency (stability) factors are considered dominant in deriving the periodicity (stability) of the cycles. A set of pathways that close the loops are considered the dominant structure [63].

EEA calculates how much each feedback loop influences the eigenvalue, and the one with most influence is considered the dominant loop. This influence is quantified by the loop elasticity e [49].

Dominance is defined as the loop, or minimum combination of loops of like polarity, whose (combined) impact is greater than the sum of all loops of opposite polarity [45].

Appendix B: Summary of Dominance Methods

Method	Dominance Criteria	Strengths	Limitations
Behavioral loop dominance analysis (Ford's behavioral approach; Generalised Loop Deactivation Method; Extension of deactivation method)	Deactivating loop causes change in behavior mode of specific variable (counterfactual test)	Intuitive relationship between structure and behavior. Identifies when structure determines behavior. Can identify multiple dominant loops.	Limited insight into how mechanisms cause behavior. May not always be able to isolate effect of individual loop through deactivation. Time domain only, not applied to state-space.
Loop Eigenvalue Elasticity Analysis (LEEA)	Loop with greatest eigenvalue elasticity with respect to overall system behavior.	Takes into consideration all system parameters. Shown to be appropriate for quasi-linear models that exhibit transient or oscillatory behavior. Identifies the relative influence of each loop (and thus, can produce a rank order of loops).	Computationally intensive. Non-intuitive relationship between structure and behavior and interpretation of elasticities. Does not detect whether structure determines behavior. Existence and uniqueness of independent loop sets. Not applied to specific variables. Relative importance of eigenvalues is subjective. Addressing phantom loops. Difficulties with chaotic systems and individual-based models. Not applied to state-space.
Eigenvector analysis or Dynamic decomposition weights analysis (DDWA)	Loop with greatest influence measure with respect to a specific variable.	Similar strengths as LEEA. Extends LEEA method to apply to specific variables.	Similar as LEEA, but solves issue of applying to specific variables. Confounds effects of initial conditions and structural elements.
Pathway Participation Metric (PPM)	Pathway or loop with greatest total PPM with respect to a specific variable.	Computationally simple. Identifies how influence shifts over time.	Does not detect whether structure determines behavior (relies on TPPM as proxy for behavior). Always identifies a single dominant loop using depth-first search. Metric undefined for zero first derivative. Time-domain only, not applied to state-space.
Loop Impact Method	Minimum combination of loops with like polarity with loop impact greater than opposing loops (with respect to specific variable).	Accounts for initial conditions. Handles higher order loops that change polarity, hidden (phantom) loops, and loop that self-cancel.	Not a direct link between structure and behavior (uses PPM as proxy for behavior). Time-domain only, not applied to state-space.

Table 3: Summary of strengths and limitations of dominance analysis methods.