Employment-Centred Stabilisation Policy Propelling the Economy to "Escape Velocity"

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Abstract. The paper refines and generalizes the Fanti and Manfredi Goodwinian model with delayed profit-sharing allowing capital investment lower than profit. Although periodic dynamics arise via simple Andronov – Hopf bifurcation for large "humped" delays, the opponents' proposition that the wage-profit indexation triggers persistent economic cycles is incorrect. The paper reveals detrimental effects of the profit-sharing rule for economic reproduction in the long run even when it alleviates oscillations.

This paper revises the equations for profit-sharing and bargained wage terms from the opponents' model in two encompassing non-linear four-dimensional models. The previous model enabled extreme condition tests for them. In the first, before second-order delay is added, a growth rate of profit is proportional to a gap between the indicated and current employment ratios. This policy rule with a great margin of safety stabilises capital accumulation being fuzzier for stretched "humped" delays. In the second model, deviations of employment ratio and delayed profit rate from their stationary magnitudes define net change of relative wage. This proportional control already present in the first model is reinforced in the second shortening a transient to a distant target employment ratio. Parametric optimization for both models is supported by *Vensim*.

Introduction

Broadly held view in economics is expressed recently by L. Summers (2013): "Growth and job creation are, after all, the ultimate ends of economic policy". Still economists, who often agree on these ends, dispute on means of their achievement.

System dynamics both as the field of research and research method can bridge the gap between theoreticians by inviting them with their models to some kind of a computer supported intellectual forum like an annual system dynamics conference. There is no other scientific approach matching system dynamics in capability of touching the mind and heart, moving the soul forward.

A challenger should present an encompassing model to opponents that not only reproduces their main results as a special case but offers new profound insights into growth and job creation absent in a model encompassed and refined. Next time the roles can be reallocated between contenders that learn and benefit from each other's work as in a positive feedback loop.

Let us turn our attention to a base model developed by Fanti and Manfredy (1998) paper. The divide it establishes between long term steady state growth and jobs creation deserves careful consideration. One of the model main paradoxes resides in stabilisation policy that governs economy to lower employment ratio in the long term than before the policy onset. This outcome with account of austerity measures and their consequences, especially mass unemployment in Greece, Italy and other EU countries, is not to be a priory rejected as a logical mistake.

P. Krugman (2013) used a simple regression to demonstrate that austerity was costly for the afflicted economies: the greater the tightening between 2009 and 2012, according to the International Monetary Fund, the bigger the fall in real GDP. M. Wolf (2013) wrote: "Tens of millions of people are suffering unnecessary hardship. It is tragic".

It is really an apologetics-breaking moment of truth. Capital-led stabilisation through austerity measures ended in unnecessary suffering of tens of millions of human beings. *Cui bono*?

Swiss economic researchers in Zurich have conducted a global network analysis of the most powerful transnational corporations (TNCs). They have revealed a core of 737 firms with control of 80% of this network, and a "super entity" comprised of 147 corporations that have a controlling interest in 40% of the network's TNCs (see Vitali *et al*., 2011).

An expected logical conclusion (Ellsner, 2012: 137) follows: "It is a closed shop of mutual control, uncontrollable itself from outside. In fact, these are only several hundred institutional top-rank persons, who largely know each other, plus some hundred mega-rich private individuals as their owners and creditors. "Markets"! Any conspiracy theory of the left turns out to be a harmless bedtime story compared to "neoliberal" reality …"

R. Marx and F. Engels explained the beginning of this monopolization process (Marx, 1863–1883). V.I. Lenin (1916) outlined in Zurich the objective tendency of capitalism to evolve from the stage of free competition to stage of monopoly capitalism. After decades of the wide-spread hypocritical denial, we see confirmation of these fundamental tendencies on the global scale.

A stabilisation policy designed in Fanti and Manfredy (1998) is carried out through a particular form of profit sharing. Monopoly capitalism dominated by financial capital is not mentioned. The base model abbreviated below as FM extends the famous Goodwin model (abbreviated below as GM) by introducing the profit sharing rule (as Lordon, 1997) whereby the rate of change of wage depends positively not only on the employment ratio but also on profit rate.

The authors give an idea to the reader that in the long run distribution is left inalterable compared with GM while the employment ratio is reduced; the conservative oscillation of Goodwin (1967) is lost and the system becomes dissipative with dynamics convergent to the positive equilibrium given the profit sharing rule. However, under the assumption of "staggered" wage contracts, with an indexation to "humped" (like inverted U) distributed lags of the profit rate (instead of the current one), they show that the positive equilibrium may be destabilised and a closed orbit emerges in result of simple Andronov – Hopf bifurcation.

The authors have been among the very first researches that applied local bifurcation theory to economic systems with dimension higher or equal to four. These Italian economists underlie the importance of simple Andronov – Hopf bifurcation for three different reasons (Fanti and Manfredi, 2003: 3). Consider them briefly.

The first one is a point of agreement – "it is always the outcome of a fully endogenous interaction between (non-linear) economic forces." Still their model deserves reproach for being only weakly non-linear.

The second point is a battle field: "it is a "local" bifurcation, thus much in spirit with the common belief of our science by which economic systems are generally close to their equilibrium state".

This bifurcation is typically local in nature. Still a broader perspective for its correct interpretation is required. Here the Mosekilde and co-others (1988) paper is to be recalled: "As a particular road to deterministic chaos … involves three subsequent, independent Hopf bifurcations, each of which creates a self-sustained oscillation in the system. This can occur, for instance, in an integrated model of three commodity markets or three predator-prey systems, each of which exhibits limit-cycle behavior". It represents a qualitative change in the evolving system that can develop further from periodic orbit to diverging fluctuations stronger necessitating stabilisation policies (Mosekilde and Laugesen, 2007: 250–251).

Confidence in closeness of economic systems to their equilibrium state is not shared by Jay Forrester and other leading representatives of system dynamics approach to socio-economic and living systems in general. They argue in depth why this mechanistic understanding is far from objective reality and scientific frontiers: "Living systems operate under far-from-equilibrium conditions and they display a multitude of complicated nonlinear phenomena, including spiking and bursting dynamics, synchronization, chaos, and nonlinear wave propagation" (Mosekilde and Laugesen, 2007: 250).

The third point – is a reflection of the opponents' intellectual dependence on the «neoclassical» canons: "it implies "local" oscillations, which are the normal route through which disequilibrium manifests itself when the equilibrating forces operating in the economy are relaxed (e.g. the adjustment process of a walrasian market)."

Established rival schools generally disagree with this simplistic argumentation in line with the Newtonian mechanics. Moreover, the hardly ever existed "adjustment process of a walrasian market" masks the competitive-co-operative network of TNCs unveiled by the Swiss researchers.

FM abstracts from other important properties of capitalist economy such as induced technical change, labourers competition for jobs, non-equilibrium processes on the market for produced commodities. The state activity is reduced to implicit defence of private property without explicit modelling this function.

The present paper proves that seemingly strong conclusions of the paper (Fanti and Manfredi, 1998) implying significant practical guidelines are not incontrovertible if considered in a broader theoretical context. The system dynamics approach will shed light upon limitations of their excessively abstract model and will enable constructive alternatives favourable for social well-being.

This paper not only refines and generalizes an interesting model that has evoked a substantial resonance. It also lays bare inaccuracy of an appraisal (Mastromatteo, 2006: 246): "Fanti and Manfredi (1998) …[are] pointing out the crucial role of the time delay involved in profit-sharing in determining cyclical fluctuations." Such lavish and cosy appraisal is, in my view, detrimental for economic science and education. A more critical assessment of other models of the same authors can be found in the literature (Ryzhenkov, 2009).

The rest of the paper is organized in the following manner. Section 1 considers the FM extensive and intensive deterministic forms. The focus is on properties of the model feedback structure, features of the stationary state, destabilizing effects of second order delay in wage formation against profitability. The uncomfortable detrimental effects of stabilisation policy on long-term employment ratio are not left out of sight.

Although the mathematical proofs are mostly skipped, the reader will be well informed on main propositions of the authors and how they were able to derive them for a rather complicated 4-dimensional system of ordinary differential equations.

The second section offers an alternative Marxian approach maintained by overt application of structural control theory. For the sake of brevity, references to the alternative model apply the abbreviation AM. The arrangement of presentation of AM is basically the same as in the preceding section.

Before second-order delay is added in formation of the wage growth rate, a growth rate of profit is proportional to a gap between the indicated and current employment ratios. This policy rule with a great margin of safety stabilises capital accumulation being fuzzier for stretched "humped" delays.

Comparisons of basal properties of the alternative models are carried out in depth additionally. They reveal much more importance of non-linearity in wage formation in AM compared with FM that helps to bring employment closer to a benchmark, as well as to develop and use economic potential with greater efficiency. A suitable magnitude of a key control parameter (c_2) is found as a solution of parametric optimization problem over 64 years.

Factors responsible for destabilisation of economic growth in FM are kept mostly in check in AM. It is proved by application of Liénard – Chipart criterion that a supposed stabilisation policy has a sufficient margin of safety.

The conditions of restricted local equivalence in linear approximation at the same stationary state in both models are defined. This enables to emphasize the greater role of non-linear interactions in AM compared with FM that explains why the economic dynamics are so different in these models in spite of some common properties.

Section 3 reports on a reinforced stabilisation policy. In the modified AM, deviations of employment ratio and delayed profit rate from their stationary magnitudes define net change of relative wage. This proportional control, already given in AM, is reinforced shortening a transient to a distant target employment ratio.

The second control parameter (q) is introduced in an equation of net change of relative wage. The solving of parametric optimization problem yields most suitable estimates of both control parameters (c_2 and q) that strongly shorten the lengthy period required before for substantial reduction of unemployment ratio. Recalling L. Summers's article (2013), the employment-centred policy for propelling the economy to its "escape velocity" is sketched.

Both alternative models considered in the present paper refine and encompass FM; they are rather simplified and abstract as well. Still these models can serve as extreme condition tests for the supposed policy with a great potential social benefit. Parametric optimization for both models is supported by *Vensim*. Passing extreme condition tests (especially with one exception as shown below) does not, of cause, guarantee theoretical and empirical plausibility of these two alternative models. AM is a special case of more complex models with a greater number of endogenous variables that have been systematically statistically tested in the previous publications.

1. A model of cyclical dynamics with profit sharing

Before the relative wage delays were introduced, the initial two-dimensional model had been a special case of the model of the French economist F. Lordon (1995), critically analysed in the article (Ryzhenkov, 2012). We go directly to a detailed examination of the full model of the Italian economists that contains a second order delay in the relative wage. Terms unit value of labour power, relative wage and wage share are interchangeable in Marxian models.

1.1. The model extensive form

A variable's time derivative is denoted by a dot, its growth rate – by a hat over the variable's sign.

Labourers are advancing capitalists as they receive wage after a particular circuit of capital is finished. Having abstracted from the foreign economic relations, FM consists of the following equations:

$$P = K/s; \tag{1.1}$$

$$a = P/L; \tag{1.2}$$

$$u = w/a,\ 0 < u < 1;^{1} \tag{1.3}$$

$$\hat{a} = h > 0, \tag{1.4}$$

$$s = \text{const} > 1; \tag{1.5}$$

$$v = L/N,\ 0 < v < 1;^{2} \tag{1.6}$$

$$N = N_0 e^{nt},\ n = const \geq 0,\ \ N_0 > 0; \tag{1.7}$$

$$\hat{w} = -g + rv + e\frac{1-y}{s},\ g > 0,\ r > 0,\ e > 0; \tag{1.8}$$

$$P = Q + \dot{K} = wL + (1-k)S + \dot{K}; \tag{1.9}$$

$$\dot{K} = kS = k[(1-u)P],\ sd < k \leq 1. \tag{1.10}$$

Equation (1.1) specifies a technical-economic relationship between the fixed capital (K) and the net output (P). Capital-output ratio is denoted as s, its inverse by m. Equation (1.2) expresses net output per worker (a) as a ratio of net output (P) to employment (L). Equation (1.3) describes the relative wage as the labour share in net output (u). Equation (1.4) assumes a constant exogenous growth rate of output per worker (a) that equals to the growth rate of capital intensity (K/L), whereas capital-output ratio remains constant according to equation (1.5).

Equation (1.6) defines the employment ratio (v) as a result of the sale of the labour power. According to equation (1.7), the growth rate of labour force (N) is equal to a constant (n). Equation (1.8) links the growth rate of real unit wage (w) with employment ratio (v) and delayed profit rate ($(1-y)/s$).

The use of delayed profit reflects information lags for labourers regarding the actual relative wage. Capitalists receive information on relative wage in real time. In addition, the "staggered" labour contracts between workers and capitalists are taken into account.

A growth rate of wage is represented as the sum of bargained $\hat{w}^m$ and profit sharing $\hat{w}^b$ terms $\hat{w} = \hat{w}^m + \hat{w}^b$, where the first is determined by employment ratio (v) as in the Phillips equation

$$\hat{w}^m = -g + rv, \tag{1.8a}$$

and the second – by the delayed profit rate

[1] In the original text there is inaccuracy as in the paper (Lordon, 1995): $0 \leq u \leq 1$. The substitution of the segment [0, 1] by the interval (0, 1) takes into account that $u = 1$ and $u = 0$ are both incompatible with capitalist production relations: in the first case the labour power has no social utility (the ability to produce a surplus product) and cannot be sold, in the second the emptiness of a consumption bundle prohibits reproducing this commodity.

[2] In the original text there is inaccuracy: $0 \leq v \leq 1$. The substitution of the segment [0, 1] by the interval (0, 1) takes into account that $v = 1$ and $v = 0$ are both incompatible with capitalist production relations: in the first case there is no reserve of labour power vital for the capitalist mode of production, in the second the latter is simply impossible.

$$\hat{w}^b = e\frac{1-y}{s}, \tag{1.8b}$$

where $e > 0$ is a profit sharing index. Some reasonable bounds on this parameter are defined below.

For representing "staggered labour contracts", the distributed lag for profit is introduced. The second order delay in relative wage and profit rate is reflected by use of a distribution function from the Erlang family that has the form of a "humped" curve with mean value $2/b$ and variance $2/b^2$ (Fanti and Manfredi, 1998: 385, Sterman, 2000: 464–467).

Balance equation (1.9) shows the end use of the net output (P), where Q is a private and public consumption, $\dot{K}$ is net fixed capital formation. Investment delays are not taken into account.

It is expected that the surplus product (S) that equals total profit (M) can be not only invested, but also be used to cover personal expenses of the bourgeoisie, as well as parts of government spending not directly related to the reproduction of the labour power. Consequently, the rate of accumulation k, or share of investments in surplus product, is such that $sd < k \leq 1$. The left boundary is set to avoid a non-positive stationary relative wage.

The presence of sd as a lower boundary for the rate of accumulation is a drawback of both GM and FM, since in reality, relative wage remains positive even when $sd \geq k$. This means they do not pass this particular extreme condition test (cf. Sterman, 2000: 337). Models in the articles (Ryzhenkov, 2008 and 2010) contain endogenous capital-output ratio and endogenous rate of accumulation in the absence of the specified lower bound as a real necessity. Long term decline in this ratio mitigates the tendency of profit rate to fall in Italy and in the USA as well.

Table 1.1 and Figure 1.1 present a causal loop structure of FM. Loop B1 is inherited from GM, loop B2 is due to the profit sharing rule. Besides these, FM includes two minor negative feedback loops for delayed relative wage z, for relative wage delayed twice y, and two minor feedback loops with alternating polarity for relative wage u and employment ratio v.

Table 1.1. Two main negative feedback loops in FM

Loop B1 of length 9	Loop B2 of length 9
Wage share u $\xrightarrow{-}$ Profit rate Growth rate of fixed capital Growth rate of employment ratio Net change of v Employment ratio v Growth rate of bargained wage term Growth rate of wage Growth rate of wage share Net change of u	Wage share u Net change of z Wage share delayed z Net change of y Wage share delayed twice y $\xrightarrow{-}$ Profit rate delayed twice Growth rate of profit sharing wage term Growth rate of wage Growth rate of wage share Net change of u

Note. Only a negative first partial derivative is explicitly shown as an arrow. All other first partial derivatives are positive.

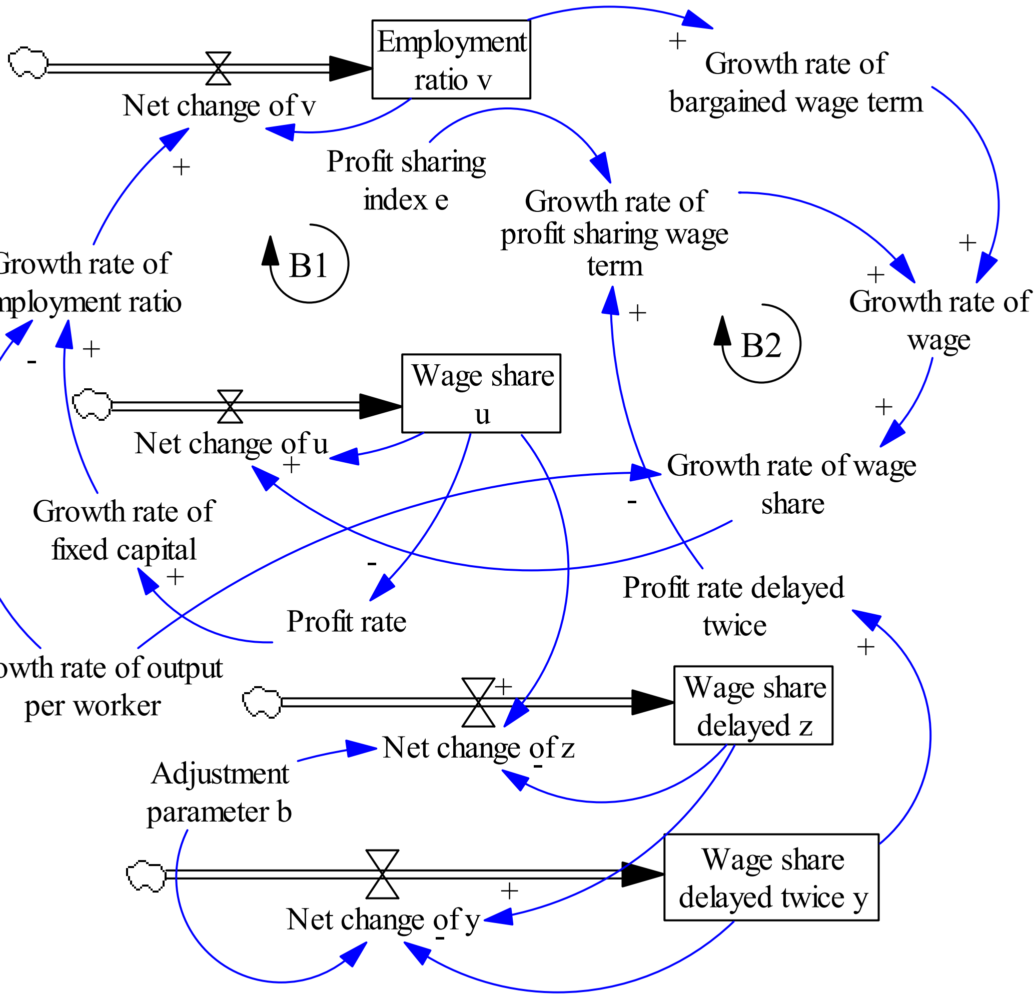

Figure 1.1. A causal structure of FM with second order delay in relative wage

1.2. The model intensive form and properties of a stationary state

An integro-differential equation for net change of relative wage (u) contains the delaying kernel assumed to be a specific probability density function (see equation (2.2) in Fanti and Manfredi, 1998: 382). It is omitted in the present paper for the sake of brevity.

As shown in (Fanti and Manfredi, 1998: 385), an intensive form of deterministic FM consists of four ordinary differential equations (including two non-linear). Here is this system in a generalised form for $sd < k \leq 1$ in relation to the original form (for $k = 1$):

$$\dot{z} = b(u - z) \tag{1.11}$$

$$\dot{y} = b(z - y) \tag{1.12}$$

$$\dot{u} = \left(-g + rv + e\frac{1-y}{s} - h\right)u \tag{1.13}$$

$$\dot{v} = \left(k\frac{1-u}{s} - h - n\right)v\,. \tag{1.14}$$

The equation (1.14) for the net change of employment ratio remains the same as it was before the introduction of the second order delay for the relative wage. The previous three equations result from the transformation of the mentioned integro-differential equation.

Equations (1.11) and (1.12) include new variables representing the cascading (second-order) delay in relative wage (z relative to u and y relative to z). In these equations the rate parameter b of the distribution density function of the Erlang-2 distribution takes on added significance: it is the adaptation parameter determining the speed of adjustment.[3] Thus, this parameter besides the probabilistic meaning gains an important dynamic characteristic.

A positive stationary state of the system (1.11–1.14) is defined as

$$E_a = (z_a, y_a, u_a, v_a), \tag{1.15}$$

where $z_a = y_a = u_a = 1 - \dfrac{sd}{k}$, $v_a = v_G - \dfrac{e}{k}\dfrac{d}{r}$.

Here a growth rate of output per worker and growth rate of wage equals h. A growth rate of fixed capital and net output is $\hat{K}_a = \hat{P}_a = d = h + n$, $d \geq h$. A stationary rate of surplus value is $m_a' = (1 - u_a)/u_a$. A stationary profit rate is $(1 - u_a)/s_a = d/k$. Economic restrictions on the model parameters are presented in Table 1.2. It should be noted that even for $k = 1$, the boundary T_3 is not mentioned in (Fanti and Manfredi, 1998).

Properties of the stationary state

For the same parameters, the stationary employment ratio is lower in FM than in GM: $v_a < v_G = (g + h)/r$. The relative decline of stationary employment ratio after onset of the stabilisation policy is $\dfrac{v_a - v_G}{v_G} = -\dfrac{e}{k}\dfrac{d}{g+h}$ in agreement with definition (1.15). This has other negative consequences (see the end of section 1).

The profit sharing rule does not alter the stationary relative wage $u_a = u_G$. Other stationary magnitudes (ratios and growth rates) also coincide. There is an important statement in the analysed paper (Fanti and Manfredi, 1998: 381): "Proposition [1.1]: the existence of a profit-sharing rule within Goodwin-type economies does not modify the long term distribution (that is: every short term effort of workers aimed to gain either higher wages or a greater power of the workers as a class, as represented by a higher V [u – as denoted in this paper], will be useless) but reduces the equilibrium level of the employment rate."[4]

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[3] In the Erlang-2 distribution the shape parameter of the corresponding probability density function equals 2.

[4] The same proposition is in the paper (Fanti, 2003: 14, 24) based on a similar model with a different profit sharing rule. Our critique of the model from the paper (Fanti and Manfredi, 1998) extends basically on the latter model that is, regrettably, closer to the «neoclassical» paradigm than the previous one being analyzed.

It's easy to see the increase in the exogenous growth rate of output per worker is not favourable for stationary relative wage but it is favourable for stationary employment ratio (for $e < k \le 1$): $\frac{\partial u_a}{\partial h} = -\frac{s}{k} < 0$, $\frac{\partial v_a}{\partial h} = \frac{k-e}{rk} > 0$ since $e < k$ (see Table 1.2). The higher growth rate of labour force is detrimental for stationary employment ratio $\frac{\partial v_a}{\partial n} = -\frac{e}{kr} < 0$ and for stationary relative wage $\frac{\partial u_a}{\partial n} = -\frac{s}{k} < 0$.

Table 1.2. Economically determined restrictions on FM parameters

Economic requirement	Restriction
$1 > v_G > 0$	$r > g + h > 0$
$1 > u_a > 0$	$sd < k \le 1$
$1 > v_a > 0$	$e < kT_2 = k\frac{h+g}{h+n}$
$\hat{w}_a^m = h - \frac{e}{k}d > 0$	$e < kT_3 = \frac{kh}{h+n} \le k < kT_2$

Let's pay also attention to positive dependence of the stationary relative wage and employment ratio on the rate of accumulation: $\frac{\partial u_a}{\partial k} = \frac{sd}{k^2} > 0$, $\frac{\partial v_a}{\partial k} = \frac{ed}{rk^2} > 0$. Table 1.3 shows additional properties of the stationary state in FM.

Table 1.3. Properties of the stationary growth rates of wage and its components

Indicator	Restriction or property
$\frac{\partial \hat{w}_a^m}{\partial e}$	$-\frac{d}{k} < 0$
$\hat{w}_a^b$	$\frac{e}{k}d > 0$
$\frac{\partial \hat{w}_a^b}{\partial e}$	$\frac{d}{k} > 0$

For the stationary state (1.15) equation (1.16) defines the Jacoby matrix.

$$J(E_a) = \begin{array}{|c|c|c|c|} \hline -b & 0 & b & 0 \\ \hline b & -b & 0 & 0 \\ \hline 0 & -emu_a & 0 & ru_a \\ \hline 0 & 0 & -kmv_a & 0 \\ \hline \end{array} \qquad (1.16)$$

This matrix will be compared with the similar matrices for AM and modified AM for determining conditions of their local equivalence in the linear approximation in Sections 2.2 and 3.

Using the Liénard – Chipart criterion, the authors proved analytically for $k = 1$ (Fanti and Manfredi, 1998: 388–389) the following mathematical statement which is reinforced by this paper for the more general case of $sd < k \leq 1$.

Proposition 1.2. Let the positive steady state E_a exists, then it is asymptotically locally stable if the principal minor of third order in the Hurwitz matrix is positive

$$\Delta_3 = b^3 emu_a(2b^2 - bmeu_a - 2mrv_a u_a k) > 0. \tag{1.17}$$

1.3. Andronov – Hopf bifurcation

Proposition 1.3. When a magnitude of the control parameter b becomes critical, inequality (1.17) turns into equity, formerly steady state E_a loses stability and a closed orbit is born as a result of a simple Andronov – Hopf bifurcation.[5] A mathematical proof of the leading Proposition 1.3 applies the results from (Liu 1994).

Proposition 1.4. The critical magnitude is

$$b_h = \frac{eu_a m + \sqrt{(eu_a m)^2 + 16 r v_a u_a k m}}{4}. \tag{1.18}$$

Proposition 1.5. The approximation for the period of a closed orbit is

$$T_{LC} = \frac{2\pi}{b_h} = \frac{2\pi}{\frac{eu_a m}{4} + \sqrt{\left(\frac{eu_a m}{4}\right)^2 + r v_a u_a k m}}. \tag{1.19}$$

Given the delayed relation between net change of relative wage and profit rate, the stationary state E_a is locally asymptotically stable provided that the average delay for profit rate is sufficiently small, that is, if $b > b_h$. The stationary state E_a is undergoing a simple Andronov – Hopf bifurcation if $b = b_h$ as defined earlier.

The average delay of second order $T_h = 2/b_h$, corresponding to the critical value of the bifurcation parameter (b_h), called a critical lag. The expression of critical lag and the closed orbit's period differ only by a constant in the numerator (2 and 2π, respectively). The reciprocal of the critical lag ($b_h/2$) represents one half of the angular frequency (b_h).

[5] The literature on applications of Andronov – Hopf bifurcation in economic modeling is fast growing and cannot be fully reviewed here. Few references must suffice. Brøns and Sturis (1991), Lordon (1995), Ryzhenkov (2008, 2009, 2012) applied nonlinear Hopf bifurcation in analysis of the economic long waves and other fluctuations in models reduced to two- and three-dimensional systems of nonlinear ordinary differential equations. The additional contributions deserve examination beyond the limited scope of the present paper.

I complement the above assertions with next important propositions absent in the original paper. Proposition 1.6. The period of closed orbit is shorter than period of conservative fluctuations in GM under the same constellation of common parameters (including the rate of accumulation, *k*) $T_{LC} < T_G = 2\pi/\sqrt{kmv_G ru_a}$. It is proved by comparison.

Proposition 1.7. If the rate of accumulation (*k*) increases, the critical lag and period of a cycle are reduced, respectively. It is proved by calculating the first partial derivatives of T_h and T_{LC} with respect to *k*.

Importance of accumulation rate in determining cyclical dynamics is emphasised by Figure 1.2.

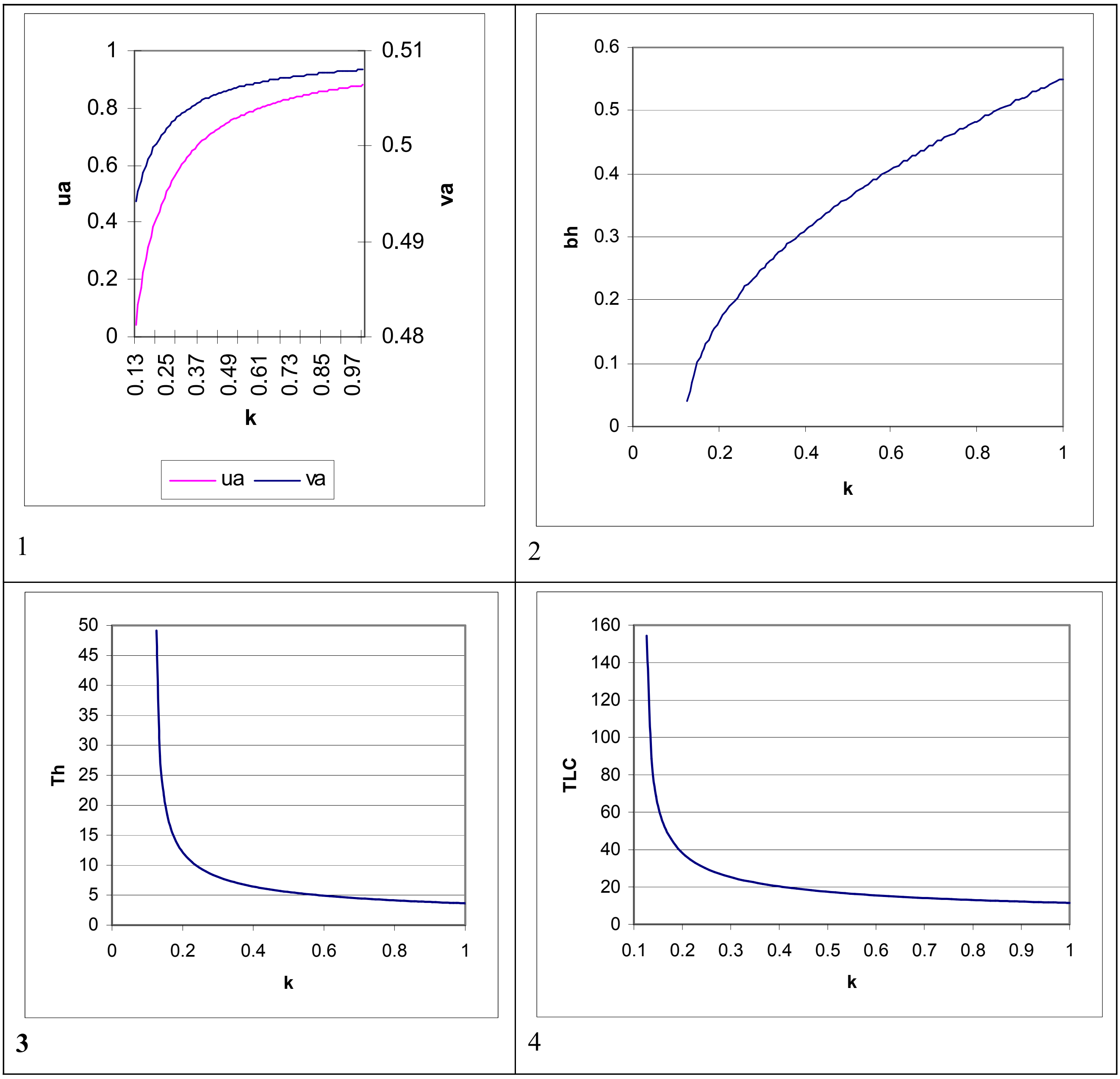

Figure 1.2. Dependence of main characteristics of cyclical dynamics on accumulation rate *k*:
Panel 1 – stationary relative wage u_a and employment ratio v_a, Panel 2 – critical magnitude of adjustment parameter b_h, 3 – critical delay T_h, 4 – approximate period of closed orbit T_{LC}

For a more realistic rate of accumulation (about 12.62–30 per cent of profit, as opposed to 100 per cent in the original FM), critical lag is too high being in an economically unrealistic range (about 8 – 49.12 years) when magnitudes of the other parameters remain the same as basal.

Simulation runs based on the FM equations (1.1) – (1.10) apply the following basal magnitudes from the original source: $m = 0.33$, $v_a \approx 0.5080$, $u_a \approx 0.8788$, $g = 1$, $k = 1$, $e = 0.1$, $h = 0.02$, $n = 0.02$, $r = 2$, that yield, according to my calculations, following estimates: $b_h \approx 0.5501$, $T_h = 3.635$, $T_{LC} = 11.42 < T_G = 11.55$ for $v_G = 0.51$.

Proposition 1.8. The claim is unwarranted that the trajectories, in result of the simple Andronov – Hopf bifurcation at E_a, approach a unique (stable) limit cycle for a very wide set of initial values (Fanti and Manfredi, 1998: 392–393).[6]

Simulation experiments maintained by *Vensim* reveal that a closed orbit in result of a simple Andronov – Hopf bifurcation at E_a is not unique and is not globally asymptotically stable, some of the closed orbits, formed depending on the initial conditions, represent limit cycles for their respective basins of attraction with a limited scope.

Let us look at the left panel of Figure 1.3. The first year is denoted as 1958 for a pure illustrative purpose; this has no practical connotation.

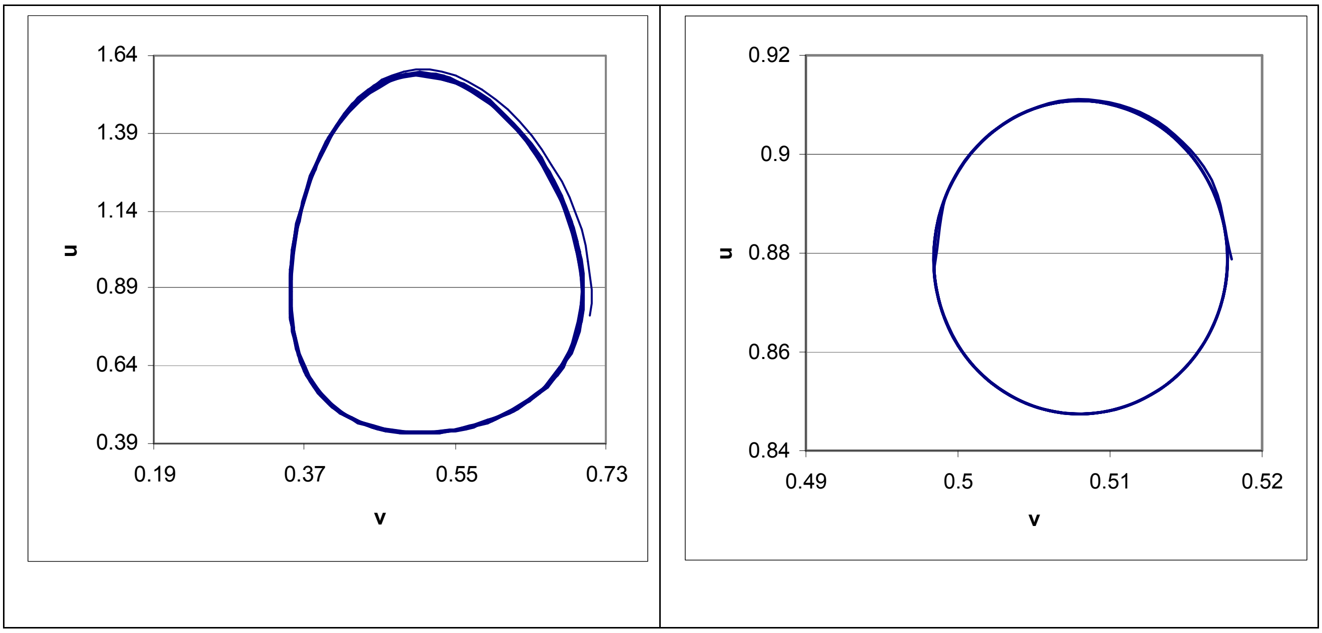

Figure 1.3. Closed orbits planar projections, counter clock-wise, 1958–2158: on the left, my simulation of convergence to the inappropriate "limit cycle" suggested in the original source (Fanti and Manfredi, 1998: 392–393); on the right, an alternative simulation of convergence to closed orbit in economic region (initial conditions, respectively, $v_0 = 0.71$, $u_0 = 0.8$, $z_0 = 0.95$, $y_0 = 0.99$; $v_0 = 0.5180$, $u_0 = z_0 = y_0 = 0.8788$; all parameters magnitudes are the same: $e = 0.1$, $g = 1$, $h = 0.02$, $k = 1$, $m = 0.33$, $n = 0.02$, $r = 2$, $b_h = 0.5501$, $T_{LC} = 11.42 < T_G = 11.55$)

[6] The authors write (Fanti and Manfredi, 1998: 392): "This phenomenon, convergence to a unique (stable) limit cycle, was actually found for a very wide set of initial values. This leads us to conjecture that the involved periodic orbit be also globally asymptotically stable rather than simply local."

We see the candidature offered in the paper (Fanti and Manfredi, 1998) as the limit cycle goes far beyond economic region (the relative wage for almost one half of the cycle exceeds one). Accordingly, the reasoning about a very broad basin of attraction of this “limit cycle” is economically shallow. Now turn our attention to the right panel. It reflects a quite different closed orbit generated via a similar bifurcation and existing within the economic region for the given initial magnitudes of the phase variables.

Finally, the above conclusion (Proposition 1.1) that the long run distribution is left inalterable in FM compared with GM while the employment ratio is reduced, requires refinement: the long run distribution is left inalterable only in *relative* terms! As the proposed stabilisation policy reduces long run employment ratio (v) of steady growing labour force (N), the employment (L), output (P), surplus value (S/a), total wage (wL), consumption per head (wv) and profit (M) are, as a rule, lower that they would be in GM. This policy worsens reproduction and use of economic (first of all – labour) potential in the long term and typically even in the middle term. In particular, the higher profit sharing index e, the lower are the long term and usually even middle term output and employment.[7]

2. Stabilising capital accumulation

The present paper adhered to the Marxian economic theory accepts the term profit sharing for the following reasons. First, it expresses an objective contradictory form of socio-economic consciousness similar to wage. The latter is basically the transformed form of the value of labour power commonly perceived as the price of labour. Second, profit is the money form of surplus value created by labourers still it appears as the result of the advanced capital as a whole in the common perception. Third, profit sharing means that the labourers may appropriate a certain part of net output created by them as a part of wages that would be otherwise distributed to capitalists in the form of profit.

This section is based on a simplified version of the models of cyclic dynamics built for the economy of the United States (Ryzhenkov, 2005, 2007 and 2010). In essence, this (and next) section presents a thought experiment at the same level of abstraction, as chosen by the opponents. Similar to their undertaken, a simplified version of that model for the US is complicated by adding the second order delay in net change of relative wage relative to profit rate for the same economic reasons. The alternative model (AM) demonstrates how to alleviate accumulation cycles attaining much higher employment and better use of economic potential than in FM.

[7] Notice that for a particular closed orbit in GM, the average magnitudes of relative wage and employment ratio are practically the same as their stationary counterparts. The same attribute is typical for FM not only for a particular closed orbit but even for a usual transient to this orbit. Therefore my comparison of the stationary magnitudes (instead of the average ones) in GM and FM is correct due to weak non-linearity of both.

2.1. The extensive deterministic form of the alternative model

According to the AM key assumption, owners of capital, state officials under pressure of workers' parties, trade-unions and grass-root organisations set a target growth rate of profit depending on the difference between the indicated (X_1) and current (v) employment ratios:

$$\hat{M} = -\frac{\dot{u}}{1-u} + \hat{K} = c_2(X_1 - v), \tag{2.1}$$

where $c_2 > 0$, $v < X_1 = X + \frac{d}{c_2}$, X denotes a target employment ratio, d is a stationary economic growth rate as in FM.[8] This policy rule with a great margin of safety stabilises capital accumulation even being fuzzier for stretched "humped" delays as the reader will see soon.

An equation for net change of relative wage follows from equation (2.1)

$$\dot{u} = [\hat{K} + c_2(v - X_1)](1-u). \tag{2.2}$$

After the second order delay has entered into action, the latter equation is modified into

$$\dot{u} = \left[\frac{k(1-y)}{s} + c_2(v - X_1)\right]\frac{1-y}{y}u. \tag{2.3}$$

An equation for the growth rate of wage is derived from equation (2.3):

$$\hat{w} = \left[\frac{k(1-y)}{s} + c_2(v - X_1)\right]\frac{1-y}{y} + h. \tag{2.4}$$

The growth rate of total wage is the sum of bargained and profit sharing terms

$$\hat{w} = \hat{w}^m + \hat{w}^b, \tag{2.5}$$

where the first term is determined by the employment ratio (v) and by the delayed rate of surplus value $\left(\frac{1-y}{y}\right)$

$$\hat{w}^m = c_2(v - X_1)\frac{1-y}{y} + c_1, \tag{2.6}$$

the second term is determined by the growth rate of output per worker (h) and by relative wage delayed twice (y)

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[8] This presentation is not complete since changes in the wage-setting and other relevant institutions implied by supposed closed-loop control over capital accumulation as a whole are not discussed. In particular, it is not yet clear, first, whether such a closed-loop control is to be achieved through coercive and/or voluntary cooperation; second, what arrangement of coincidence, coercion and co-adjustment is mostly suited to provide superior social outcomes. The paper Ryzhenkov (2007) gives a preliminary answer to these complicated questions. It introduces excess labour compensation levy and subsidy on pre-levy primary profit. For simplicity, it is the state that can levy surcharges on excessive income of labourers (or capitalists) and pay equivalent subsidies to capitalists (or labourers). The levy year and base are sliding. A socially desirable transition to a target employment ratio, profit enhancement and an extending of proved reserves can be realised through excess income levy as that paper demonstrates.

$$\hat{w}^b = h + \frac{k(1-y)}{s}\frac{1-y}{y} - c_1 . \tag{2.7}$$

For certainty (in the light of the economic requirements) $c_1 = dm_b' + h/2$ where the stationary rate of surplus value is defined below. The remaining equations coincide with equations (1.1) – (1.7), (1.9) and (1.10) in FM.

Figure 2.1 and Table 2.1 display a causal loop structure of AM. Loop B1 is inherited from the GM and FM; loop B2 is due to the profit sharing rule like in FM. Besides these, AM includes, quite similar to FM, two minor negative feedback loops for delayed relative wage *z*, for relative wage delayed twice *y*, and two minor feedback loops with alternating polarity for relative wage *u* and for employment ratio *v*.

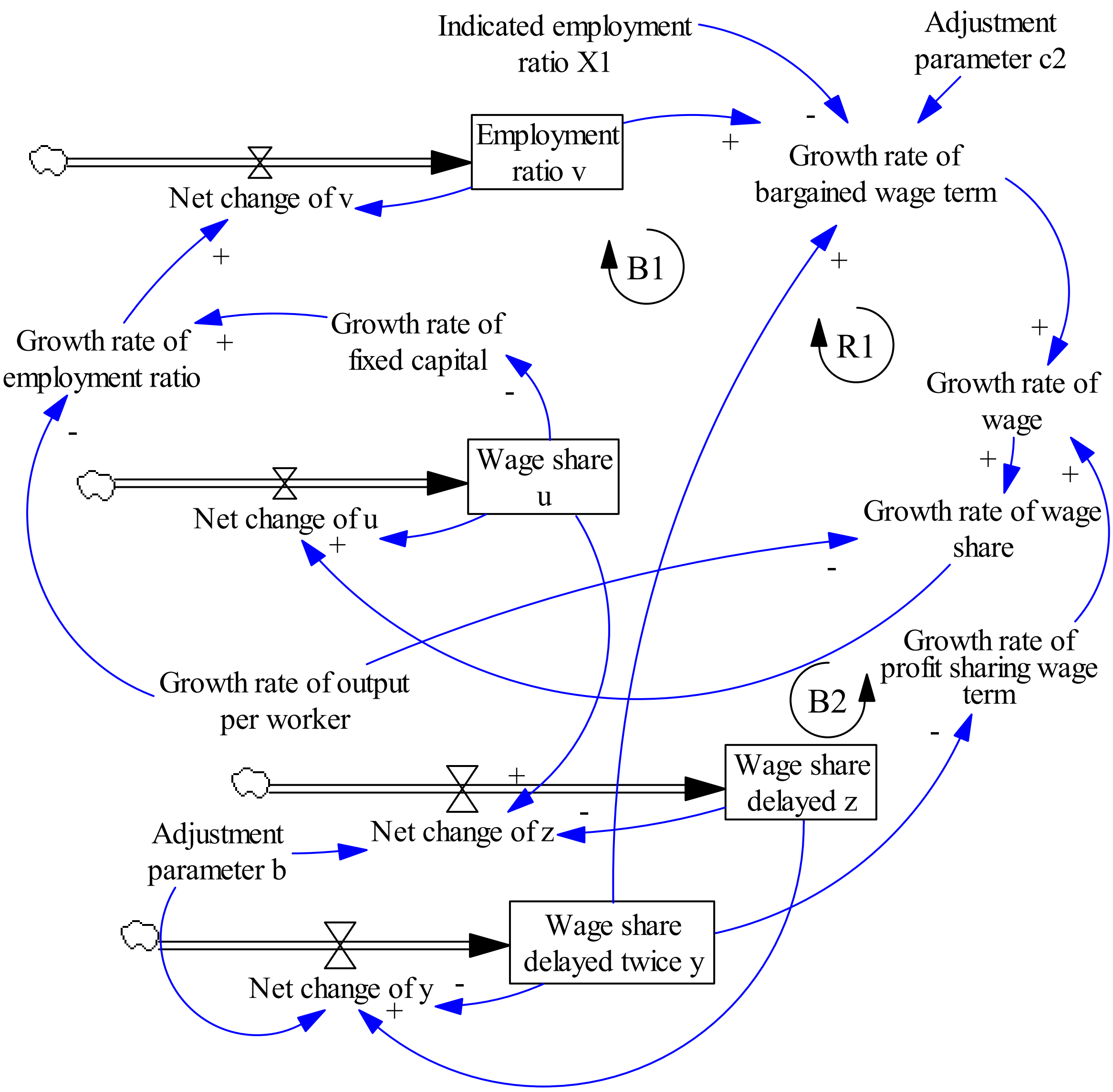

Figure 2.1. A causal structure of AM with second order delay in relative wage

Table 2.1. Two main negative and one positive feedback loops in AM

Loop B1 of length 8 – negative	Loop B2 of length 8 – negative	Loop R1of length 8 – positive
Wage share $u \xrightarrow{-}$ Growth rate of fixed capital Growth rate of employment ratio Net change of v Employment ratio v Growth rate of bargained wage term Growth rate of wage Growth rate of wage share Net change of u	Wage share u Net change of z Wage share delayed z Net change of y Wage share delayed twice $y \xrightarrow{-}$ Growth rate of profit sharing wage term Growth rate of wage Growth rate of wage share Net change of u	Wage share u Net change of z Wage share delayed z Net change of y Wage share delayed twice y Growth rate of bargained wage term Growth rate of wage Growth rate of wage share Net change of u

Note. Only a negative first partial derivative is explicitly shown as an arrow. All other first partial derivatives are positive.

The completely new structural feature of AM, compared with both preceding models (FM and GM), is positive feedback loop R1 that helps to stabilise the growing system. AM contains positive feedback loop R1 since the bargained wage term positively depends on delayed relative wage.

The inquiry digs infinitely deeper in Table 2.2. It highlights important qualitative differences between FM and AM not visible on these causal-loop diagrams that unintentionally conceal subtle properties reflected by the first, second and higher order partial derivatives.

In FM, all four first order derivatives of both wage terms are constant. Three first order derivatives of wage terms in AM, unlike their counterparts in FM, depend on relative wage delayed twice, and one of them – on employment ratio additionally. Besides that, both bargained wage term and profit sharing wage term have partial derivatives of higher orders (from two to infinity) with respect to delayed relative wage (with alternating polarity) in AM, unlike FM.

D. Ricardo, the great classical economist, discovered importance of relative wage, his outstanding successor K. Marx exposed the essence of the rate of surplus value. Both would be surprised that all powers of relative wage (from one to infinity), that enter into derivatives of the rate of surplus value and consequently into partial derivatives of the wage growth terms (Table 2.2), are significant determinants of capitalist reproduction as a whole.

After these detailed comparisons, it is crystal clear that non-linearity in AM is much stronger than in FM. Therefore their local equivalence at the same stationary state in linear approximation is rather restricted and it is not supplemented by same qualitative dynamic properties at a broader scale (see Section 2.2). This higher degree of non-linearity in AM allows a socially beneficial transition to a distant fixed point attractor in the phase space as demonstrated below.

Table 2.2. Qualitative and quantitative differences between FM and AM

Attribute	FM	AM
First partial derivative of bargained wage term with respect to relative wage delayed twice $\frac{\partial \hat{w}^m}{\partial y}$	0	$-c_2(v-X_1)\frac{1}{y^2}>0$ for $v<X_1$
Second partial derivative of bargained wage term with respect to relative wage delayed twice $\frac{\partial^2 \hat{w}^m}{\partial y^2}$	0	$2c_2(v-X_1)\frac{1}{y^3}<0$ for $v<X_1$
Higher order partial derivative of bargained wage term with respect to relative wage delayed twice $\frac{\partial^i \hat{w}^m}{\partial y^i}$, $i = 3, 4, ...$	0	Non zero for $v<X_1$ with alternating sign $(-1)^i \frac{i!}{y^{i+1}} c_2(v-X_1)$ and with ever growing absolute magnitude (for $i \to \infty$)
First partial derivative of bargained wage term with respect to employment ratio $\frac{\partial \hat{w}^m}{\partial v}$	$r>0$	$c_2\frac{1-y}{y}>0$
First partial derivative of profit sharing wage term with respect to relative wage delayed twice $\frac{\partial \hat{w}^b}{\partial y}$	$-\frac{e}{s}<0$	$-\frac{k}{s}\frac{(1-y^2)}{y^2}<0$
Second partial derivative of profit sharing wage term with respect to relative wage delayed twice $\frac{\partial^2 \hat{w}^b}{\partial y^2}$	0	$2\frac{k}{s}\frac{1}{y^3}>0$
Higher order partial derivative of profit sharing wage term with respect to relative wage delayed twice $\frac{\partial^i \hat{w}^b}{\partial y^i}$, $i = 3, 4, ...$	0	Non zero with alternating sign $(-1)^i \frac{i!}{y^{i+1}} \frac{k}{s}$ and with ever growing absolute magnitude for $i \to \infty$

2.2. The intensive form and properties of its stationary state

An intensive deterministic form of AM includes three equations of intensive form of FM (1.11) (1.12), (1.14) and replaces its equation (1.13) with equation (2.3). A positive stationary state in AM is defined as

$$E_X = (z_b, y_b, u_b, X), \tag{2.8}$$

where $z_b = y_b = u_b = u_a$, $v_b = X = X_1 - \frac{d}{c_2}$, $m_b' = m_a'$.

The control parameter c_2 in AM has a higher degree of freedom than parameter e in FM. As a rule, we set $X_1 > X > v_a$. The stationary bargained and profit sharing wage terms are equal: $\hat{w}_b^m =$

$\hat{w}_b^b = h/2 > 0$. The stationary growth rates of labour force, employment, output per worker, capital intensity, net output, fixed capital, wage, profit and surplus value are the same as in FM. Still there are significant differences between the stationary states laid bare in Table 2.3.

Table 2.3 shows, in plain words, that FM may offer several excuses or complaints for eroding social objective: achieving high employment ratio did not happen for particular reason or combination of elementary reasons (lower growth rate of output per worker, or lower rate of accumulation, or higher profit sharing index, or higher growth rate of labour force as expected). Quite differently, AM firmly resists against eroding social objective (X) and adjusts indicated employment ratio (X_1) to external influences appropriately.

Table 2.3. Qualitative and quantitative differences between FM and AM stationary states

Exogenous parameter	First partial derivative	
	FM	AM
Growth rate of output per worker	$\frac{\partial v_a}{\partial h} > 0$	$\frac{\partial X}{\partial h} = 0$, $\frac{\partial X_1}{\partial h} > 0$
Growth rate of labour force	$\frac{\partial v_a}{\partial n} < 0$	$\frac{\partial X}{\partial n} = 0$, $\frac{\partial X_1}{\partial n} > 0$
Rate of accumulation	$\frac{\partial v_a}{\partial k} > 0$	$\frac{\partial X}{\partial k} = 0$, $\frac{\partial X_1}{\partial k} = 0$
Profit sharing index	$\frac{\partial v_a}{\partial e} < 0$	...
Control parameter	...	$\frac{\partial X}{\partial c_2} = 0$, $\frac{\partial X_1}{\partial c_2} < 0$

For the stationary state (2.8) equation (2.9) defines a Jacoby matrix.

$$J(E_X) = \begin{array}{|c|c|c|c|} \hline -b & 0 & b & 0 \\ \hline b & -b & 0 & 0 \\ \hline 0 & -d & 0 & c_2(1-u_b) \\ \hline 0 & 0 & -\frac{k}{s}X & 0 \\ \hline \end{array} \qquad (2.9)$$

Comparison of Jacobi matrices (1.16) and (2.9) allows for finding the terms and conditions of FM – AM local equivalence near the same stationary state in linear approximation (Table 2.4). Full local equivalence does not take place, because, as we already know, non-linearity is expressed in AM stronger than in FM. Similar to GM and FM, AM (also constructed with exogenous capital-output ratio and exogenous accumulation rate) does not pass the extreme condition test not permitting $u_b > 0$ for $k \leq sd$.

Table 2.4. Conditions of FM – AM local equivalence in linear approximation

FM – prototype for AM	AM – prototype for FM
$1 > X_1 = v_G > 0$, $1 > X = v_a > 0$, $c_2 = \dfrac{r}{m_a'}$, $e = km_a'$, $c_1 = h$	$v_a = X$, $e = km_a'$, $r = c_2 m_a'$, $c_1 = h$, $v_G = X_1$ or consistently $g = m_a'(d + c_2 X) - h$

Notice that under restricted local equivalence the stationary growth rate of profit sharing term is $\hat{w}_a^{\,b} = \hat{w}_b^{\,b} = e\dfrac{1-u_a}{s} = km_a'\dfrac{1-u_a}{s}$. The multipliers are three fundamental concepts in "*Das Kapital*" of Karl Marx: rate of accumulation, rate of surplus value and profit rate! Now this scalar product has gotten a clear economic rationale.

The characteristic equation related to the matrix $J(E_X)$ is written as

$$a_0\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0. \tag{2.10}$$

Using the Liénard – Chipart criterion, the conditions of asymptotic local stability are determined after routine calculations (Table 2.5). The parameter c_2 is selected as control parameter.

Table 2.5. Concretisation of Liénard – Chipart criterion for E_X in AM

$a_0 = 1$
$\Delta_1 = a_1 = 2b > 0$
$a_2 = b^2 + c_2 dX > 0$
$a_3 = 2c_2 dXb + db^2 > 0$
$a_4 = c_2 X d b^2 > 0$
$\Delta_3 = a_1a_2a_3 - a_0a_3^2 + a_1^2a_4 = db^3(2b^2 - db - 2c_2dX) > 0$, if inequality (2.11) is satisfied

Proposition 2.1. Let the positive stationary state E_X (2.8) exists, then it is asymptotically locally stable if the principal minor of third order in the Hurwitz matrix is positive ($\Delta_3 > 0$), when a magnitude of the control parameter is positioned within limits

$$0 < c_2 < \frac{b(2b-d)}{2dX}. \tag{2.11}$$

Proposition 2.2. The positive stationary state E_X is not stable any more if $\Delta_3 \le 0$ for $c_2 \ge \dfrac{b(2b-d)}{2dX}$.

The information in Table 2.5 clearly contains all the necessary elements proving both propositions.

Proposition 2.3. When a magnitude of the control parameter b becomes critical the previous inequality turns into equality, the stationary state E_X (2.8) loses stability and a closed orbit is born as a result of a simple Andronov – Hopf bifurcation.

Proposition 2.4. This critical magnitude is

$$b_1 = \frac{d + \sqrt{d^2 + 16c_2 dX}}{4}. \qquad (2.12)$$

Proposition 2.5. The approximation for the period of a closed orbit is

$$Tc = \frac{2\pi}{b_1} = \frac{2\pi}{\frac{d}{4} + \sqrt{c_2 dX + \left(\frac{d}{4}\right)^2}}. \qquad (2.13)$$

These both (b_1 и T_c) do not depend on a particular magnitude of the rate of accumulation (k).

Proposition 2.3 can be proved with the knowledge about FM–AM restricted local equivalence or without recourse to it. A proof is omitted because its mirrors the similar proof in (Fanti and Manfredi, 1998), with the difference that instead of $k = 1$ more general relation $sd < k \leq 1$ holds so all original relations are generalised.

Uniqueness of a limit cycle as result of a simple Andronov – Hopf bifurcation for this model (AM) has been neither proved nor even supposed. Self-sustained oscillations in AM do not have economic interest when they happen since their period is much greater than that of a medium-term economic cycle. The very existence of the latter is not causally related to any simple Andronov – Hopf bifurcation in this model.

Finally, let's formulate the decisive conclusion. It relates to a FM originally unintended role – to provide an extreme condition test for AM.

Proposition 2.6. The second order delay of relative wage, equal to the critical lag for FM ($T_h = 2/b_h$), poses no threat to stability of stationary state E_X in AM if a magnitude of the control parameter c_2 is selected within (non-empty) interval (2.14).

Proof. The requirements of Liénard – Chipart criterion (Table 2.5) are satisfied for T_h, with the possible exception of requirement $\Delta_3 = db_h^{\,3}(2b_h^{\,2} - db_h - 2c_2 dX) > 0$. However, to meet this requirement it is sufficient to select the value of $c_2 = c_2(b_h)$ from a nonempty interval

$$0 < c_2(b_h) < \frac{b_h(2b_h - d)}{2dX}. \qquad (2.14)$$

The possibility of such a choice is guaranteed by the fact that $b_h >> d/2$ that completes the proof.

A satisfactory magnitude of c_2 should not only allow for smooth transition to an immediate local area of E_X but also guarantee dynamics of model variables in the economic space. For the baseline values of relevant parameters of FM, the main difficulty is satisfying the requirement $v < 1$, especially for $v_0 << X$ as in our case.

A specific magnitude of c_2 is to be found for the established selection area (2.14) as a solution to a parametric optimization problem according to the criterion of minimum integral of the absolute deviations of v from X, taking into account the initial values of variables. This optimization criterion is extended by a penalty function, the values of which are rapidly increasing whenever variable v exceeds X. The solution found is consistent with the requirement $v < 1$ throughout the transitional period without a single violation.

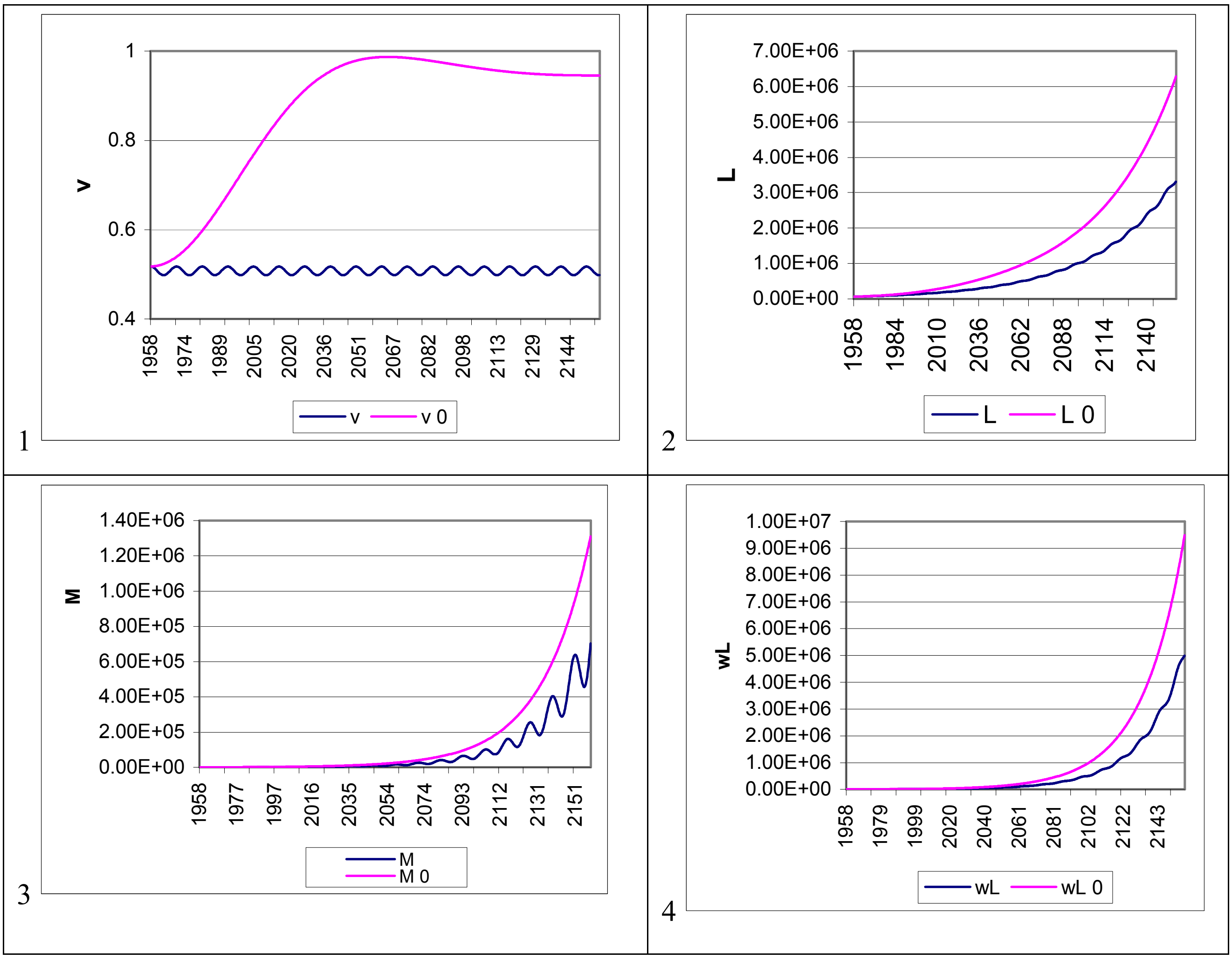

Figure 2.2. Better use of labour force (v 0, L 0) and higher profit and total wage (M 0, wL 0) in AM than in FM, respectively, (v, L) and (M, wL), 1958–2158

For the basic magnitudes of the parameters and particular initial values of the phase variables ($u_0 = z_0 = y_0 = 0.9087$, $v_0 = 0.518 < X = 0.95$), using the critical values of $b = b_h \approx 0.5501$ from FM, the optimization period covered 64 years (1958 – 2021) with extrapolating over 2022–2158. As the solution of parametrical optimization problem for AM, the best magnitude of the key control parameter is found: $c_2 = 0.0381$. This magnitude is possibly not truly optimal being sub-optimal (still satisfactory for AM compared with FM).

Figure 2.2 illustrates a much better use of the labour potential in AM than in FM, judging from indicators of employment ratio (v) and the number of persons employed (L). Because output per worker is the same in both models at any moment of time, the advantage of AM in net output vis-à-vis FM becomes more and more significant over the years. AM has comparative advantages in unit

(w), and especially in the total wage (wL). The basic cycle, with fairly constant amplitude and period of about 11.4 years, generated in FM, is virtually disappeared in AM.

An abrupt drop of the rate of accumulation can be destroyer for this stabilisation policy. So models with endogenous rate of accumulation are required. Papers of Lordon (1995), Ryzhenkov (2008, 2010, 2012) offered such Goodwinian models.

3. A reinforced stabilisation policy

Decades are required for transiting in AM from v_0 =0.518 in 1958 to v = 0.95 in 2036 and v = 0.9857 in 2058 under the stabilisation policy in the preceding section. A reinforced stabilisation policy enables substantial reduction of a transient period to a benchmark compared with the previously described policy. Still we move back for a while.

FM

An equivalent for equation (1.13) takes the form of combined proportional control over the net change of relative wage defined by deviations of employment ratio and delayed profit rate from their stationary magnitudes

$$\dot{u} = r(v - v_a)u + e\left(\frac{1-y}{s} - \frac{d}{k}\right)u \;. \tag{3-1.13}$$

The equations for two terms of the growth rate of wage can be equivalently presented as manifestation of combined proportional control in the respective elementary forms

$$\hat{w}^m = r(v - v_a) + h - e\frac{d}{k}\,, \tag{3-1.8a}$$

$$\hat{w}^b = e\left(\frac{1-y}{s} - \frac{d}{k}\right) + e\frac{d}{k} \;. \tag{3-1.8b}$$

Modified AM

An equivalent for equation (2.2) also takes the form of combined proportional control over the net change of relative wage defined by deviations of employment ratio and profit rate from their stationary magnitudes

$$\dot{u} = c_2(v - X)(1-u) + k\left(\frac{1-u}{s} - \frac{d}{k}\right)(1-u) \;. \tag{3-2.2}$$

Missed is an appropriate multiplier at the second term. The *reinforced* stabilisation policy modifies the latter equation by adding such an element

$$\dot{u} = c_2(v - X)(1-u) + qk\left(\frac{1-u}{s} - \frac{d}{k}\right)(1-u)\,, \tag{3-2.3}$$

where $q \geq 1$. In equations (2.2) and (3-2.2) $q = 1$ implicitly like $k = 1$ in the original FM.

After the second order delay has entered into action, the latter equation is transformed into

$$\dot{u} = \left[c_2(v - X) + qk\left(\frac{1-y}{s} - \frac{d}{k} \right) \right] \frac{1-y}{y} u \,. \tag{3-2.4}$$

An equation for the growth rate of wage is derived from equation (3-2.4):

$$\hat{w} = \left[c_2(v - X) + qk\left(\frac{1-y}{s} - \frac{d}{k} \right) \right] \frac{1-y}{y} + h. \tag{3-2.5}$$

Correspondingly the two terms of the wage growth rate are presented as manifestation of combined proportional control in the respective elementary forms

$$\hat{w}^m = c_2(v - X)\frac{1-y}{y} + c_1, \tag{3-2.6}$$

$$\hat{w}^b = h + qk\left(\frac{1-y}{s} - \frac{d}{k} \right)\frac{1-y}{y} - c_1, \tag{3-2.7}$$

where c_1 = const > 0 can be specified, for example, as $c_1 = h/2$, then $\hat{w}_b^m = \hat{w}_b^b = h/2$ as implied in definition of E_X (2.8) initially (although for c_1 defined differently).

These modifications produce no impact on the stationary state $E_X = (z_b, y_b, u_b, X)$. The Jacoby matrix for E_X does not remain the same: the new element J_{32} equals the former J_{32} multiplied by new parameter q: $J_{32}^{new} = -qd$, whereas the other elements remain the same as before.

A new characteristic equation for the new Jacoby matrix remains the same as the equation (2.10). Still the parameter a_3 of this characteristic equation and the principal minor of the third order Δ_3 in the corresponding Hurwitz matrix are modified (Table 3.1).

One particular requirement of restricted local equivalence from Table 2.4 is also modified: its more general form is $e = qkm_a'$ now. Similar to GM, FM and AM, this modified AM (with exogenous capital-output ratio and accumulation rate) also does not pass the extreme condition test not allowing $u_b > 0$ for $k \le sd$. For curing this drawback an equation for the net change of employment ratio has been revised in the author's more advanced models mentioned above.

Proposition 3-2.1. Let the positive stationary state E_X (2.8) exists, then it is asymptotically locally stable if the principal minor of third order in Hurwitz matrix is positive ($\Delta_3 > 0$) when a magnitude of the control parameter is located within the interval

$$0 < c_2 < \frac{b(2b - qd)}{2dX} \tag{3-2.11}$$

for $2b/d > q > 1$.The interval shrinks compared with the initial one for $2b/d > 1$ in AM.

Proposition 3-2.2. The positive stationary state E_X is not stable any more if $\Delta_3 \le 0$ for $c_2 \ge \dfrac{b(2b - qd)}{2dX}$.

The information in Table 3.1 clearly contains all the necessary elements of evidence for both propositions. A proof of the following one is more difficult. It is omitted for the same reasons as before.

Table 3.1. Concretisation of Liénard – Chipart criterion for E_X in modified AM

$a_0 = 1$
$\Delta_1 = a_1 = 2b > 0$
$a_2 = b^2 + c_2 dX > 0$
$a_3 = 2c_2 dXb + qdb^2 > 0$
$a_4 = c_2 Xdb^2 > 0$
$\Delta_3 = a_1 a_2 a_3 - a_0 a_3^2 + a_1^2 a_4 = qdb^3(2b^2 - qdb - 2c_2 dX) > 0$, if inequality (3-2.11) is satisfied

Proposition 3-2.3. When a magnitude of the control parameter b becomes critical the previous inequality $\Delta_3 > 0$ turns into equality, the stationary state E_X (2.8) loses stability and a closed orbit is born as a result of a simple Andronov – Hopf bifurcation.

Proposition 3-2.4. This critical magnitude is

$$b_1 = \frac{qd + \sqrt{(qd)^2 + 16c_2 dX}}{4}. \tag{3-2.12}$$

The critical magnitude becomes higher for $q > 1$ than the initial one in AM for the same c_2.

Proposition 3-2.5. The approximation for the period of a closed orbit is

$$Tc = \frac{2\pi}{b_1} = \frac{2\pi}{\frac{qd}{4} + \sqrt{c_2 dX + \left(\frac{qd}{4}\right)^2}}. \tag{3-2.13}$$

The period gets shorter for $q > 1$compared with the initial one in AM for the same c_2.

Proposition 3-2.6. The second order delay of relative wage, equal to the critical lag for FM ($T_h = 2/b_h$), poses no threat to the stability of stationary state E_X in the modified AM if a magnitude of the control parameter c_2 is selected within (non-empty) interval (3-2.14).

Proof. Basic requirements of Liénard – Chipart criterion (Table 3.1) are satisfied for T_h, with the possible exception of requirement $\Delta_3 = qdb^3(2b^2 - qdb - 2c_2 dX) > 0$. However, to meet this requirement it is sufficient to select the value of $c_2 = c_2(b_h)$ from a nonempty interval

$$0 < c_2(b_h) < \frac{b_h(2b_h - qd)}{2dX}. \tag{3-2.14}$$

This interval also shrinks for $q > 1$compared with the initial one in AM for $2b_h/d > 1$.

Simulation experiments

The initial magnitudes of phase variables and parameters values (except q and c_2) are the same. For the basic magnitudes of the parameters and particular initial values of the phase variables ($u_0 = z_0 = y_0 = 0.9087$, $v_0 = 0.518 < X = 0.95$), using the critical values of $b = b_h \approx 0.5501$ from FM, optimization period covered 64 years (1958 – 2021) with extrapolating over 2022–2158 again.

As the solution of parametrical optimization problem for modified AM, the best magnitudes of the control parameters are found: $q = 3.0411$ and $c_2 = 0.3028$. These best magnitudes are possibly sub-optimal (still quite satisfactory). Figures 3.1 and 3.2 as well as Table 3.2 demonstrate that the reinforced stabilisation policy substantially reduces take-off period for employment growth.

Moderation of growth in wage in the beginning of the transient is as expected. Parity of wage (relative wage) first achieved in 1978 (1980) is later saved. The reinforced stabilisation policy provides advantage in total wage (wL) almost from the very beginning, in spite of higher profit and surplus value (1959–2006).

The paper (Ryzhenkov, 2010) develops and elaborates the above stabilisation policy as in AM within a system dynamics model with endogenous rate of accumulation, endogenous capital-output ratio, induced technical change, economy of scale and endogenous supply of labour force based on the US official statistics. This paper argues that the supposed stabilisation policy may help to overcome the structural crisis of the American economy and to transform the industrial cycle into growth cycle without an absolute decrease of net output. Scenarios of development of the American economy in 2008 – 2020 have taken into account the targets defined by the US Congressional Budget Office in attaining an increased relative wage at a higher employment ratio. This paper demonstrates that less intensive labourers' competition for jobs, as well as higher domestic rate of accumulation can facilitate approaching to these socio-economic objectives.

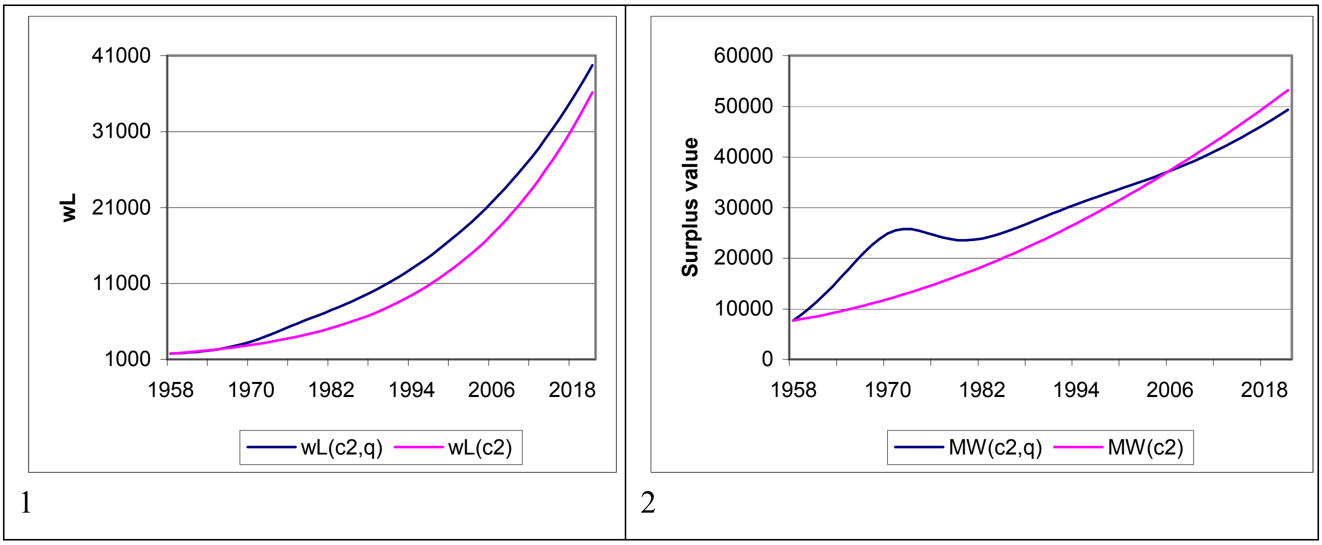

Figure 3.1. Dynamics for the initial and reinforced stabilisation policies:
Panel 1 – total wage (wL), Panel 2 – surplus value (MW = S/a)

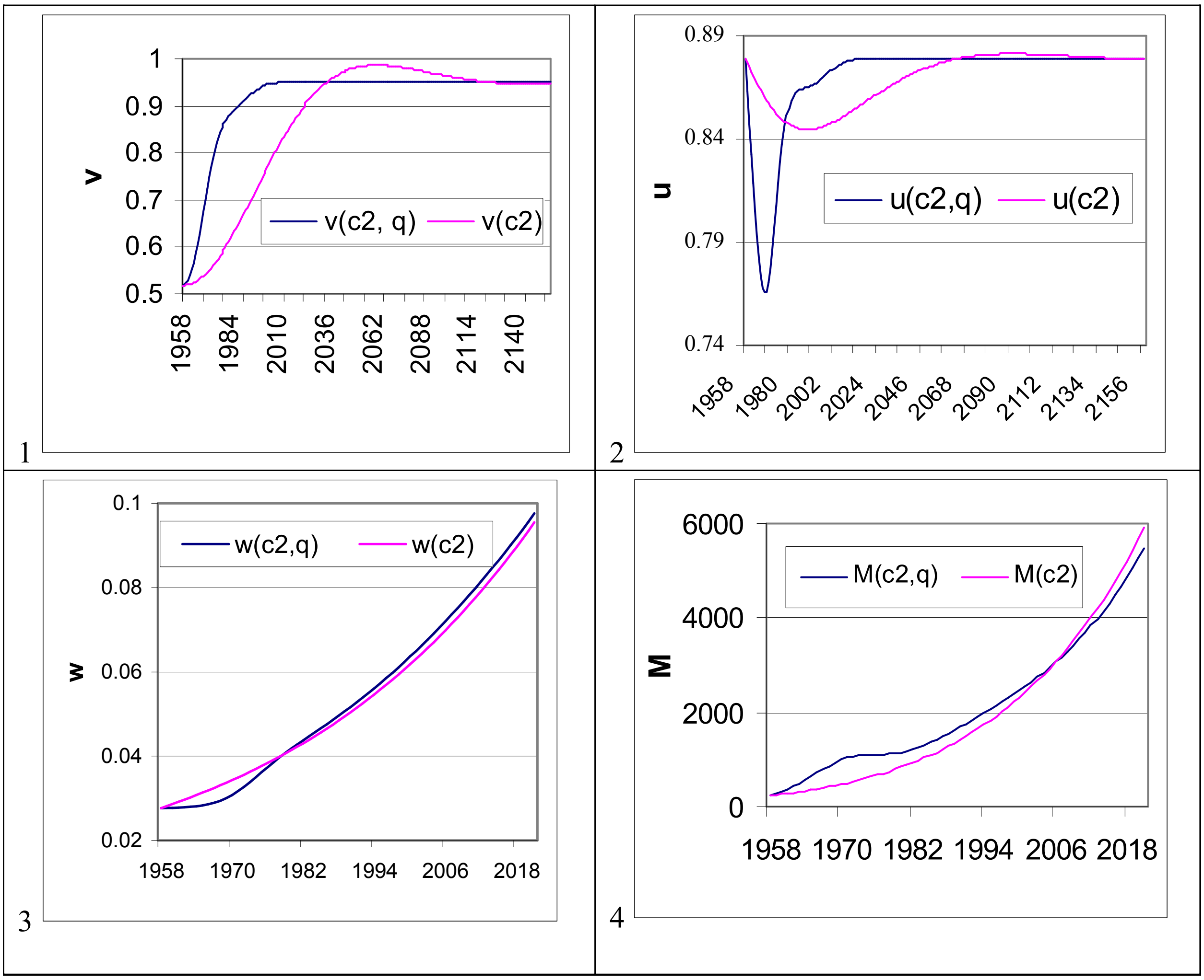

Figure 3.2. Dynamics for the initial and reinforced stabilisation policies: Panel 1 – employment ratio (v), Panel 2 – relative wage (u), Panel 3 – wage (w), Panel 4 – profit (M)

Table 3.2. The year of employment ratio attainment in AM and in modified AM

Employment ratio (v)	AM ($q = 1$, $c_2 = 0.0381$)	Modified AM ($q = 3.0411$, $c_2 = 0.3028$)	Lead due to reinforcement of the stabilisation policy (years)
0.518 (initial)	1958	1958	0
0.6	1982	1966	16
0.7	1996	1970	26
0.8	2009	1975	34
0.9	2025	1990	35
0.95	2037	2010	27

Conclusion

The valuable achievement of Fanti and Manfredi paper (1998) is (even if not deliberate and comprehensive) warning on possible detrimental effects of profit sharing. The applications of local bifurcation theory to the Goodwinian predator-prey models with dimension of four (and higher degree in their subsequent research) are skilful.

The model built for closed economy (FM) for a specific accumulation rate ($k = 1$) is generalised in this paper for $sd < k \leq 1$; consequently, the original propositions are reconsidered. This paper revises the equations for profit-sharing and bargained wage terms in the two substantially non-linear four-dimensional models of capital accumulation.

In the first (AM) before adding the second order delay, a growth rate of profit is proportional to a gap between the indicated and current employment ratios. This policy rule with a great margin of safety stabilises capital accumulation even being fuzzier for stretched "humped" delays. In the second model (modified AM), deviations of employment ratio and delayed profit rate from their stationary magnitudes define net change of relative wage. This proportional control already present in AM is reinforced in the modified model shortening a transient to a distant target employment ratio. Parametric optimization for both models is supported by *Vensim*.

It is irony that this paper, as often happens in science, that would be not written without a careful and grateful study of the opponents' contributions, turns the opponents' undisputable formal achievements into refuting a core of their theoretical interpretations. This paper, first of all, rejects the wide-spread belief that stabilisation policies, involving the workers' profit sharing, inevitably cause a lasting reduction of employment ratio in a Goodwinian model. Also, after the rate of accumulations has been explicitly introduced, rebutted is the duo's argumentation that the mechanism of profit indexation with second order delay in the equation of net change of relative wage is the main catalyst (or trigger) of the persistent medium-term economic cycle.

AM and its modification imply stabilisation policies through linking class distribution of national income with employment benchmarks. It is shown that reproduction and use of economic (especially labour) capacity improves in AM and in modified AM; accordingly over long and middle-term, well-being of working class rises, compared to FM with its ill-defined stabilisation policy.

The conditions of AM–FM and modified AM–FM local equivalence in linear approximation are determined. Uncovered restricted local equivalence permits extension of the FM local stability analysis, including existence part of a simple Andronov – Hopf bifurcation, onto AM and its modification. Still qualitative properties of these models at a broader scale differ substantially as revealed through calculation of the variables' partial derivatives and via computer simulations.

Serendipity has entered into the process of this paper preparation. The definition of the profit sharing index, under restricted local equivalence, turns to be the scalar product of the main Marx's notions, namely: the rate of accumulation, stationary rate of surplus value and stationary profit rate – combination hardly previously explicitly stated in the economic literature.

The Italian economists have offered a valuable opportunity to expose the supposed stabilisation policies to serious extreme condition tests (whereby about a half of labour force being unemployed at the very beginning of the stabilisation policy). The successful completion of these tests in AM and modified AM shows that the invented stabilisation policies are robust and have a large margin of safety; they withstand a "staggered" nature of labour contracts as well as the information lags. The enhanced proportional control over a net change of relative wage in modified AM will be used for upgrading models built earlier.

This paper besides purely theoretical has also practical relevance. Remember the need to reduce mass unemployment – its seasonally-adjusted rate has been 12.2% in Italy, the highest one in at least

36 years (EU27 at 10.9%, 12.1% in Euro area – EA17) in May 2013.[9] Targeting a high employment ratio in conjunction with an appropriate growth rate of total profit can be placed at the very heart of nation-wide and local wage bargaining (if some other factors not yet taken into explicit account do not destroy this setting).

In-depth research on stabilisation policies focusing on the labourers' interests and the needs of social development is to be continued. *Economic Report of the President 2013* strengthens this position (p. 21): "Although economics has long been called "the dismal science," it is more appropriately viewed as a "hopeful science." The right mix of economic policies and leadership can help a country to recover from a deep recession and point to the investments and reforms that will build a stronger, more stable, and more prosperous economy that works for the middle class."[10]

Growth of profit ought to be not mainly determined by powerful TNCs. The organised working class' supremacy over capitalist reproduction, including these oligopolistic entities, could be a transitory alternative to state-monopoly capitalism on the revolutionary road to socialism.[11] It is necessary at least to make target employment ratio a key factor of growth rate of profit for the labourers' and unemployed benefit. Developing working class cohesion, strength and consciousness is prerequisite.

Is this idea a progressive dream only? No. It is principally an objective requirement touching the hearts and minds, elevating working masses across the world. This idea becomes a material force.

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[9] The Eurostat News Release 102/2013–1 July 2013; see also Reuters of May 31 2013 at URL: http://www.reuters.com/article/2013/05/31/us-eurozone-economy-idUSBRE94U0DJ20130531

[10] The notion of the American middle class in this context is mostly based on a reasonable amount of discretionary income and/or college education. Multiple definitions of middle class by different schools deserve a special analysis beyond the scope of this paper.

Marxism defines social classes essentially according to their relationship with the means of production. This approach is applied in my present and previous papers.

[11] V. I. Lenin (Lenin, 1917: 343, 361) wrote: "Large-scale capitalist economy, by its very technical nature, is socialised economy … socialism is merely the next step forward from state-capitalist monopoly. *Or, in other* words, socialism is merely state-capitalist monopoly *which is made to serve the interests of the whole people* and has to that extent *ceased* to be capitalist monopoly."

Whether socialism is possible and /or (already) necessary at the current stage of development of productive forces in advanced capitalist countries is highly controversial issue even in the Marxian, neo-Marxian and post-Marxian literature. The history remains the best judge on this "apple of discord". The paper limits do not allow the author to delve deeper into this subject matter. Still it debunks *TINA* (Mrs. M. Thatcher's slogan "There is no alternative") as false neoliberal perception and unveils the material interests behind this.

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