Absent or distant? On the use of indirect causal links and chains of causal links to compare mental models of dynamic systems

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Abstract

The topic of eliciting and comparing mental models of dynamic systems has been present in system dynamics since many years. Elicitation and comparison methods have been used from other social science disciplines and have been adapted to the specific needs of system dynamics. Most of them focus on variables and causal links; the most recent methods also account for feedback loops. However, this focus on variables and causal links has challenges in dealing with differences stemming from diverse degrees of aggregation. We propose to use the method *chain of causal links* to include the perspective of the level of analysis. A simple example is used to demonstrate the method *chain of causal links*. First, a reference model is defined with a standard length for each causal chain; then, a distance matrix representation of causal loop diagrams is used and two new indicators – *relative length difference* and *relative content difference* – are shown to provide useful information for interpreting different levels of aggregation in the causal connection between variables.

Keywords: mental models of dynamic systems, model comparison, indirect causal links, causal chain

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1 Introduction

Qualitative diagrams, i.e., causal loop diagrams (CLD) and stock-and-flow diagrams (SFD), can help to structure problems (Lane, 2008; Sterman, 2000). They can also be interpreted as representations of mental models of dynamic systems (MMDS) of individuals or groups (Groesser & Schaffernicht, 2012; Schaffernicht & Groesser, 2011). However, when individuals or groups represent their reasoning in visual form, some diagrams are more explicit than others. In one of the most widely used textbooks on system dynamics, the reader is to "choose the right level of aggregation" (Sterman, 2000: 154), which is a trade-off between the two issues: first, "having too much detail makes it hard to see the overall [...] structure [...], and second, having too little detail makes it hard for your audience to grasp the logic and evaluate the plausibility and realism [...]." While this is reasonable advice for developing diagrams as communicative means, two other aspects emerge where different levels of aggregation may be problematic: first, when mental models are compared to compute the degree of similarity of the MMDSs of individuals at a given moment or the learning steps accomplished by an individual over a time period, then all traditional methods (e.g., Markóczy & Goldberg, 1995) and also the currently developed method for MMDS (Groesser & Schaffernicht, 2012) only account for direct links connecting neighbouring variables. Therefore, these methods would conclude that diagrams displaying the same case at different levels of aggregation (as in Sterman, 2000: 154, Figure 5-15) are dissimilar. And second, the number of feedback loops increases with the number of variables (Mojtahedzadeh, 2011). This may lead to a proliferation of apparently different loops that correspond to the same underlying feedback loop. This implies that when modelling according the rule "adapt the level of aggregation to your audience", the number of loops may considerably vary, and so does the feedback complexity of the resulting CLD or SFD. Therefore, if one expects that individuals with little systems diagramming experience require more disaggregate diagrams about the same subject than experts, but at the same time need less complex feedback diagrams in order to understand them, there is paradox.

This paper deals with the first of the aforementioned aspects. Schaffernicht and Groesser (submitted; 2012) have compared MMDS at the level of individual variables and causal links, feedback loops, and the overall model. They show that MMDS can be very similar at the level of feedback loops despite considerable differences at the level of variables and causal links. While this allows reducing the impact of differences between MMDS at this level, it leaves without consideration the question if such differences between MMDS are meaningful and in case the difference are meaningful, what do they indicate.

Most comparison methods for causal maps, cognitive maps, concept maps, and mental models (traditional and dynamic systems alike) interpret the diagrams as graphs and apply graph-theoretical algorithms to analyse and compare them. In the system dynamics literature, Oliva (2004) has shown how stock-and-flow diagrams can be represented by an adjacency matrix and then used it to compute a reachability matrix. The method for mental model comparison developed by Markóczy and Goldberg (1995) and adapted to dynamic systems by Schaffernicht & Groesser (2011) and Groesser & Schaffernicht (2012) also uses a matrix representation of the mental models. We argue here that longer causal chains in disaggregated models can be approximately equivalent to direct causal links in aggregated models because the disaggregated causal chains detail the direct links by multiple indirect causal links.

The paper is organized as follows: Section 2 introduces the illustrative example consisting of one stock-and-flow diagram (SFD) and four causal loop diagrams (CLD) with different levels of aggregation. The concept of *indirect links* is introduced. The following subsection discusses the use of adjacency matrices to represent CLDs; it is followed by a subsection on the distance matrix representation. Both types of matrices are then used in the fourth subsection to compute the two new indicators - *relative length difference* and *relative content difference*. Section 3 discusses the indicators' meaning and their use to interpret differences between MMDS. The conclusion addresses limitations of our approach, the necessary next steps, and a connection to how system dynamics modelling might take advantage of different levels of aggregation of causal chains to improve their usability.

2 Analysing chains of causal links

2.1 System dynamics diagrams represented as adjacency matrices

For the sake of simplicity, a deliberately small case based on the market growth model (Forrester, 1975; Morecroft, 2007). It is used as an illustrative example. The manager of a furniture company has the mission to achieve a high sales growth rate assuming that the company uses advertisements to stimulate demand. Advertisement presence influences demand, which yields orders; as orders are fulfilled—limited only by production capacity—sales revenues increase available funds. A given fraction of these funds is considered as the advertisement budget; ad spending is converted into new advertisements by a given unit cost. The following stock-and-flow diagram (Figure 1) contains all the elements of this description:

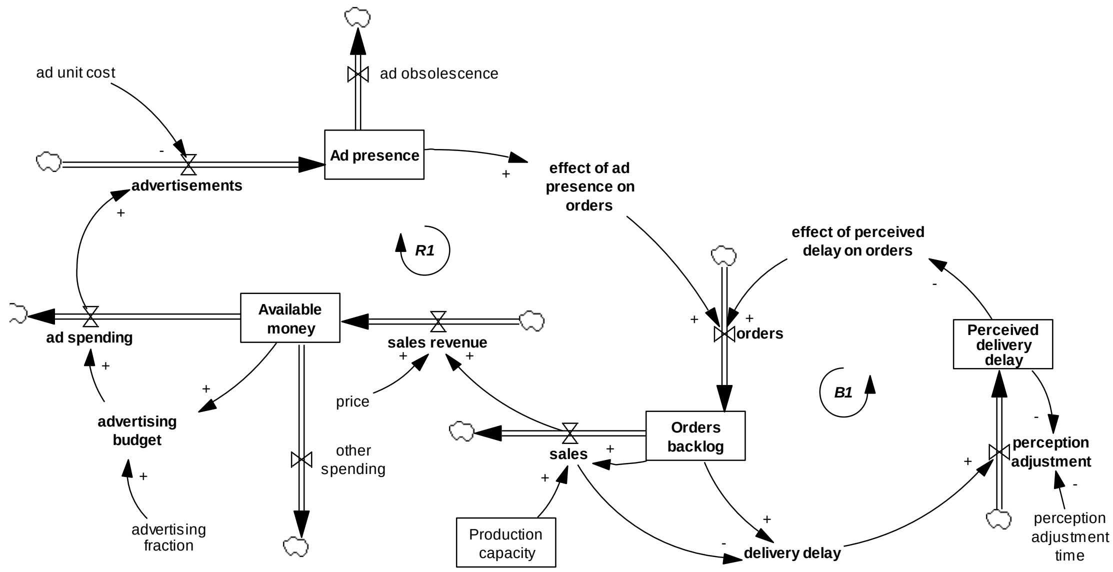


Figure 1: Stock-and-flow diagram of the exemplary case

A direct translation of the SFD yields a CLD with a high degree of disaggregation (Figure 2):

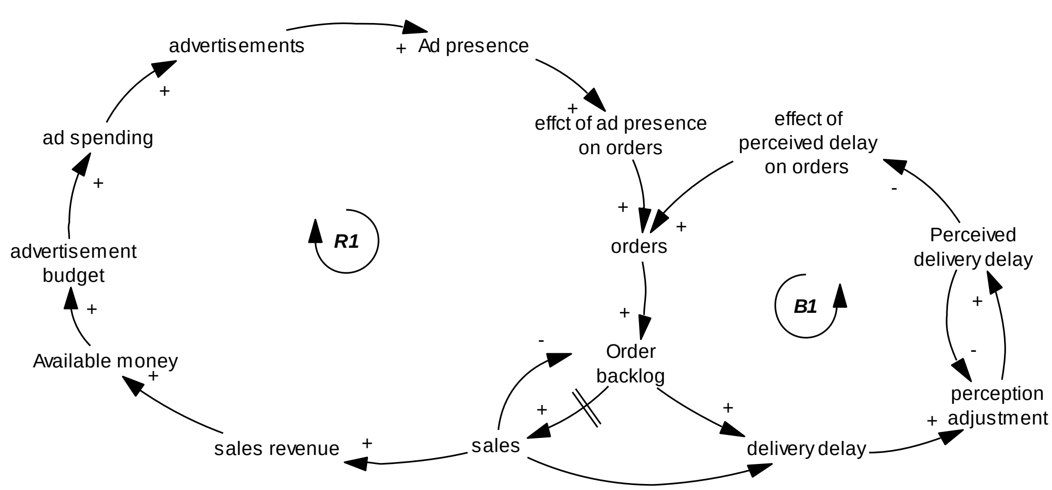
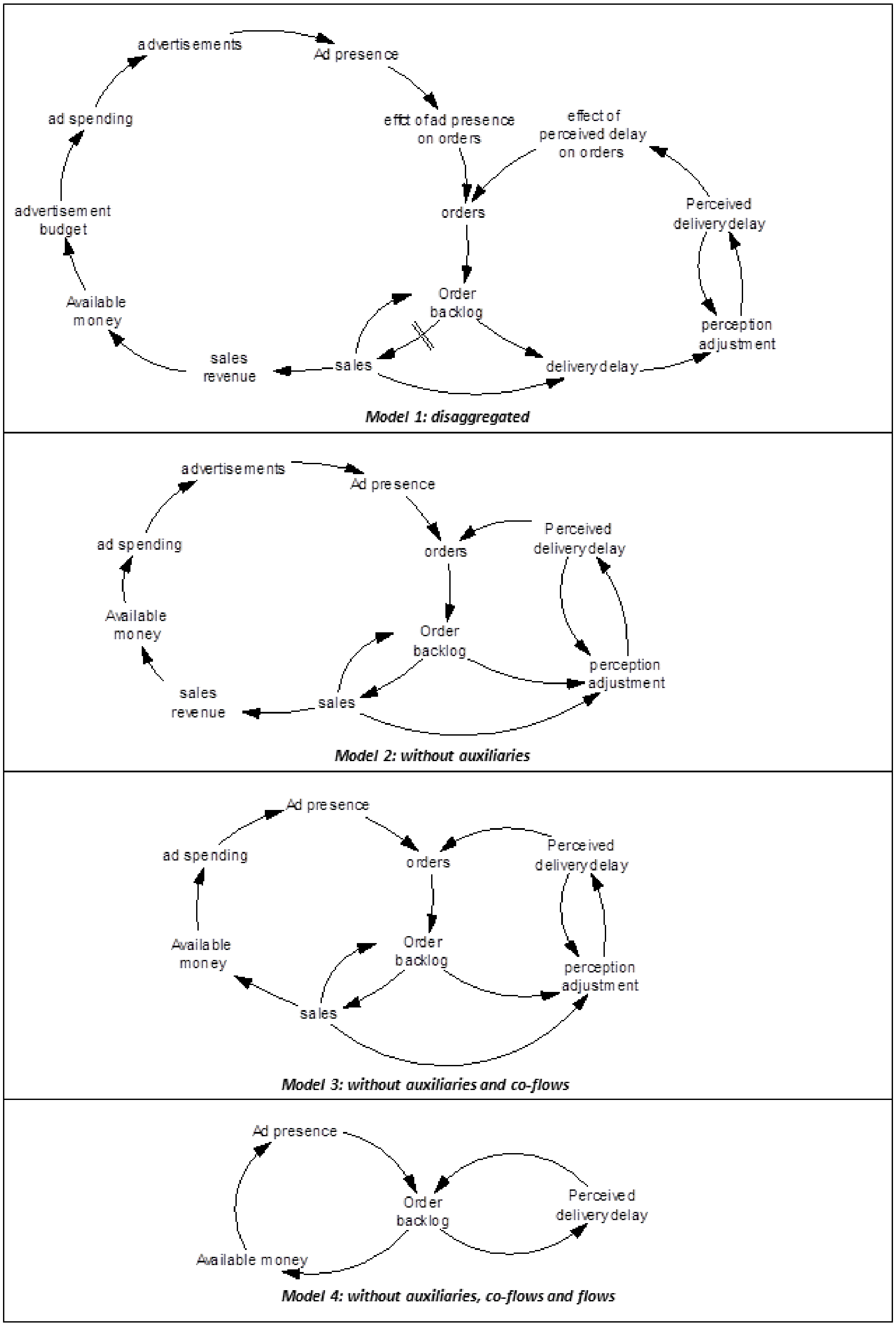


Figure 2: The corresponding CLD

In the CLD shown in Figure 2, the variables which represent stocks in Figure 1 start with a capital letter to make them salient. Even if in this paper we use causal diagrams – which are widely used and can serve for model conceptualization, model communication (Lane, 2008), and articulation of mental models – from a more scientific point of view, such diagrams are simplified representations of a stock-and-flow structure. In this structure, stock variables are essential, particularly because they are the only part of the structure which is directly observable. For this reason, we will use the stock variables to define a reference model, and we have also used them to generate the illustrative diagrams sued as examples.

When considering Figure 2, we ask: does the absence of a causal link *sales* → *Ad presence* mean that *sales* do not influence *Ad presence*, or should we rather interpret this as an indirect link? Since our discussion centres on direct and indirect causal links, we will abstract away the links' polarity and delays as well as the feedback loops in the following diagrams. We look at several versions of the diagram shown in Figure 1, which are the results of successive steps of aggregation (Figure 3). Model 1 corresponds to the "literal translation" shown in Figure 2—just without polarities and loops. Model 2 contains only the stocks and flows, while Model 3 arbitrarily leaves out one flow of each co-flow. For Model 4, only stock variables from Figure 1 are represented. Figure 3 shows the four models.



1. Figure 3: Successive steps of aggregation

These CLD may appear to be different when looked upon as sets of variables and causal links. We call the set of variables *V*, with *v* referring to the number of variables in *V* (*V*[1] through *V*[*v*]). We call the causal links *CL*, and each of them will be a connection between two elements of *V*. Table 1 displays the inclusion of the different variables in each of the diagrams:

V[]	Name	Model 1	Model 2	Model 3	Model 4
1	Order backlog	1	1	1	1
2	Aales	1	1	1	-
3	Aales revenue	1	1	-	-
4	Available money	1	1	1	1
5	Advertisement budget	1	-	-	-
6	Ad spending	1	1	1	-
7	Advertisements	1	1	-	-
8	Ad presence	1	1	1	1
9	Effect of ad presence on orders	1	-	-	-
10	Orders	1	1	1	-
11	Delivery delay	1	-	-	-
12	perception adjustment	1	1	1	-
13	Perceived delivery delay	1	1	1	1
14	Effect of perceived delay on orders	1	-	-	-

Table 1: Variables of each MMDS

From a lecturer's point of view, there are two questions:

- Which of these possibilities should be used as reference model for assessing a learners' work?
- How should such an assessment evaluate the differences between the learners' models and the reference model?

There are several options to respond to the first question:

- Use the most detailed model, so all compared models will be subsets of the variables and links; this has been done in Model 1.
- Use the model with only those variables and links which are needed for the model's behaviour pattern, aggregating the intermediate variables into the flows (Mojtahedzadeh, 2011), like done in Model 2.
- Use the most simplified model (Model 3 or 4), so the comparison is not cluttered by details which would make the degree of difference appear larger than it actually is.

In this paper, we use the second choice for the following reasons: The dynamics of the studied situation are governed and described by situation's relevant stocks and flows. Therefore, any auxiliary variable will not be essential for the dynamic characteristics and can be thought of as "collapsed" in the flow rates (Mojtahedzadeh, 2011). If decision policies are part of the "system", then they should also have been represented by stocks and flows. Auxiliaries should be used only for components which are neither stock nor flow. This means that Model 2 would be the reference model to which Models 1, 3, and 4 are compared to in order to assess which degree they are useful for discussing the dynamic problem under study.

Let us now turn to model comparison. A quick visual inspection of the four models reveals increasing differences as shown in Figure 3. However, does aggregation in a causal chain as in "sales → sales revenue → Available money" into "sales → Available money" delete "sales revenue" or does it hide it from the visual representation? Beholden from the other perspective: Does the disaggregation of "sales → Available money" mean this causal link is substituted by "→sales revenue →"? Or would it be more meaningful to suggest that in chains of variables the presence of direct links like "→ sales revenue →" imply the existence of indirect links “sales → Available money”? Figure 4 displays how such indirect links, if used in a diagram, depict the implicit inclusion of more aggregated models as subsets of more disaggregated models.

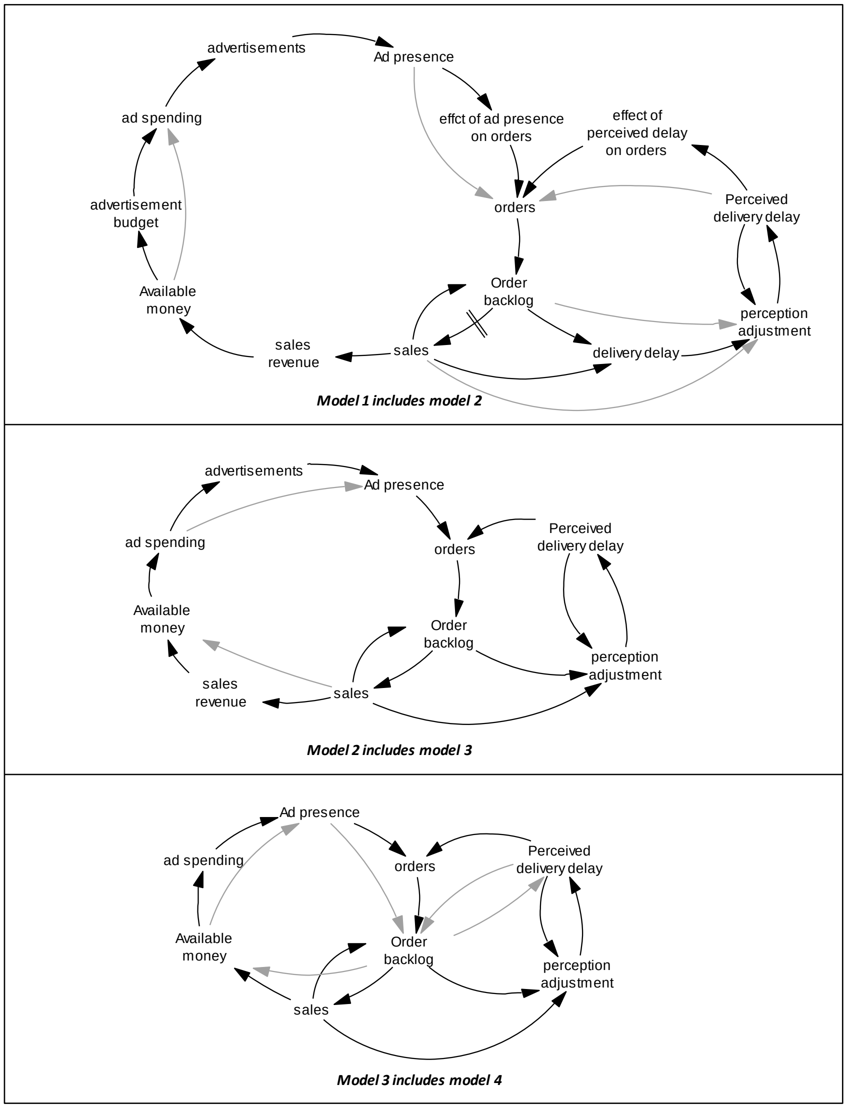


Figure 4: Indirect links depict the inclusion of more disaggregated models

2.2 The use of adjacency matrices

Many methods used to analyse the structure of simulation and mental models interpret the sequences of variables and links as graphs. In these cases, the first step is to construct an *adjacency matrix* of the model(s). An adjacency matrix A of this model has r rows and c columns, where $r = c = v$. Each of its $r*c$ cells contains one piece of information about the variables $V[r]$ and $V[c]$: if there is a link $CL[r,c]$ then the cell is equal to 1; otherwise it is set to 0. Since v is different for each of our four models (14, 10, 8 and 4) we will use extended adjacency matrices, which take the union of the models' variables sets as $V[]$; since in our case, Models 2 through 4 are simplifications of Model 1, the extended adjacency matrices will all have a 14 * 14 structure. Table I shows how the two CLDs are represented as adjacency matrices:

Adjacency matrix model 1

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	1	0	0	0	0	0	0	0	0	1	0	0	0
2	0	0	1	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	1	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	1	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	1	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	1	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	1	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	1	0	0	0	0
10	1	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	1	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	1	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	1
14	1	0	0	0	0	0	0	0	0	0	0	0	0	0

Adjacency matrix model 2

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	1	0	0	0	0	0	0	0	0	0	1	0	0
2	0	0	1	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	1	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	1	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	1	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	1	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	1	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	1	0
13	1	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Adjacency matrix model 3

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	1	0	0	0	0	0	0	0	0	0	1	0	0
2	0	0	0	1	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	1	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	1	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	1	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	1	0
13	1	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Adjacency matrix model 4

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	0	0	1	0	0	0	0	0	0	0	0	1	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	1	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	1	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	1	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table I: Adjacency matrix representation

The number of differences between Model 2 and Models 3 and 4 increases when the decreasing number of variables is taken into account in the more aggregated models. Also, since Model 1 contains more variables than the reference model, there are also differences between the Models 1 and 2 as Table III shows:

Differences Models 2 vs 1 12

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	0	0	0	0	0	0	0	0	0	1	1	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	1	1	0	0	0	0	0	0	0	0
5	0	0	0	0	0	1	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	1	1	0	0	0	0
9	0	0	0	0	0	0	0	0	0	1	0	0	0	0
#	0	0	0	0	0	0	0	0	0	0	0	0	0	0
#	0	0	0	0	0	0	0	0	0	0	0	1	0	0
#	0	0	0	0	0	0	0	0	0	0	0	0	0	0
#	1	0	0	0	0	0	0	0	0	0	0	0	0	1
#	1	0	0	0	0	0	0	0	0	0	0	0	0	0

Differences Models 2 vs 3 6

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	1	1	0	0	0	0	0	0	0	0	0	0
3	0	0	0	1	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	1	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Differences Models 2 vs 4 14

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	1	0	1	0	0	0	0	0	0	0	1	1	0
2	0	0	1	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	1	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	1	0	1	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	1	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	1	0	0	0	0	0	0
8	1	0	0	0	0	0	0	0	0	1	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	1	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	1	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table II: Differences between the models

Does the increasing number of differences mean that the models are becoming ever more different? Established model comparison methods (Groesser & Schaffernicht, 2012) and methods specifically designed to compare MMDS (Schaffernicht & Groesser, 2011) do indeed determine the degree of difference on the number of actual differences, divided by the number or potential differences. However, in the light of the argument provided above, it seems questionable to consider that the causal chains "$a \rightarrow b \rightarrow c$" and "$a \rightarrow c$" lead to two differences: "$a \rightarrow b \rightarrow c$" indirectly contains "$a \rightarrow c$". The two components are not identical, because in the first case, *c* is further away from *a* than in the second case. Above, it has been argued that we will use a reference model with only stocks and flows. If *b* is an auxiliary variable in any articulated MMDS, then aggregating it away will not make a significant difference to the behavioural implications of the causal structure displayed. It then appears that comparison methods that focus on individual elements are not well suited for this case, where we are comparing *chains* of variables and links.

Adjacency matrices only contain direct information about direct links (all other information has to be computed). However, other constructs like the *distance matrix* might be a useful for answering the question if a direct link from *a* to *c* in the reference model has an equivalent link or chain of links in another model, or if a direct link in one of the compared models has an equivalent link or chain of links in the reference model.

2.3 The use of distance matrices

A *distance matrix D* has the same $r*c$ dimensions as the adjacency matrix. However, its cells do not contain the "link vs. no-link" information, but the distance from $V[r]$ to $V[c]$: how many intermediate links have to be followed in order to send a signal from $V[r]$ to $V[c]$? Graph theory has algorithms for transforming adjacency matrices into distance matrices, and the commercially available mathematics software packages offer these as standard functions or as additional libraries. The distance matrices of our exemplary models are as follows:

Distance matrix model 1

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	1	2	3	4	5	6	7	8	9	1	2	3	4
2	9	0	1	2	3	4	5	6	7	8	10	11	12	13
3	8	9	0	1	2	3	4	5	6	7	9	10	11	12
4	7	8	9	0	1	2	3	4	5	6	8	9	10	11
5	6	7	8	9	0	1	2	3	4	5	7	8	9	10
6	5	6	7	8	9	0	1	2	3	4	6	7	8	9
7	4	5	6	7	8	9	0	1	2	3	5	6	7	8
8	3	4	5	6	7	8	9	0	1	2	4	5	6	7
9	2	3	4	5	6	7	8	9	0	1	3	4	5	6
10	1	2	3	4	5	6	7	8	9	0	2	3	4	5
11	4	5	6	7	8	9	10	11	12	13	0	1	2	3
12	3	4	5	6	7	8	9	10	11	12	4	0	1	2
13	2	3	4	5	6	7	8	9	10	11	3	4	0	1
14	1	2	3	4	5	6	7	8	9	10	2	3	4	0

Distance matrix model 2

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	1	2	3	0	4	5	6	0	7	0	1	2	0
2	7	0	1	2	0	3	4	5	0	6	0	8	9	0
3	6	7	0	1	0	2	3	4	0	5	0	7	8	0
4	5	6	7	0	0	1	2	3	0	4	0	6	7	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	4	5	6	7	0	0	1	2	0	3	0	5	6	0
7	3	4	5	6	0	7	0	1	0	2	0	4	5	0
8	2	3	4	5	0	6	7	0	0	1	0	3	4	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	1	2	3	4	0	5	6	7	0	0	0	2	3	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	2	3	4	5	0	6	7	8	0	9	0	0	1	0
13	1	2	3	4	0	5	6	7	0	8	0	2	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Distance matrix model 3

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	1	0	2	0	3	0	4	0	5	0	1	2	0
2	5	0	0	1	0	2	0	3	0	4	0	6	7	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	4	5	0	0	0	1	0	2	0	3	0	5	6	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	3	4	0	5	0	0	0	1	0	2	0	4	5	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	2	3	0	4	0	5	0	0	0	1	0	3	4	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	1	2	0	3	0	4	0	5	0	0	0	2	3	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	2	3	0	4	0	5	0	6	0	7	0	0	1	0
13	1	2	0	3	0	4	0	5	0	6	0	2	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Distance matrix model 4

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	0	0	1	0	0	0	2	0	0	0	0	1	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	2	0	0	0	0	0	0	1	0	0	0	0	3	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	1	0	0	2	0	0	0	0	0	0	0	0	2	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	1	0	0	2	0	0	0	3	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table III: Distance matrices

To facilitate comparison, these matrices have a row and column for each of the variables that are contained in at least one of the compared models. Therefore, the more aggregated models have an increasing number of zero rows and columns (for each of the variables they do not contain). Also, the main diagonals have only zero elements, because in system dynamics a variable is never directly linked to itself. All variables are linked with all other variables, which is a necessary consequence of the fact that the models consist of two connected feedback loops. As compared to a *reachability matrix R* (Oliva, 2004) the difference is that instead of the $R[2,1]=1$ indicating that one can reach $V[1]$ from $V[2]$, we now have $D[2,1]=9$ meaning that we can reach $V[1]$ from $V[2]$ in nine steps.

2.4 Relative length difference and relative content difference

Therefore, for "$a \rightarrow b \rightarrow c$" the distance from *a* to *b* is 2, and for "$a \rightarrow c$" it is 1. We can use this distance to weight the degree of difference between these two cases. To perform this calculation, we use the distances between stock variables in the reference model as norm. Stocks are what can be directly measured and what is used to describe the state of the system as well as its changes over time. Therefore, it seems that using stocks to punctuate the chains of causal links yields meaningful chains. The length of these chains in the reference model, i.e., where the auxiliary variables have been aggregated into the flow variables as argued above, will serve as a denominator for comparing the equivalent chains in the other models. For instance, consider the chain from *Order backlog* (V[1]) to *Ad presence* (V[8]) in our 4 models:

Model	**Distance from *Order backlog* to *Ad presence***	**Difference: model - reference**	**Relative Length difference**
Model 2 (reference)	6	0	**0**
Model 1 (more details)	8	2	**0.33**
Model3 (less details)	4	-2	**-0.33**
Model 4 (even less details)	2	-4	**-0.67**

Table IV: Distance comparison for one chain

The absolute difference of chain lengths between models, divided by the length of the reference chain, indicates the degree of disaggregation of each chain as compared to the reference chain as justified norm. Positive values are more disaggregated, negative values stand for more aggregated chains.

A second question is how different are the variables which conform the chains. We can construct a list with the variables in each chain by starting with the first stock[3] and recursively following through the successor elements in the adjacency matrix. Then the degree of difference for the causal chain from model *m* is computed as the number of variables in only model *m* plus the number of variables in the reference model but not in model *m*, divided by the total number of variables in the reference model's causal chain. For example, in the same case of *Order backlog* (V[1]) to *Ad presence* (V[8]), the adjacency matrix of the reference model:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	1	0	0	0	0	0	0	0	0	0	1	0	0
2	0	0	1	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	1	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	1	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	1	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	1	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	1	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	1	0
13	1	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 5: Following a causal chain in the adjacency matrix

Application of this tactic leads to the following results:

[3] Even though one might just start with any variable, recall that only stocks are directly observable. Using them as starting point will tend to yield comparable chains at diverse levels of aggregation, because stock variabes are the only type of variables to be found in models of any level of aggregation.

Model	Variables								Total
Reference	1	2	3	4		6	7	8	
1	1	2	3	4	5	6	7	8	
3	1	2		4		6		8	
4	1			4				8	
Union	1	2	3	4	5	6	7	8	8

Variables only in the compared model		Total
1	5	1
3		0
4		0

Variables from reference model not in the compared model		Total
1		0
3	3 5 7	3
4	2 3 5 6 7	5

Total differences per model

Model	Count	Degree
1	1	0,1
3	3	0,4
4	5	0,6

Table V: Differences between causal chains

Only Model 1 has a variable which is not contained in the reference model; Models 3 and 4 do not contain all variables mentioned in the reference model. Counting the number of such differences and dividing by the number of variables in the reference model yields the results displayed at the bottom of Table VII. Joining the two parts of comparison—*relative length of the causal chain* and *relative content difference*—we now have the following situation:

Model	Relative length difference RLD	Relative content difference RCD
1	0.33	0.1
3	-0.33	0.4
4	-0.67	0.6

Table VI: Relative difference indicators

3 Discussion

To interpret the meaning of these indicators, let us consider them in the following figure:

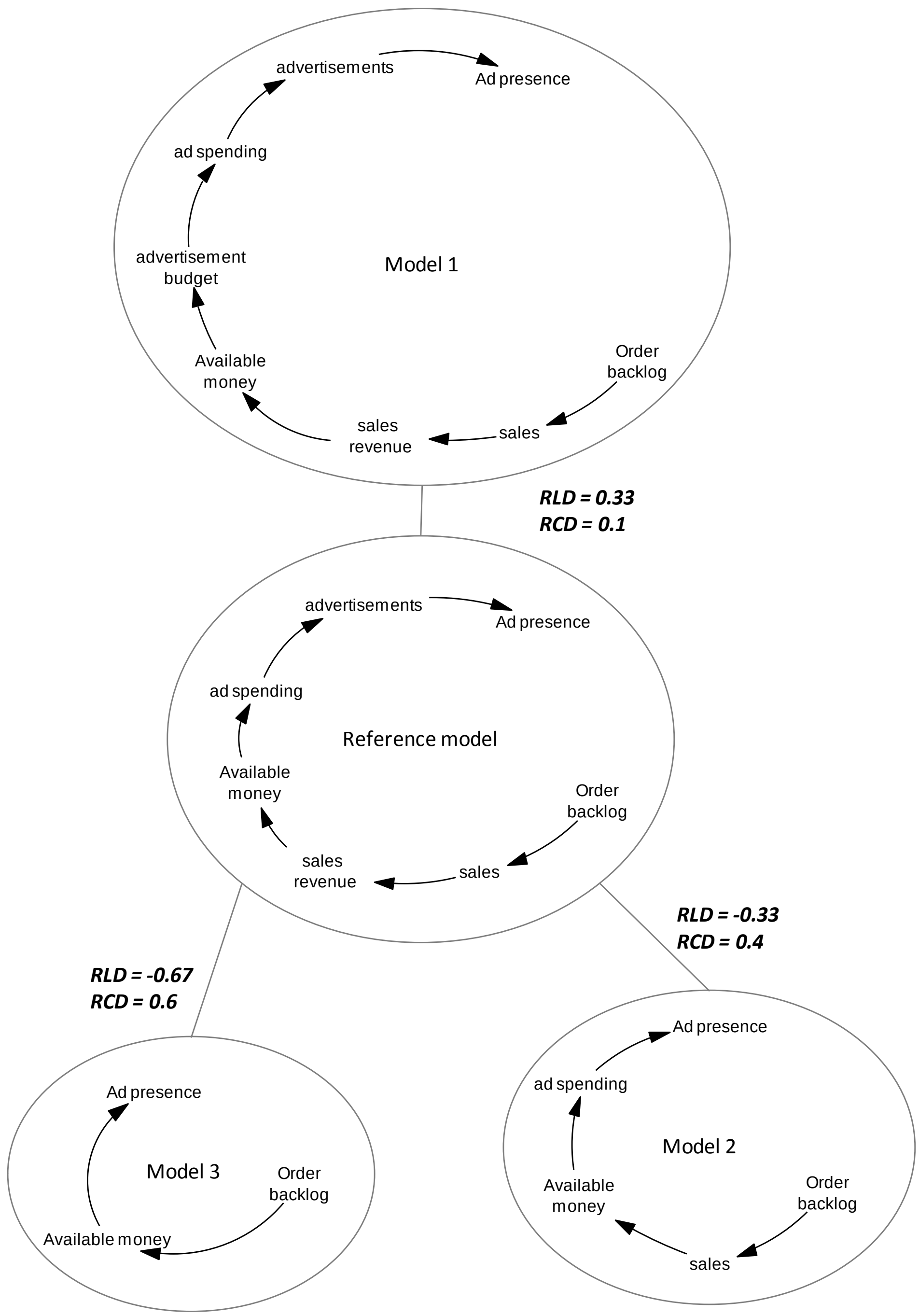


Figure 6: Content differences between the causal chains

Each of the causal chains in the Models 1, 3, and 4 contain the same set of feedback loops which can also be found in the reference model. In terms of system behaviour, the models are assumed to be equivalent. Nevertheless, Model 1 has slightly more details in variables which changes influences, e.g., from *sales* to *Ad presence*, more indirect and an individual proposing this representation of the studied situation has a more explicit argument. Individuals arguing according to the structure that is represented in Models 3 and 4 would state a simpler more direct way for the *Order backlog* to impact *Ad presence,* and their argument would be less explicit.

Neither of these models is “wrong” when compared to the reference model; therefore, when evaluating the elaborations made by different individuals as compared to a reference model, the quality of the articulated model does not automatically improve by increasing the number or details or diminish by decreasing the number of details: the number of variables should not be graded directly. Still, different lengths of the causal chain should be inquired and diagnosed, for it may be the consequence of different differences. Individuals who propose a shorter causal chain may be either a domain expert who chunks the whole chain together in a reduced number of steps or a domain novice who does not distinguish several of the intervening factors. Individuals who propose a longer causal chain may be either a domain specific advanced novice who has reflected and articulated stepwise or a domain expert who is a novice in the diagramming language and therefore articulated step by step.

To summarize, the two indicators developed in this paper are useful for indicating relevant *qualitative* differences, and greater values correspond to a higher importance of detecting which of the abovementioned possibilities is the case.

4 Conclusion

Different levels of aggregation in different MMDS may lead to biases in the differences at the elementary level of description of MMDS. The paper shows that a disaggregate model contains indirect links which are equivalent to the shorter causal chains in more aggregate models. Moreover, it shows how the use of adjacency and distance matrices allows computing indicators for the difference in the length of causal chains and for the content difference. Since this is a conceptual contribution, only a simple illustrative example was used.

The paper offers several methodological contributions. First, the fact to define the situation in terms of stocks and flows and then apply the standard rule "aggregate the auxiliaries into the flow variables" for defining the reference model brings objectivity to each instructional setting and increases the comparability of assessments across instructional settings. It also avoids taking into account detail differences which do not impact the implied behavioural patterns. Last not least it assures that the set of feedback loops taken into account for comparison is not unnecessarily increased in its complexity by inserting more auxiliary variables. Second, shifting the focus of comparison from individual variables and links to the chains of causation helps to avoid the undesired effect of counting intermediate variables in the models articulated by individuals who did not follow the standard rule mentioned above. Third, taking the previous contributions together, we can now interpret such detail differences between models as differences between the modelling of the respective individuals and – instead of attributing different grades to the models – concentrate on the development state of individuals in their learning about a content domain and/or the problem structuring language used. An additional implication is that existing methods for the comparison of MMDS (Schaffernicht & Groesser, 2011), which focus on individual variables/links and then on feedback loops, may be enriched by inserting the intermediate description level of “causal chains”.

Since this is only an initial step, there are several limitations. First, this was only one exemplary case which did not involve real people. It was very simplified to bring out the essential points. Therefore it is, at this moment, not more than a logically coherent possibility waiting for the test of practical application. A second limitation is that the processing steps have no automatic support and require the repetitive execution of many steps. However, introducing the required computations in existing tools like SEXTANT (Schaffernicht & Groesser, submitted) can be achieved easily.

In conclusion, it seems worthwhile to mention that the ability to collapse or aggregate and to expand or disaggregate selected parts of system dynamics diagrams in the software packages we typically use, would arguable improve the usability of these tools to elaborate and communicate diagrams of complex systems. In stock-and-flow diagrams, one could hide/unhide parts of the diagram according to the situation. Additionally, CLDs could be automatically generated at different levels of aggregation.

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