Money Multiplier Dynamics and Banking Liquidity Cycles

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Abstract

Banking crises are commonly assumed to be driven by external events such as speculative bubbles, imposed on a normally stable banking system. To the contrary, this paper presents evidence that liquidity cycles and crises can arise naturally from the most basic structure of the banking system itself. The stability of a banking system can depend not only on the reserve requirements, but also on the relative magnitudes of other fundamental parameters, such as the lifetime of loans, the lifetime of deposits, and the time banks take to convert available liquidity into loans. Under realistic parameter values, a banking-regulatory system itself may generate spontaneous oscillations of sufficient amplitude to cause crises.

The Money Multiplier: Fractional Reserve Banking

Introductory textbooks describe how fractional reserve banking creates money. The reader is likely already familiar with the process, but we briefly describe it here for clarity.

Most national banking systems conform to the idea of fractional reserve banking, by which banks are allowed to convert their liquid assets into loans, with the requirement to reserve some liquid assets (as deposits with the central bank) to better cope with unexpected withdrawals of deposits. The required reserves are expressed as a fraction of the bank's customer deposits; the required fraction is called the reserve ratio. The reserve ratio varies significantly from country to country and from time to time. Some major countries have reserve ratios of zero, in Spain and Switzerland it is now about 2%; in the US it is about 10%; in some countries (e.g. Turkey) it has been at times higher than 60%. These numbers are approximate (even when precise), because not all deposits contribute equally to basis to which the reserve ratio is applied. For example, in the US term deposits do not count toward reserve requirements, and some deposits are counted at 5% instead of 10%.

Imagine that you deposit 100 Euros in a bank that is part of a fractional reserve banking system. That bank must retain some fraction of your deposit "in reserve", but the rest may be loaned out to someone else.

Let us assume the reserve ratio is 10%. That means the bank in which you deposited 100 Euros is then authorized to issue an additional loan of 90 Euros. Here comes the important part: the loaned 90 Euros is likely to be deposited in another bank in the same banking system. (It may even be deposited in the bank that made the loan.) So from the perspective of the banking system (if not the same actual bank), your 100 Euros has now become 190 Euros. Under normal conditions, the process iterates. The 90-Euro deposit itself is then converted into a loan of 81 Euros (with 10%, or 9 Euros) held in reserve. When the 81 Euros are deposited, they enable yet another loan of 90% * 81 = 72.9 Euros, which is then deposited somewhere in the banking system, and triggers yet another loan. Each loan is 90% the size of the previous one, so we have an infinite geometric series, depending only on the initial deposit (ID) and the reserve ratio (RR). Because (1-RR) must be less than one, the series converges:

$$ID(1 + (1 - RR) + (1 - RR)^2 + (1 - RR)^3 + \cdots) = \frac{ID}{1 - (1 - RR)} = \frac{ID}{RR}$$

That is, the fractional-reserve banking system can create enough money to grow the initial deposit (ID) into $\frac{\mathrm{ID}}{\mathrm{RR}}$. Therefore, in the above example, where RR = 0.1, the initial deposit grows by a factor of 10. This money-creation ratio is called the money multiplier.

Sometimes (surprisingly rarely) the money-multiplier process is presented as a graph, like the following one from Wikipedia, which assumes an initial deposit of 100 USD and a variety of reserve ratios (called "reserve rate" in the key to the diagram):

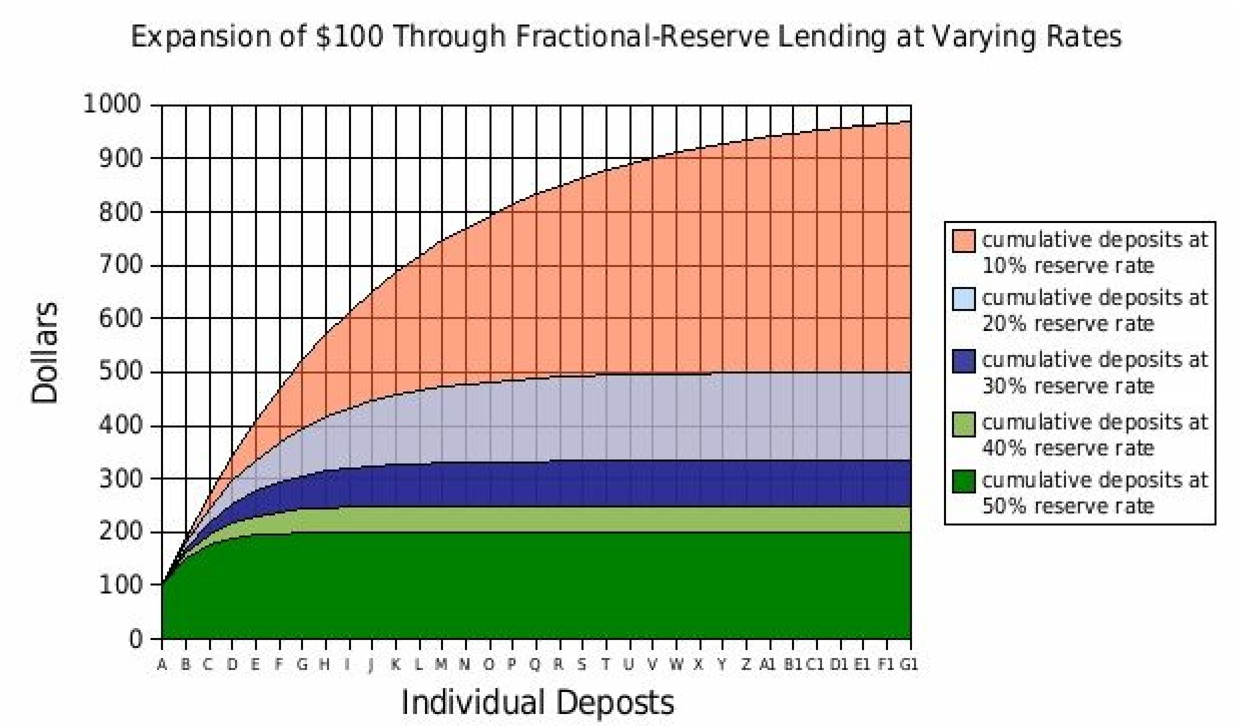


Figure 1. Money creation by a fractional-reserve banking system, as shown in Wikipedia. The horizontal axis is a sequence of deposits, starting with the initial deposit of $100, and accumulating the deposits of subsequent loans enabled by the original $100 deposit, under five different reserve ratios. Implicitly, the graph suggests the money multiplier is a stable dynamic process. Source: http://en.wikipedia.org/wiki/Money_multiplier

The converging series is presented in many introductory textbooks. The convergence of the series, and graphs like the one in Figure 1 create the impression that the money multiplier is a stable process, rising smoothly and monotonically to a stable equilibrium.

Preview of the dynamics of the money multiplier

The apparent stability of the money multiplier, as described in textbooks, is misleading. A simple dynamic simulation can reproduce the standard behavior seen in Figure 1, but this same dynamic structure, for many possible values of its parameters, instead oscillates. Figure 2 shows behaviors typical of the dynamic money multiplier.

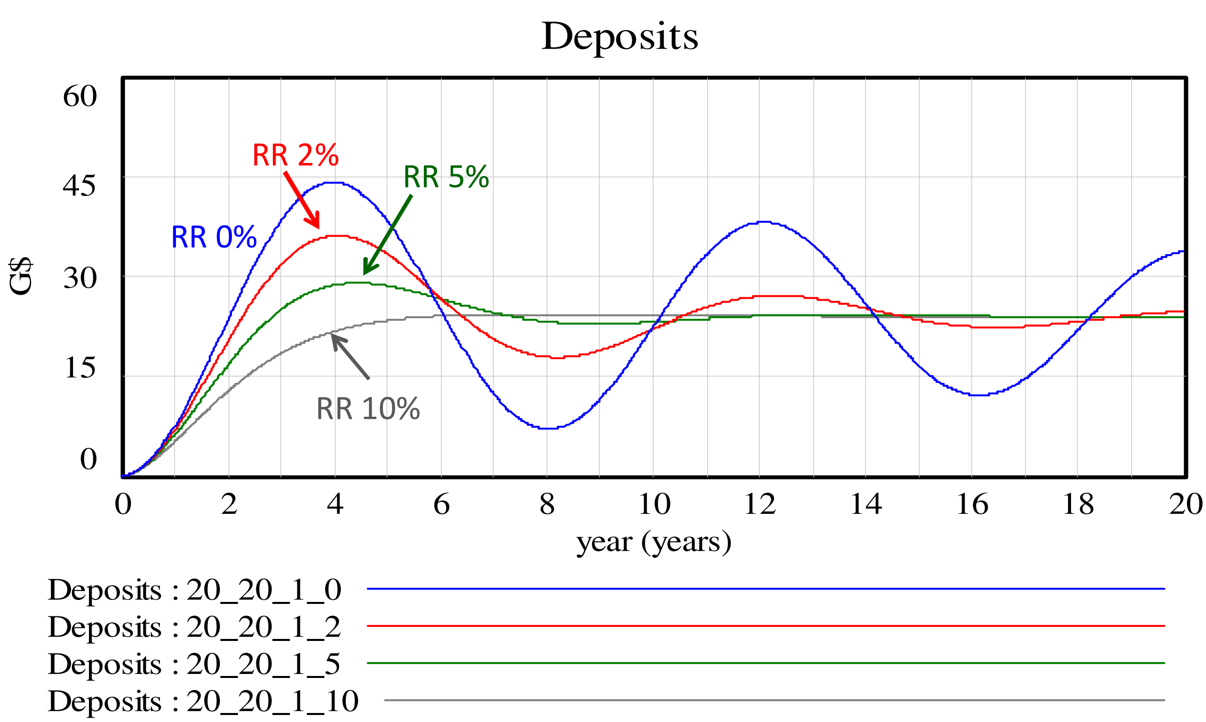


Figure 2. Simulations of the basic dynamics of the money multiplier, illustrating the tendency of the system to oscillate for low reserve ratios. The four simulations in this figure have different reserve ratios, but all share the same loan lifetime (20 years), deposit lifetime

(also 20 years) and time to convert liquidity into loans (1 month). These terms are defined in the analysis section of this paper.

The oscillations shown in Figure 2 arise spontaneously from inside the banking system, based only on the simple dynamics of the money multiplier, and the parameters that characterize the behavior of the customers, banks, and regulators. It suggests that one might expect to see oscillations in any banking system with a low reserve ratio. Several countries have, in fact, operated for decades with low reserve ratios. Figure 3 shows data for one such country, Switzerland (reserve ratios < 3% over the period), compared with the United States, which has usually maintained reserve ratios around 10%. The data qualitatively support the idea that higher reserve ratios tend to reduce oscillations in banking systems.

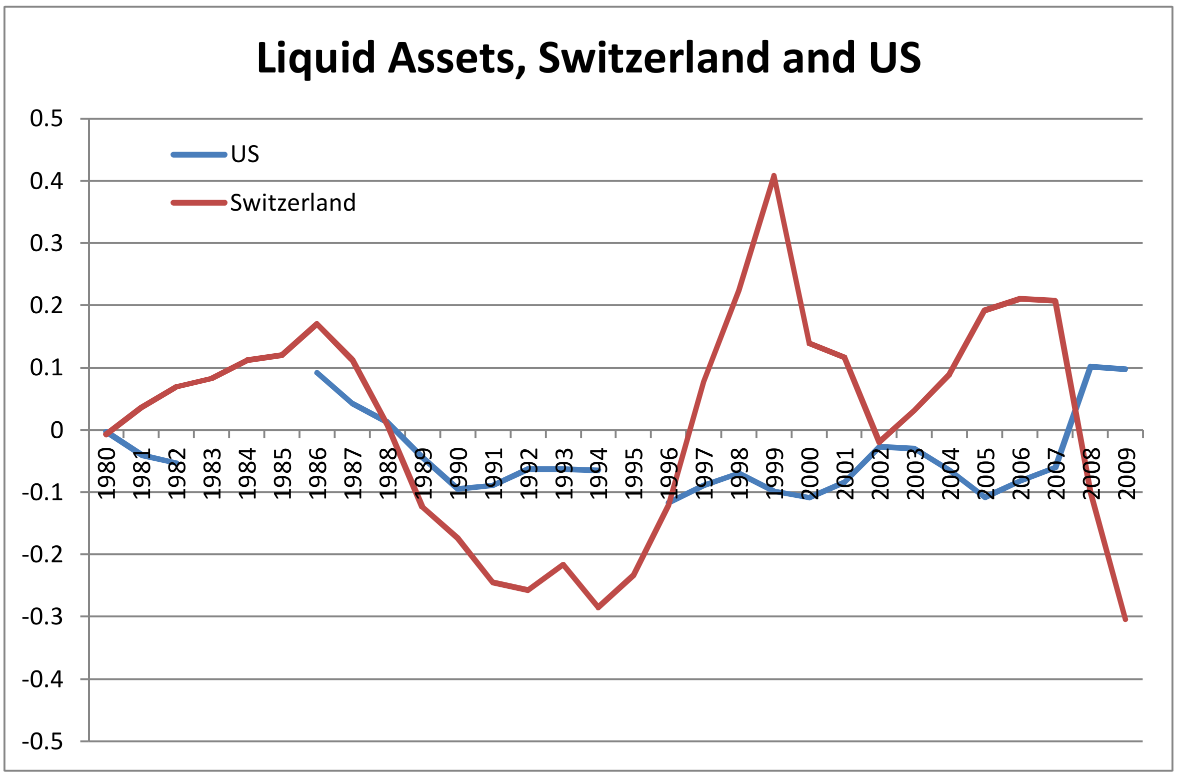


Figure 3. Liquid assets for the banking systems of Switzerland and the United States, 1980-2008. The vertical axis gives shows the liquid assets each year as a fraction of the long-term exponential trend. The data qualitatively support the expectation that countries with low reserve ratios (e.g. Switzerland) might expect wider oscillations than countries with higher reserve ratios (e.g. US). Source: OECD (http://stats.oecd.org/Index.aspx).

The remaining sections of this paper show that a simple dynamic extension of the money multiplier is sufficient to create oscillations. Examination of that simple dynamic structure and its behavior yields additional insights about internal conditions affecting banking system stability.

Before describing the dynamic model structure and analyses, we offer a few brief sections clarifying terminology and perspective.

Bankers, Banks, and the Banking System: these are not the same

When reading about the performance of a financial system, one commonly encounters the words Bankers, Banks, and Banking System employed as if they were interchangeable. In fact they represent different realities.

Bankers are individual persons, very often professional managers. The relationship with the banks is one of principal-agent. The interests of bankers and banks may coincide or may differ. This is obvious in the many accidents with derivatives investments, in which banks were harmed by decisions made by banking officials.

Banks are institutions which take deposits and lend money. They play a role in the working of the whole financial system but have neither complete information from it, nor control of it. Their decisions are made with limited information. Individual banks may create money, but have no way to know whether this is happening or to what extent it is happening.

A *banking system* is the total collection of banks interacting with a common set of customers. The system may generate systemic behaviors not anticipated by individual banks or bankers.

This paper deals with instability in the money multiplier process and attributes that instability to the dynamic behavior of the banking system as a whole, rather than the conscious initiative of individual bankers or individual banks.

Terminology

Although the model developed in this paper is deliberately as simple as possible, it addresses elements from more than one traditional field, and must cope with the sometimes conflicting vocabularies of accounting, banking, and economics. We have been unable to find in the literature a consistent vocabulary for this integrated system, simple though it is. The most challenging phrases are the following.

Dynamic Model: The results of this paper are based on extending the money multiplier analysis to include explicit times for banks to create loans, for loans to be repaid, and for deposits to endure. This results in a differential equation, which is commonly called a "dynamic model". We prefer this term to "simulator" because we are not trying to accurately reproduce the behavior of the entire banking system, but rather explore qualitative behavior modes of the money-multiplier process.

Liquid Assets: The technically correct term would be "cash and cash equivalents". To make diagrams and equations more readable, we prefer to use the shorter term "liquid assets", which we informally redefine to include cash and all other highly liquid assets that banks can use to satisfy deposit withdrawals and the issuing of loans.

Loan Lifetime and *Deposit Lifetime*: Common synonyms and near-synonyms (more often applied to loans than deposits) are maturity, tenor, tenure (in India), and duration. Because both loans and deposits take many different forms, we intend the term “lifetime” to be less contractual and more behavioral. For example, if a loan is refinanced we are interested in the entire span of the effective loan, not the separate maturities of the two constituent loans.

Money Multiplier: Oddly, the banking textbooks we have seen make no mention of the money multiplier, under any name. It seems to be more in the purview of economics, but even there the treatment is uneven. Some economic textbooks describe the process without naming it; it is simply described as the process by which banks can create money. Many texts call it the money multiplier; some refer to it as the credit multiplier.

Multiplier Money: The money created by the money multiplier is rarely given a name. We call it “multiplier money”.

This is not the Austrian Business Cycle Theory

The reader may be aware that several authors, among them scholars of the Austrian school of economics, have contended that the business cycle originates in the monetary side of the economy. The Austrian Business Cycle Theory (ABCT) asserts that cycles arise from changes in interest rates causing shifts in the balance between saving and consumption.

This paper agrees that cycles can arise from inside the banking system, but not in any way related to the ABCT theory. Our simple money-multiplier model omits interest rates, savings, and consumption altogether, yet it generates oscillatory behavior. The point of this paper is that the fractional reserve banking system is capable of endogenous cycles, even without changes in interest rates or external macroeconomic influences. Furthermore, the amplitude of these oscillations is sufficient to cause crises, and the cycles can be explained by combinations of parameter values that occur in real-world banking systems.

Constructing a dynamic model of the money multiplier

The descriptions of the money multiplier in economic textbooks invariably focus on the balance sheets of one or more banks. Therefore, a natural starting place for a dynamic model of the money multiplier would be a simplified combined balance sheet of the commercial operations of an entire banking system. Figure 4 shows the minimum requirements.

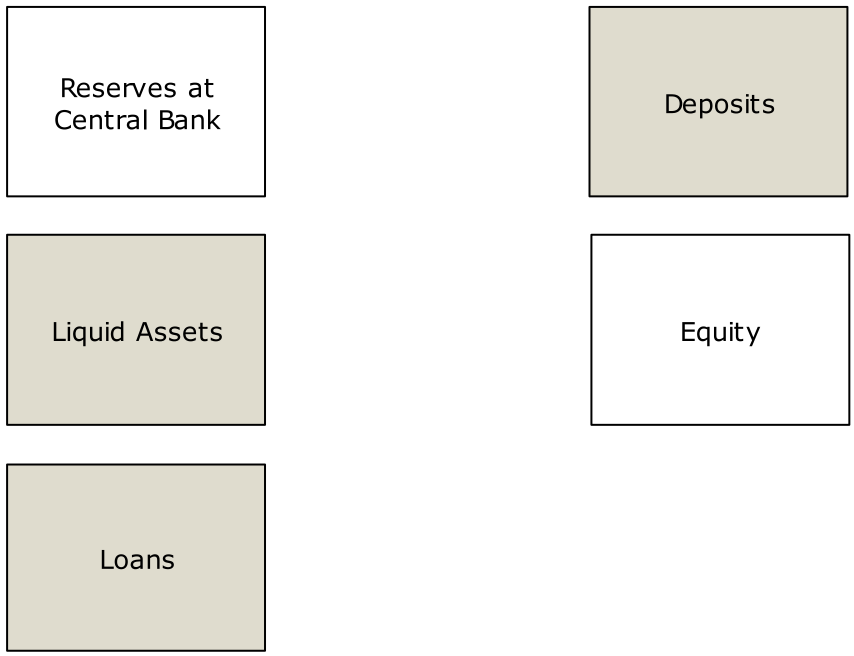


Figure 4. Minimal balance sheet of a banking system. The banks' assets are on the left; their liabilities and equity are on the right. The shaded variables will be shown to be sufficient as state variables of the dynamic model.

An accountant would point out that these five state variables do not have enough information to be considered a proper balance sheet, and the accountant would be correct. Our goal here is to use only those aspects of the balance sheet that are needed to explain the money multiplier. Missing here, for example, are retained earnings. More complete (and more complex) treatments have been developed, but are outside the scope of this paper.

These five balance-sheet variables translate easily into a fourth-order dynamical system, in which the state variables are all of the above, except Equity. Because the balance sheet must always balance (especially if we allow negative values), the Equity can be deduced algebraically from the other four variables:

$$Equity = Reserves\ at\ Central\ Bank + Liquid\ Assets + Loans - Deposits$$

Equilibrium Reserves: It is also possible to eliminate Reserves at Central Bank as a state variable by the following logic. The bank is obligated to maintain reserves at the central bank equal to a fixed fraction of the bank's deposits. In practice this is done by the bank measuring its deposits (or, many cases, deposits of a size or term for which the central bank requires reserves), and quickly adjusting its central-bank reserves so that the ratio of reserves to deposits matches the required Reserve Ratio. This approximation is minor, because the real-world adjustments happen so quickly. There are three time scales in the money multiplier: loans and deposits endure for years, and the generation of loans takes weeks or months, but equilibrating the reserves in the central bank takes only hours or days. If we assume that the Reserve Ratio is constant during any one scenario, then we may approximate the situation by assuring that the central bank reserves are always in equilibrium with the combined banks' deposits. This can be done by simultaneously shifting the right amount of money in or out of the central bank reserves whenever bank customers make a deposit or withdrawal. Specifically, when deposits are made, a fixed fraction of the deposits (the Reserve Ratio) must be put on reserve at the central bank, and when deposits are withdrawn, the corresponding reserves may be retrieved from the central bank. By this "equilibrium reserves" logic, we reduce the system to three state variables, with Reserves at Central Bank maintained by carefully deducting or restoring the required reserves from the appropriate flows among the three state variables of Liquid Assets, Loans, and Deposits. Because all of the flows are then independent of the variable "Reserves at Central Bank", it no longer is a state variable. Like Equity, it has become a useful supplementary output variable, no longer participating in the dynamics of the money multiplier.

We are now prepared to define the model, by writing the equations for the various flows that affect the surviving three state variables. Figure 5 shows the complete structure of the model.

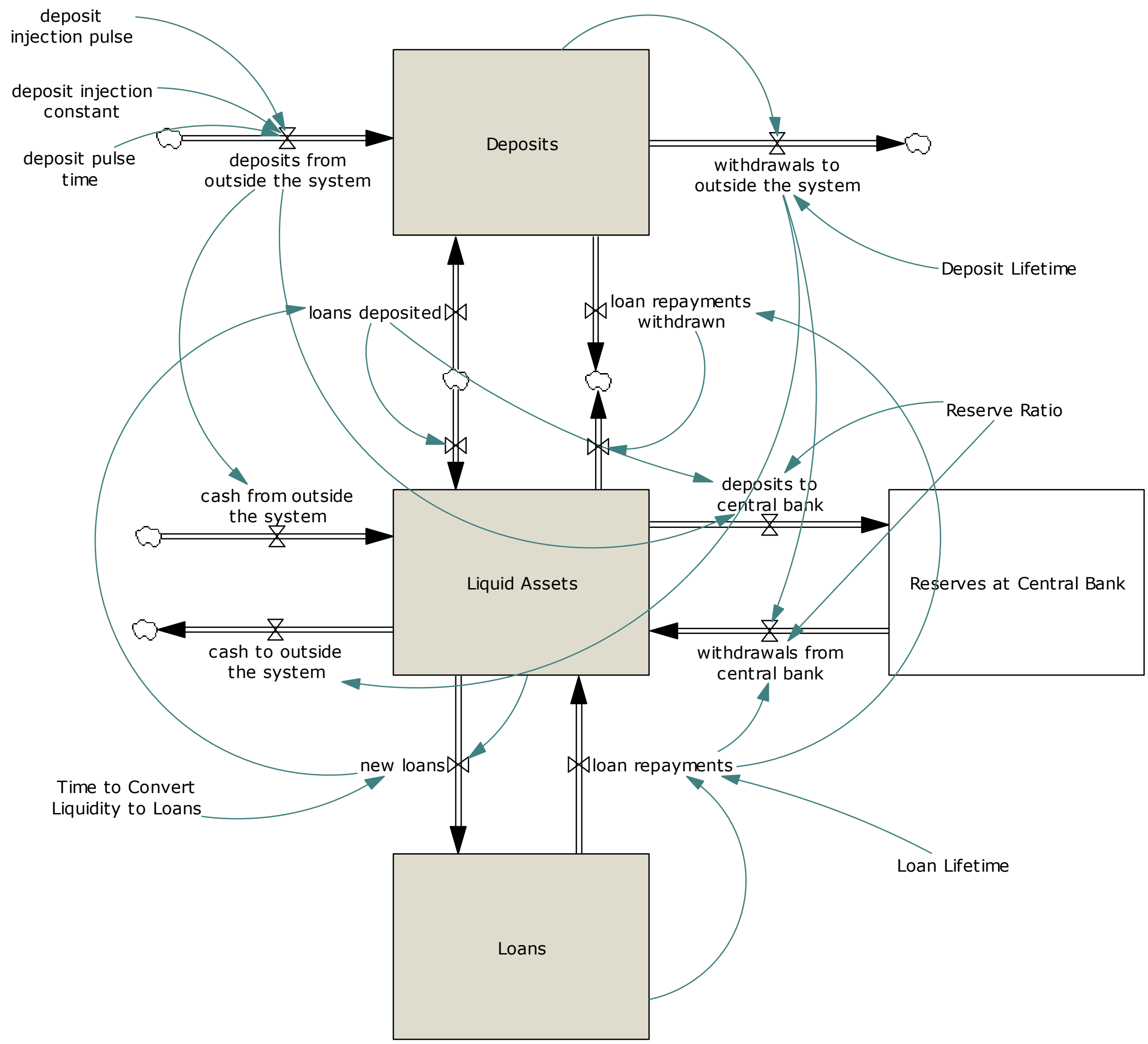


Figure 5. The dynamical structure of the money multiplier. The three state variables are shaded; all the flows of money are functions of those three variables, and the flows in turn change the values of the state variables. Reserves at Central Bank is a supplementary variable, deduced the flows defined by three shaded state variables. The double arrows ("pipes") indicate flows of money; the thin arrows pointing to each flow variable identify the causes of that flow.

Even though the model shown in Figure 5 is, we believe, the simplest possible, it may still seem complex. Some complexity arises from the accounting rules that coordinate the flows affecting deposits with those affecting loans and liquid assets. Additional complexity arises from the need to deduce the flows in and out of the reserves at the central bank. The entire process is based on a few basic flows.

The state variables are denoted in billions of US dollars (gigadollars, or G$); the flows of money are denoted in G$ per month.

The key flow equations of the model are described in the following sections; all model equations are listed in Appendix B.

Basic flows

Deposits from outside the system are exogenous, depending only on time. They may be set to a pulse of money entering the system, or an constant flow of money. They are the only source of "seed" money, because the initial conditions of all the state variables are set to zero.

Cash from outside the system is the asset flow that corresponds to the liability flow of deposits from outside the system. When a customer makes a deposit, it simultaneously boosts not only Deposits but also Liquid Assets, via the flow cash from outside the system.

$$cash\ from\ outside\ the\ system = deposits\ from\ outside\ the\ system$$

Withdrawals to outside the system represent the flow of money out of the banking system. The standard money multiplier analysis in textbooks implicitly assumes this flow is zero, so we discuss it in some detail in a section below. The equation is

$$withdrawals\ to\ outside\ the\ system = \frac{Deposits}{deposit\ lifetime * months\ per\ year}$$

and

$$cash\ to\ outside\ the\ system = withdrawals\ to\ outside\ the\ system$$

The term months per year appears in the above equation because deposit lifetime is denoted in years, but the resulting flow is per month.

$$months\ per\ year = 12$$

New loans transfer cash from Liquid Assets to Loans. From the perspective of an individual bank, the money may be transferred out of the bank to the recipient of the loan. But from the perspective of the banking system (and this model of it), the loaned money immediately appears as a deposit somewhere in the banking system. If some of the loaned money is not redeposited within the system, the phenomenon is represented by withdrawals to outside the system.

$$new\ loans = \frac{Liquid\ Assets}{time\ to\ convert\ liquidity\ into\ loans}$$

The new loans immediately induce new deposits

$$loans\ deposited = new\ loans$$

The resulting loans deposited adds to Deposits and simultaneously restores the liquid assets lost when the loan was made. This is the seeming miracle of the money multiplier: when a loan is made, it is not really lost. The loan appears as a deposit somewhere in the system, so there is no net loss of system liquid assets, but there is a real net gain in system deposits.

Loan repayments return cash from Loans to Liquid Assets.

$$loan\ repayments = \frac{Loans}{loan\ lifetime * months\ per\ year}$$

The loan repayments immediately induce loss of deposits

$$loan\ payments\ withdrawn = loan\ repayments$$

These loan payments withdrawn are the "antimatter" to loans deposited (above). The loan payments withdrawn cancel the gain in system liquid assets, and destroy money by depleting Deposits. Loan repayments are how the money multiplier moves in reverse.

Reserve flows

In addition to the above basic flows, there are two flows that move money between Liquid Assets and Reserves at Central Bank.

$$\begin{aligned} &deposits\ to\ central\ bank \\ &\quad = reserve\ ratio * (\ deposits\ from\ outside\ the\ system + loans\ deposited) \end{aligned}$$

$$\begin{aligned} &withdrawals\ from\ central\ bank \\ &\quad = reserve\ ratio * (withdrawals\ to\ outside\ the\ system \\ &\quad + loan\ repayments) \end{aligned}$$

This completes the description of the model. Complete equation listings are in Appendix B.

The four key parameters

The equations defining the flows have required the use of only four important parameters:

Loan Lifetime (LL) (years): the average duration of all the loan chains in the system. A "loan chain" refers to a sequence of renewed loans. Each single loan in the chain may be nominally short-term, but the renewing gives the loan an effective longer lifetime. If the banking system's total portfolio of loans is dominated by mortgages, the loan lifetime will tend to be long; if dominated by short-term loans, the average lifetime will be shorter.

Deposit Lifetime (DL) (years): As we have shown, the textbook descriptions of the money multiplier implicitly assume that deposit lifetime is in practice infinite. Although customer deposits are often "on demand" or perhaps have an expiry date of a few months, from the banking system's perspective they tend to be renewed once and again since the money basically is kept in the system. But the dynamics of the money multiplier illustrate the serious consequences of the finite lifetime of deposits. In fact, several mechanisms make it realistic to consider finite deposit lifetimes. First, deposits may be converted in to currency (cash) and be removed from the banking system, at least temporarily. Second, suspicions of the insolvency of banks may shift financial investments from deposits to domestic government bonds in search of a more attractive risk. Third, deposits may be sent to foreign investment vehicles outside the system when the domestic sovereign risk is not attractive and the risk premiums are very high. Fourth, tax havens may attract money kept in local deposits by offering aggressively tax advantages. Fifth, government spending in foreign aid or foreign wars may transfer deposits outside the domestic banking system.

Time to Convert Liquidity to Loans (TCLL) (months): In the textbook money multiplier, this time is implicitly the unstated interval between sequential loan/deposit events. By making this parameter explicit, we can represent the effects of bank policies and technologies in granting loans quickly or slowly. It turns out to make a big difference under some realistic conditions.

Reserve Ratio (RR) (fraction): As stated above, this is the target ratio of Reserves in Central Bank, as a fraction of Deposits. In a real banking system, this target usually imposed by regulation, and it must be realized by the banks in hours or days, by the rapid movement of reserves in and out of the accounts of the central bank. In our model, to reduce the model to three state variables and make the analysis simpler, we approximate these adjustments as happening instantly (the "reserve equilibrium" approximation). In a real banking system where the reserve requirement is zero (as in Switzerland or New Zealand), the parameter can still be used to approximate the spontaneous, unregulated behavior of the banks in deciding the magnitude of reserves to be held to self-insure adequate resources to satisfy customer withdrawals.

The model is linear

Inspection of the flow equations shows that every flow in the model is the sum of one or more products, each product being a state variable multiplied or divided by one or more constants. Therefore, by definition, the model is a linear dynamical system.

The linearity arises naturally from the simple rules of accounting and the reserve requirements. The linearity is approximate in the sense that some of the variables in some simulations may exhibit negative values. But these negatives can usually be interpreted in a natural and realistic way. For example, if liquid assets are negative, it will cause loans to be recalled via negative new loans. But exactly this sort of thing happens in real banking systems. If banks find themselves illiquid, they may compensate by slowing or stopping the issuing of new loans and calling in some existing loans before they would normally be due. If such details are represented explicitly, the model becomes more complex and technically nonlinear, but it remains virtually linear, and behaves much the same.

Given the unusual gift of a realistic model that happens also to be linear, we have taken advantage by applying linear analytic techniques confirm aspects of the model structure and to gain insight into the behavior of the money multiplier. The resulting eigenvalue analysis is summarized in Appendix A.

Comments on the loop structure of the model

Written in the usual way (the way it was, in fact, developed), the model diagram of Figure 5 may seem a tangle of money flows and information links. Another valuable perspective can be gained by, for each level, combining all the flows affecting that level into a single "net change" flow. This algebraic simplification preserves exactly the same dynamics and degrees of freedom of the more complicated view, and it better reveals some of the model's structure. Figure 6 shows the result. The major information links have been color coded: red links (think "hot water") are positive, blue links ("cold water") are negative.

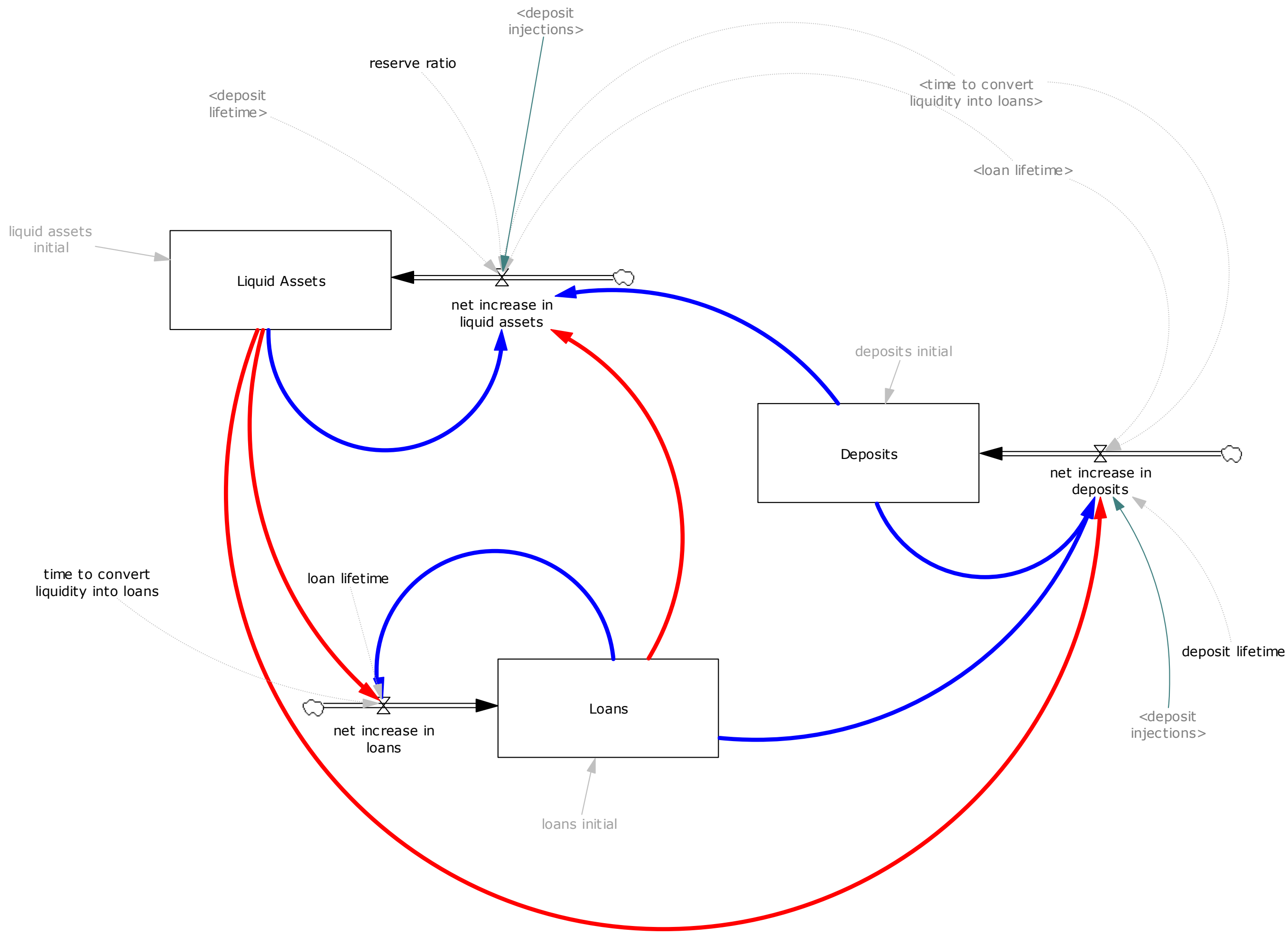


Figure 6. Reduced-form model, with a single flow (derivative) for each state variable. For any combination of positive parameter values, red links are always positive; blue links are always negative.

The outermost loop {Liquid Assets → Deposits →Liquid Assets} is negative. Two other loops are positive: {Liquid Assets → Loans → Liquid Assets} and {Liquid Assets → Loans → Deposits → Liquid Assets}. Negative loops, depending on their parameters, may oscillate; positive loops may amplify such oscillations. The combined positive and negative loops suggest the possibility of runaway growth, collapse, or oscillations.

But in addition to the three multi-state-variable loops described above, each level has a self-damping negative loop. These loops will generally tend to dampen and stabilize the system, reducing or eliminating oscillations and preventing runaway growth. Therefore, it is not obvious from the structure what the behavior will be, except that there are already hints of possibilities beyond the “Wikipedia” scenario of stable approach to equilibrium.

The dynamical system can reproduce the standard textbook behavior

We now simulate the model to explore its behavior. We have used both simulation and analysis to understand the possible range of behavior of the system; we start by demonstrating that for at least some combination of parameters, we can duplicate the standard textbook results.

The key results of the standard textbook description are seen in Figure 1: First, given a single pulse of money, the deposits grow in a smooth exponential approach to a constant level. Second that equilibrium level of deposits is the size of the initial pulse of money, divided by the reserve ratio.

To achieve these textbook properties, it is possible to prove that the following parameter values must apply:

$$loan\ lifetime = \infty$$

$$deposit\ lifetime = \infty$$

Figure 7 shows the result of a few simulations with the two infinite lifetimes.

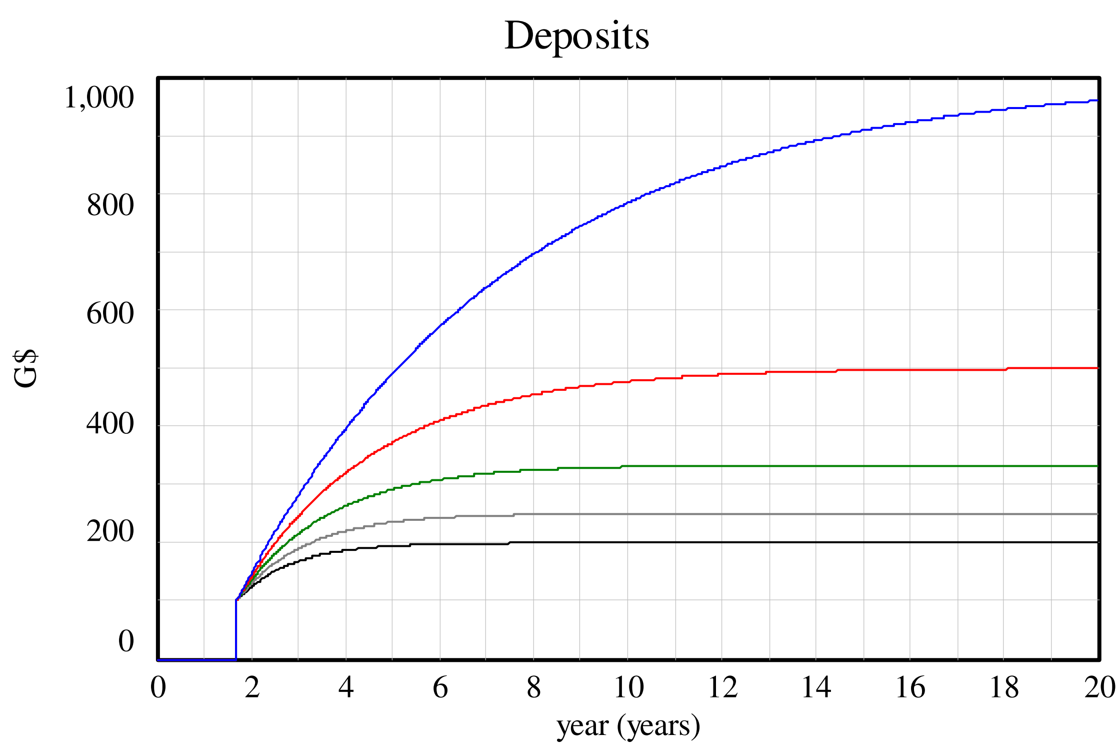


Figure 7. Five simulations of the dynamic money multiplier, with infinite lifetimes for both loans and deposits. The results duplicate the Wikipedia results shown in Figure 1, except that $ have been replaced with G$, and the horizontal axis here is explicitly time, not sequence of transactions.

The higher the reserve ratio, the faster the system approaches equilibrium, but it then creates less money.

Exploring the range of behavior: this same dynamic structure also oscillates

We now explore other values for the four key parameters, and find that for low reserve ratios, the system is prone to oscillations.

Effect of varying Deposit Lifetime and Loan Lifetime

We start with a series of simulations showing what happens if the two lifetimes take on values more realistic than the "infinity" assumption of the textbooks.

For each simulation (named in the legends beneath each graph), the values of the four key parameters are listed in a standard order. Also in the graphs, we often use abbreviations for the parameters:

LL = Loan Lifetime (years)

DL = Deposit Lifetime (years)

TCLL = Time to Convert Liquidity into Loans (months)

RR = Reserve Ratio (percent)

Each simulation is labeled in the above sequence: LL_DL_TCLL_RR For example, simulation 2_10_1_5 would have as inputs LL = 2 years, DL = 10 years, TCLL = 1 month and RR = 5%.

Figures 8-11 exercise Loan Lifetime and Deposit Lifetime over a range of four values: 2 years, 10 years, 20 years, and 100 years. Throughout, we assume TCLL = 1 month, and RR = 0%.

Effect of Increasing DL

(Base LL = 2 yrs, TCLL = 1 mo, RR = 0%)

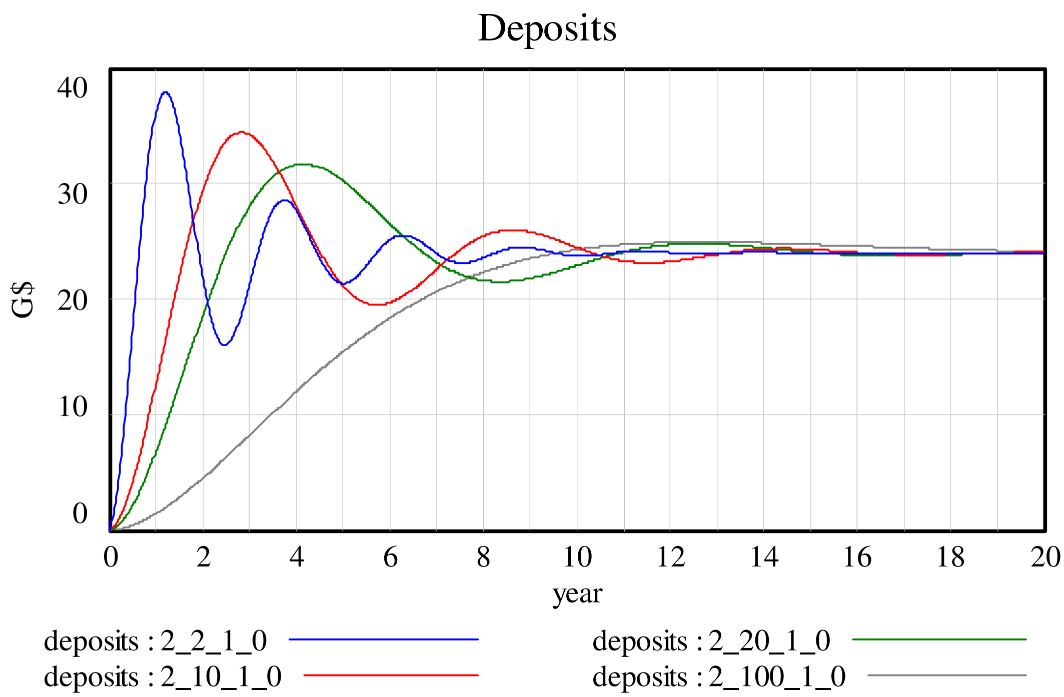


Figure 8. Loan Lifetime is held constant at 2 years, while Deposit Lifetime takes on values of 2, 10, 20, and 100 years. Time to Convert Liquidity to Loans is 1 month, and the Reserve Ratio is 0%. Each simulation is driven by a constant flow of deposits from outside the system ("step response"), and the magnitude of that flow is adjusted to make the vertical scales comparable.

Figure 8 shows that if Loan Lifetime is short (2 years), then short Deposit Lifetimes imply more rapid oscillations. The longer DL becomes, the longer the period of the oscillation, and the more rapidly the oscillation dies out.

Effect of Increasing DL

(Base LL = 20 yrs, TCLL = 1 mo, RR = 0%)

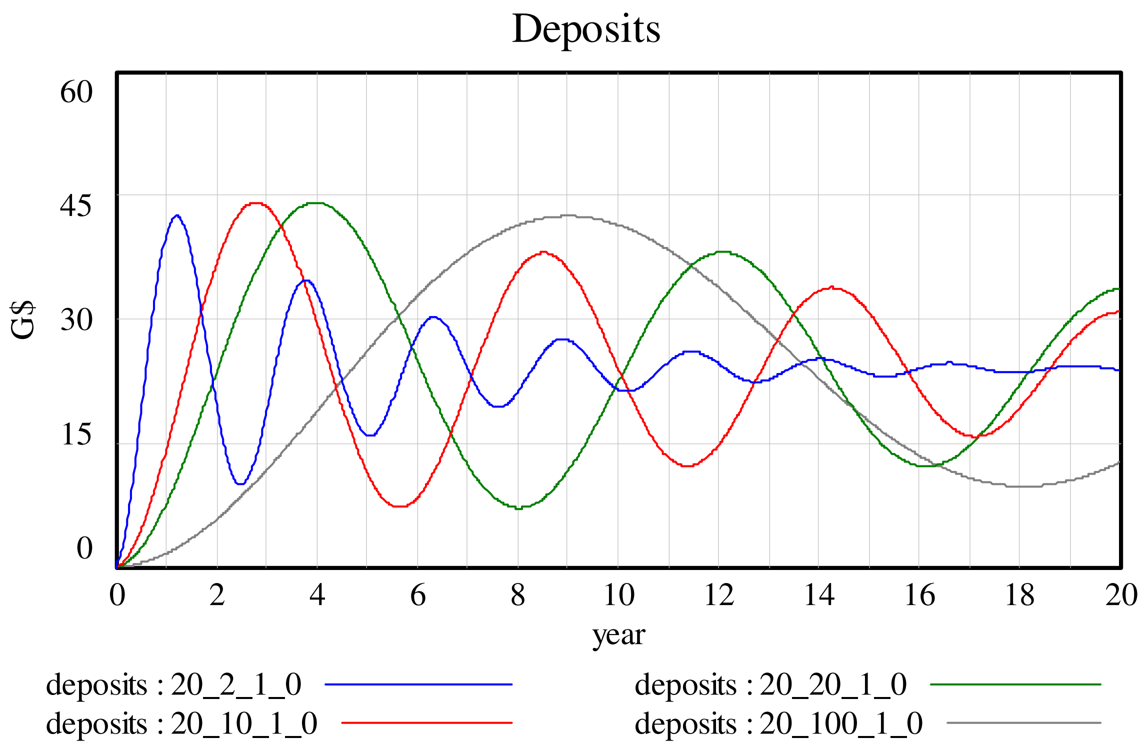


Figure 9. Same as Figure 8, but Loan Lifetime is held constant at 20 years instead of 2. As before, Deposit Lifetime takes on values of 2, 10, 20, and 100 years. Time to Convert Liquidity to Loans is 1 month, and the Reserve Ratio is 0%. Each simulation is driven by a constant flow of deposits from outside the system ("step response"), and the magnitude of that flow is adjusted to make the vertical scales comparable.

The simulations of Figure 9 show a qualitative change from Figure 8. The only difference is that in Figure 8, the Loan Lifetime is 2 years; in Figure 9 it is 20 years. The longer Loan Lifetime seriously destabilizes the system. For all values of Deposit Lifetime, the oscillations are higher in amplitude and die out much more slowly. More importantly, if Loan Lifetime is long, then longer Deposit Lifetimes no longer stabilize the system. In fact, if Loan Lifetime is long, then longer Deposit Lifetimes actually make the oscillations more sustained. As before, however, the period of the oscillation is roughly proportional to Deposit Lifetime.

There is an important implication of Figure 8 and Figure 9: It is possible that a banking system could be destabilized by a spontaneous shift to portfolios of loans with longer lifetimes. Because the longest loans are typically mortgages, this means that a subprime mortgage boom could be destabilizing, even if no loans defaulted, simply because increased selling of mortgages will bias the system's combined loan portfolio toward a longer Loan Lifetime. The system tends to be more stable if Deposit Lifetime is greater than Loan Lifetime, but this works only if Loan Lifetime is absolutely short.

We now consider a similar sequence of simulations, in which Deposit Lifetime is held constant, while Loan Lifetime is varied.

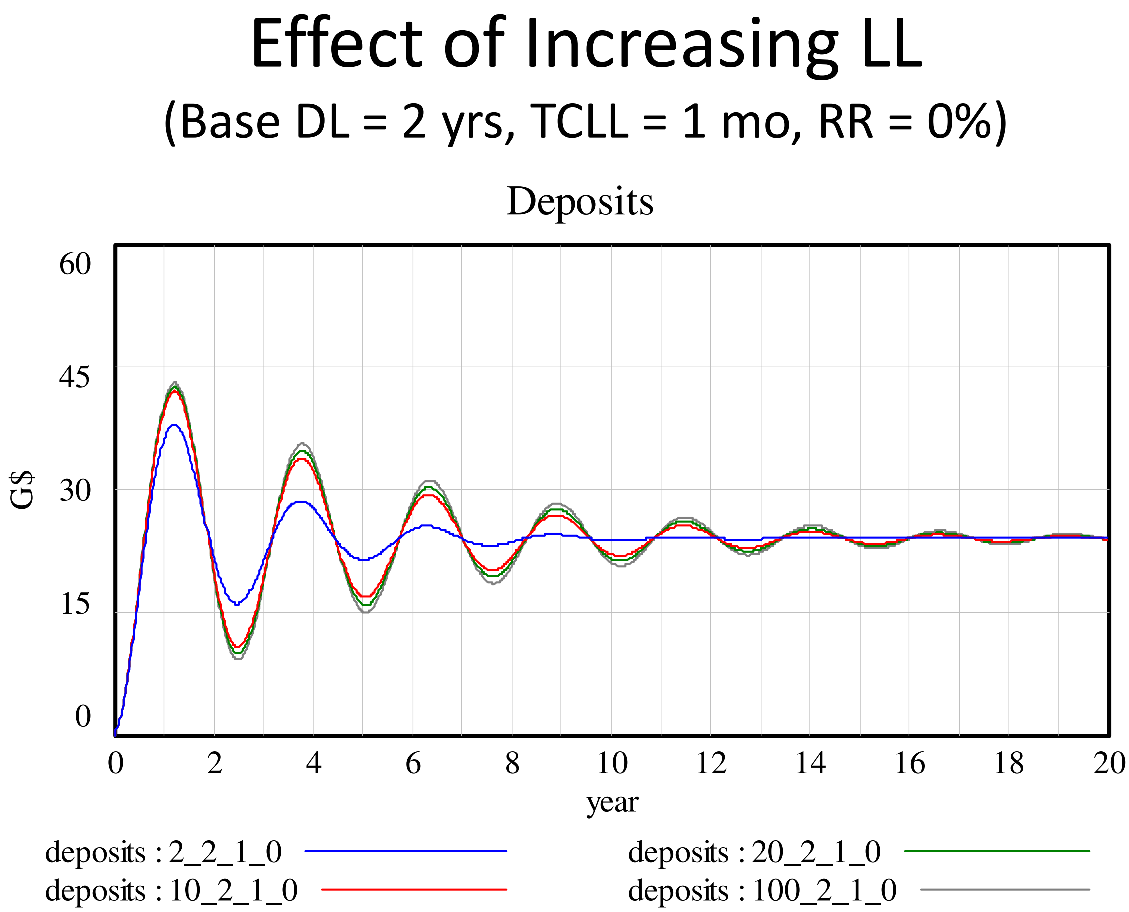


Figure 10. Deposit Lifetime is held constant at 2 years, while Loan Lifetime takes on values of 2, 10, 20, and 100 years. Time to Convert Liquidity to Loans is 1 month, and the Reserve Ratio is 0%. Each simulation is driven by a constant flow of deposits from outside the system ("step response").

The simulations of Figure 10 suggest that shorter Loan Lifetimes stabilize the system somewhat, but have no effect on the period of the oscillation.

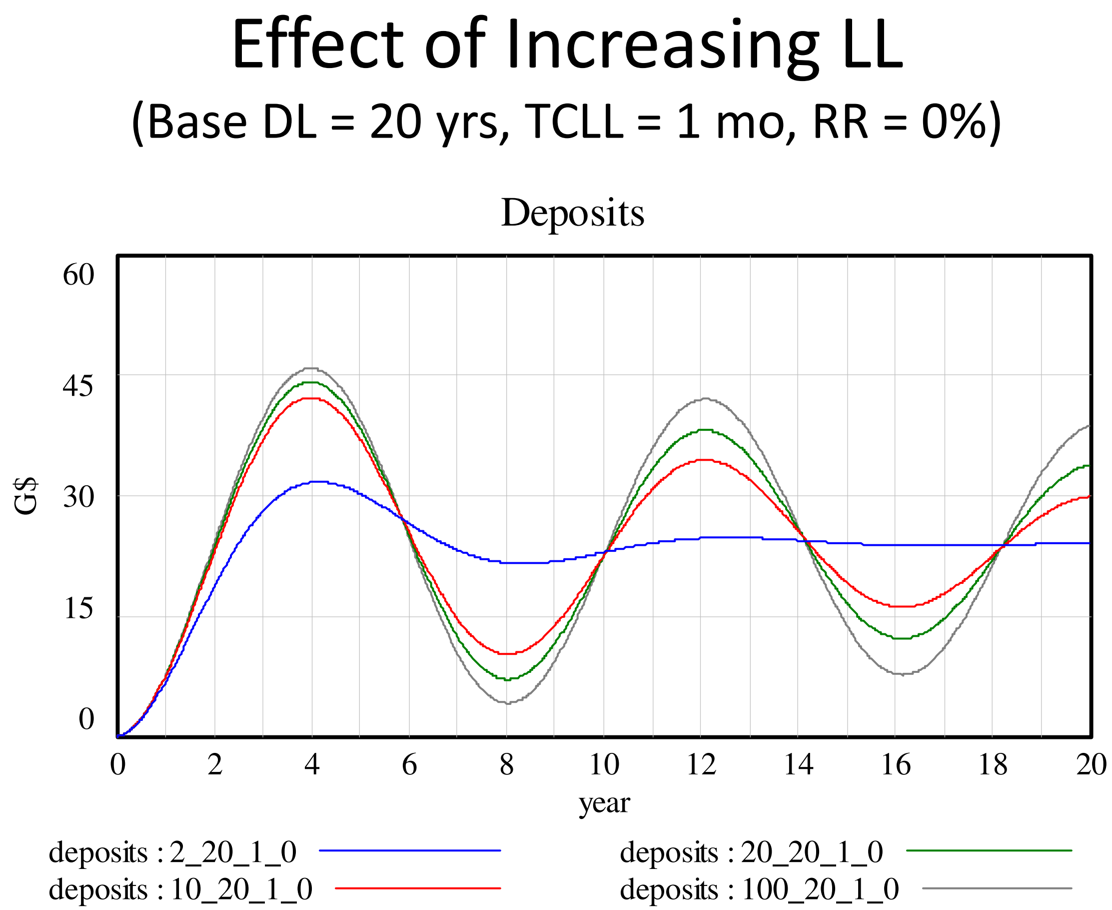


Figure 11. Same as Figure 10, but Deposit Lifetime is held constant at 20 years instead of 2. As before, Loan Lifetime takes on values of 2, 10, 20, and 100 years. Time to Convert Liquidity to Loans is 1 month, and the Reserve Ratio is 0%.

The simulations of Figure 11 confirm the pattern seen in the earlier figures: Longer Loan Lifetimes are destabilizing, but do not affect the period of the oscillation, and the period of oscillation is roughly proportional to Deposit Lifetime.

Effects of varying Time to Convert Liquidity into Loans

Figures 12 and 13 show the effect of varying the Time to Convert Liquidity into Loans, in two different contexts of Loan and Deposit Lifetimes.

Effect of Increasing TCLL

(Base LL = 2 yrs, DL = 2 yrs, RR = 0%)

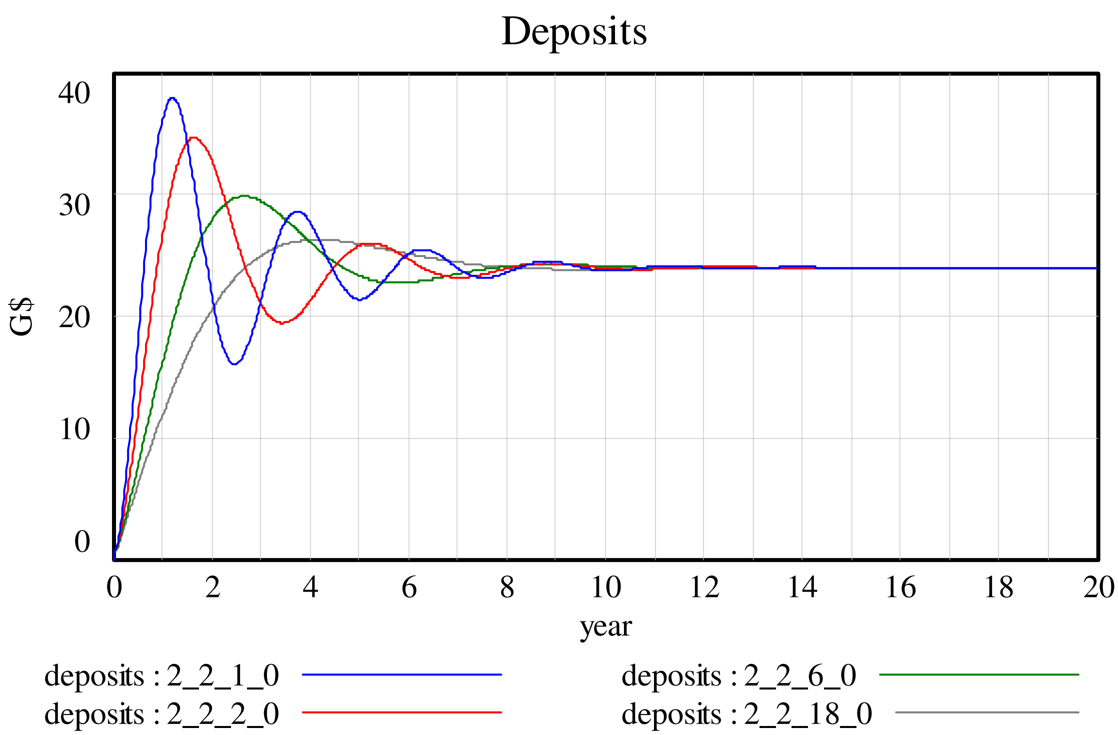


Figure 12. Loan Lifetime and Deposit Lifetime are both held constant at 2 years, while Time to Convert Liquidity into Loans takes on values of 1, 2, 6, and 18 months. The Reserve Ratio remains 0%. Each simulation is driven by a constant flow of deposits from outside the system ("step response").

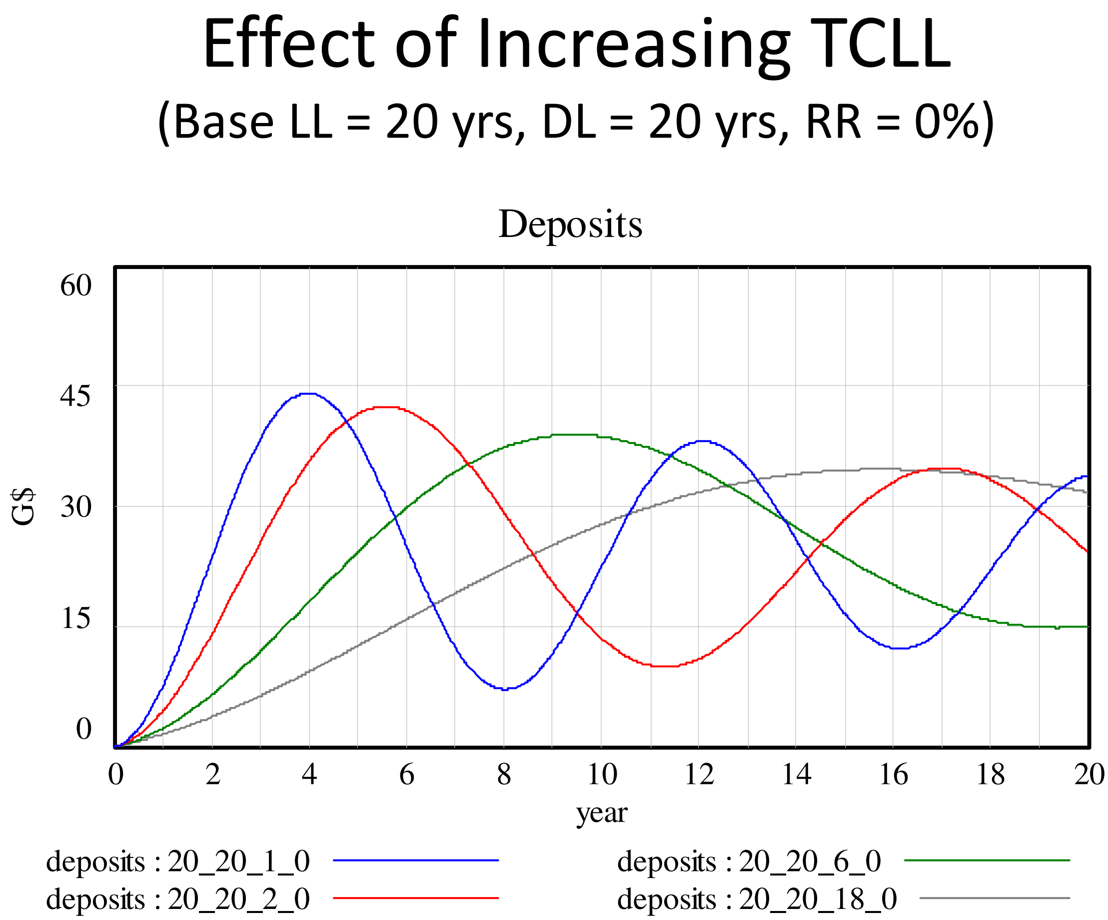


Figure 13. Same as Figure 12, but Loan Lifetime and Deposit Lifetime are held constant at 20 years instead of 2. The Reserve Ratio remains 0%. As before, Time to Convert Liquidity into Loans takes on values of 1, 2, 6, and 18 months.

Figures 12 shows the consequences of the speed with which banks process loan applications, and the fraction of loans approved. Both of these factors influence the average Time to Convert Liquidity into Loans. If the banks are fast at writing lots of loans, the system is destabilized, and the period of oscillations is shorter. But Figure 13 reminds us that long Loan Lifetimes can defeat stabilization effects. Longer Time to Convert Liquidity into Loans fails to stabilize the system *if Loan Lifetimes are long*.

The subprime mortgage boom had two effects: 1) It biased loan portfolios toward longer loan lifetimes, and 2) it sped up the process of granting loans. We have now shown that both of these effects destabilize the system and promote oscillation, so it may be said that a subprime mortgage boom could twice over destabilize a banking system, *even if none of the mortgages fail*.

Qualitative Observations and Conclusions

First, some cautions: The model used here is deliberately the simplest possible for understanding the core dynamics of the money multiplier, stopping short of the complete dynamics of the banking system.

Specifically, in focusing only on the money multiplier, the model does not address many aspects and activities of modern integrated banks. The addition of such aspects in some cases might change the conclusions.

Having issued the above caution, we now summarize what can be learned from the simple dynamics of the money multiplier (keeping in mind that these effects could be amplified or counteracted by external events or additional layers of policies and regulation outside the money multiplier).

Nevertheless, it is good to know the "physics" at the core of the money supply, as a starting framework to understand the consequences of incremental added concepts.

Effect of Loan Lifetime: Long-term loans are destabilizing, especially if Deposit Lifetime is also long. Loan Lifetime has no effect on the oscillation period, but longer loans sustain oscillations (less damping). The effect of long Loan Lifetime is more severe if Deposit Lifetime is also long. Therefore, a shift to long-term loans (e.g. mortgages) is, by itself, destabilizing, and must be compensated by an increased reserve ratio or some other stabilizing strategy.

Effect of Deposit Lifetime: Extending Depost Lifetime lengthens the period of oscillations, and damps the oscillations, but only if Loan Lifetime is short (<5 years). If Loan Lifetime is long (for example, 20 years), increasing Deposit Lifetime only increases the period of the oscillation, without stabilizing it.

Effect of Time to Convert Liquidity into Loans: rapid loan-writing can be destabilizing. Longer loan delay lengthens the period of oscillations, but damps them. But slowing loans is ineffective if Loan Lifetime is long, with the oscillations increasing in period, but do not dying away.

Effect of the Reserve Ratio: Larger reserve ratios simply damp the oscillations, with no effect on the period. Therefore, it is an effective stabilizer of the core dynamics of the system.

Summary Conclusions

- The techniques used here, extended further into the banking system surrounding the money multiplier, may suggest new approaches to more effective regulation of banking systems
- Knowledge of these fundamental dynamics may be helpful to the management of banks (e.g. Asset and Liability Management committees)

Appendix A: Eigenvalue analysis of the system

Because the model is third-order linear, it is amenable to closed-form analytic solution.

The eigenvalues of the system determine its potential behaviors. Because the system is third-order, there are three eigenvalues:

0,

$$\frac{-DL\,LL\,RR - DL\,TCLL - LL\,TCLL - \sqrt{-4\,DL\,LL\,TCLL\,(LL + TCLL) + (DL\,LL\,RR + DL\,TCLL + LL\,TCLL)^2}}{2\,DL\,LL\,TCLL}$$

$$\frac{-DL\,LL\,RR - DL\,TCLL - LL\,TCLL + \sqrt{-4\,DL\,LL\,TCLL\,(LL + TCLL) + (DL\,LL\,RR + DL\,TCLL + LL\,TCLL)^2}}{2\,DL\,LL\,TCLL}$$

Inspection of these eigenvalue expressions shows that

- Because all four parameters are normally positive, the real parts of any eigenvalue must be negative, implying at worst slowly decaying oscillations
- The expressions under the radical signs can go negative, indicating complex eigenvalues and oscillations. Complex (oscillatory) eigenvalues happen whenever $\frac{TCLL\,(DL-LL)^2}{4\,DL\,LL^2} < 1$.
- The expressions under the radical signs can also be positive, implying negative real eigenvalues, with stable asymptotic behavior
- The zero eigenvalue implies a steady-state solution consisting of zero deposits, with loans and liquid assets conforming to:

$$\frac{Loans}{Liquid\ Assets} = \frac{LL}{TCLL}$$

For the textbook case shown in Figures 1 and 7, both LL and DL are infinite. As LL and DL approach infinity, the three eigenvalues approach the following values:

0, 0, and $\frac{-RR}{TCLL}$.

The zero eigenvalues allow steady-state solutions, with the decaying exponential implied by the third eigenvalue being added or subtracted. The solution for Deposits is of the form

$$\frac{K}{RR}\left(1 - e^{\frac{-RR}{TCLL}t}\right),$$

where K is the magnitude of the initial pulse of deposits from outside the system ($100 in the Wikipedia example). This implies an asymptote of $\frac{\mathrm{K}}{\mathrm{RR}}$, approached with a time constant of $\frac{\mathrm{TCLL}}{\mathrm{RR}}$.

Inspection of Figures 1 and 7 confirm the asymptotes and the time constants, where the Wikipedia TCLL is the time for one transaction. In the dynamic model, TCLL is given explicitly.

Appendix B: Equation Listing of the Model

```
********************************
   .money multiplier dynamics 02
********************************

(01)  cash from outside the system=
            deposits from outside the system
      Units: G$/Month

(02)  cash to outside the system=
            withdrawals to outside the system
      Units: G$ / Month

(03)  deposit injection constant=
            0
      Units: G$/Month

(04)  deposit injection pulse=
            0
      Units: G$

(05)  Deposit Lifetime=
            20
      Units: years

(06)  deposit pulse time=
            20
      Units: Month

(07)  Deposits= INTEG (
            deposits from outside the system
            +loans deposited
            -loan repayments withdrawn
            -withdrawals to outside the system,
                  0)
      Units: G$

(08)  deposits from outside the system=
            deposit injection constant
             + deposit injection pulse / TIME STEP * PULSE(deposit pulse
time, TIME STEP
      )
      Units: G$/Month

(09)  deposits to central bank=
            Reserve Ratio * ( deposits from outside the system + loans
deposited )
```

```
     Units: G$ / Month

(10) Liquid Assets= INTEG (
           cash from outside the system
           +loan repayments
           +loans deposited
           +withdrawals from central bank
           -cash to outside the system
           -deposits to central bank
           - loan repayments withdrawn
           -new loans,
                 0)
     Units: G$

(11) Loan Lifetime=
           20
     Units: years

(12) loan repayments=
           Loans / ( Loan Lifetime * months per year )
     Units: G$/Month

(13) loan repayments withdrawn=
           loan repayments
     Units: G$ / Month

(14) Loans= INTEG (
           new loans
           -loan repayments,
                 0)
     Units: G$

(15) loans deposited=
           new loans
     Units: G$/Month

(16) months per year==
           12
     Units: months / year

(17) new loans=
           Liquid Assets / Time to Convert Liquidity to Loans
     Units: G$/Month

(18) Reserve Ratio=
           0
     Units: fraction

(19) Reserves at Central Bank= INTEG (
           deposits to central bank
           -withdrawals from central bank,
                 0)
     Units: G$
```

```
(20)  Time to Convert Liquidity to Loans=
            1
      Units: months

(21)  withdrawals from central bank=
            Reserve Ratio * ( loan repayments + withdrawals to outside the
system )
      Units: G$/Month

(22)  withdrawals to outside the system=
            Deposits / ( Deposit Lifetime * months per year )
      Units: G$/Month

********************************
   .Control
********************************

            Simulation Control Parameters

(23)  FINAL TIME  = 120
      Units: Month

(24)  INITIAL TIME  = 0
      Units: Month

(25)  SAVEPER  =
              TIME STEP
      Units: Month

(26)  TIME STEP  = 0.25
      Units: Month
```

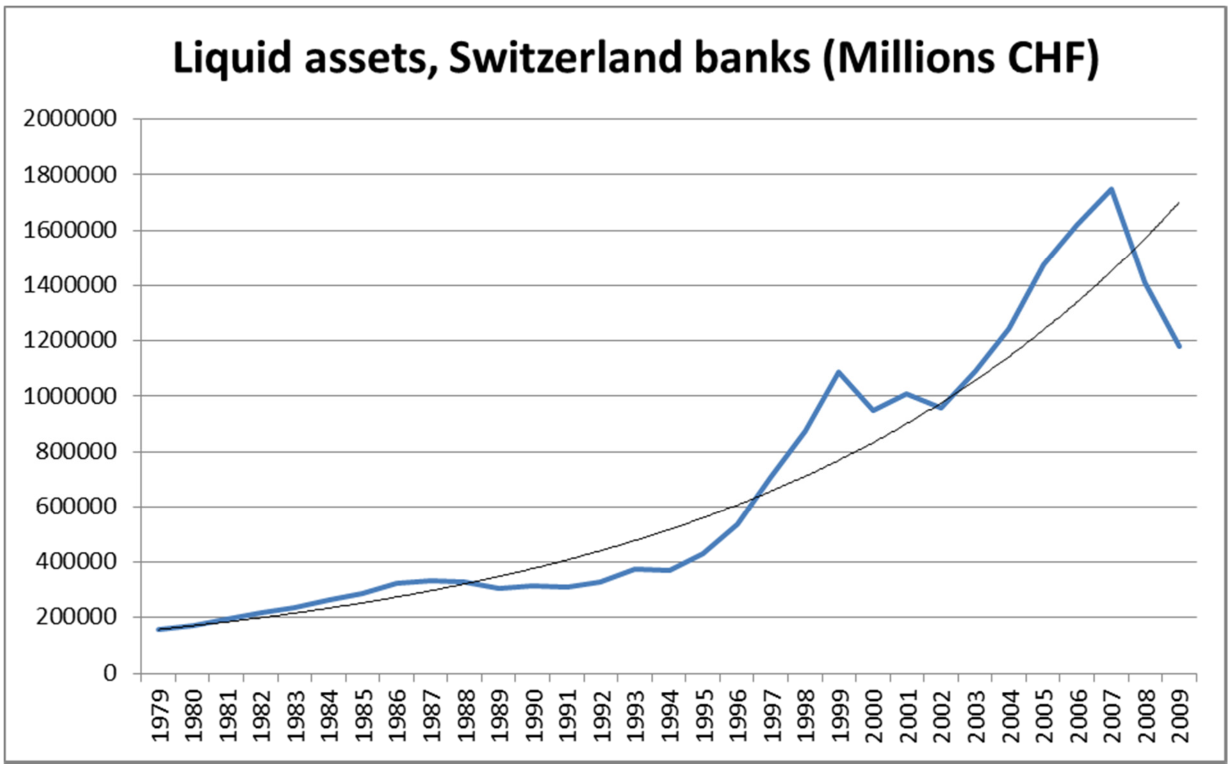
Alphabetic Index:

(01) cash from outside the system
(02) cash to outside the system
(03) deposit injection constant
(04) deposit injection pulse
(05) Deposit Lifetime
(06) deposit pulse time
(07) Deposits
(08) deposits from outside the system
(09) deposits to central bank
(23) FINAL TIME
(24) INITIAL TIME
(10) Liquid Assets
(11) Loan Lifetime
(12) loan repayments
(13) loan repayments withdrawn
(14) Loans
(15) loans deposited
(16) months per year

```
(17)     new loans
(18)     Reserve Ratio
(19)     Reserves at Central Bank
(25)     SAVEPER
(00)     Time
(26)     TIME STEP
(20)     Time to Convert Liquidity to Loans
(21)     withdrawals from central bank
(22)     withdrawals to outside the system
(26)     TIME STEP
```

Appendix C: Derivation of Figure 3

OECD data from http://stats.oecd.org/Index.aspx gave the total liquid assets for the banking systems of Switzerland and the United States. This data was plotted in Excel and fitted with an exponential trend, as shown here for Switzerland:



The curves of Figure 3 were then computed as $\frac{(assets-trend)}{trend}$.

Appendix D: References

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