Controlling the Direction of a Model Helicopter[1]

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Abstract
We developed a feedback model for the control of the angular position of a model helicopter that has a single main rotor and single tail rotor. Basically, there are two mechanisms involved in the control of angular position in the horizontal plane; the torque resulting from the rotation of the main rotor and the opposite direction torque created by the tail rotor. For the purpose of this study, the rotational speed of the main rotor is kept constant. The angular position is solely controlled through the change in the tail rotor's rotational speed. Tail rotor's rotational speed determines the net torque on the fuselage; the net torque determines the angular acceleration; the angular acceleration changes the angular velocity; the angular velocity changes the angular position. Therefore, the angular position of the fuselage is indirectly controlled by the tail rotor's rotational speed. The delay introduced by the indirect control process makes it difficult to control the angular position of the model helicopter. After modeling the physical structure of the model helicopter, we simulated it with two different decision making heuristics. The Modified-VSL heuristic that is introduced in this paper performs much better than the simple naive heuristic.

Keywords: control task; model helicopter; angular position; dynamic decision making; virtual supply line; measurement delay; Modified-VSL; indirect control.

1. Introduction

There are different types of model helicopters. Our aim is not to model all. The aim of this study is to show that a modified version of the Virtual Supply Line Heuristic, which was developed by Yasarcan and Barlas (2005), can successfully be applied to different control problems. Therefore, this study will be a part of a series of Virtual Supply Line Heuristic applications. For this purpose, we selected a model helicopter with a single main rotor and single tail rotor to be the subject of the study. We are only interested in the control of the angular position (direction) of the helicopter. Therefore, we assumed that the rotational speed of the main rotor of the helicopter is constant and the helicopter is hovering in the air.

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Main rotor rotates at a constant speed and as a result applies a constant torque[2] on the fuselage (body of the model helicopter). Tail rotor creates a torque in the opposite direction. Depending on the speed of the tail rotor, tail rotor can produce a torque greater than, equal to, or less than the torque created by the main rotor. If both the torques are equal to each other, they cancel out. In that case, if the helicopter is still, it will continue to stay still. Low and high speeds of the tail rotor is the main source for the change in the direction. Therefore, the direction of the fuselage is controlled indirectly by the rotational speed of the tail rotor.

We first modeled the case described above. In section 2, the model is explained in full detail. In section 3, we introduced two decision making heuristics and compared their performances under two different scenarios; in the absence and presence of measurement delay. Note that the fuselage can approach a target from both sides because of the circularity of the path. We also suggested a small variation in calculating the discrepancy in order to make the fuselage approach the target from the closer side.

2. The Model

2.1. Assumptions on the Physical Aspects of the Model Helicopter

- The helicopter has a single main rotor and torque canceling tail rotor.
- The fuselage of the model helicopter is assumed to consist of a sphere (the main part of the fuselage) and a thin rod attached to the sphere (the tail of the helicopter). This assumption eases the calculation of the moment of inertia of the fuselage. (See Appendix A).
- The total length of the helicopter is assumed to be 1.41 m.
- The weight of the helicopter is taken to be 4.8 kg.

Note that, in order to have realistic values, the length and weight of the helicopter is taken from Coppa et al. (2008).

2.2. Assumptions on Directions

- The counter clockwise direction is the positive direction.
- The main rotor of the helicopter rotates clockwise.
- The direction of the torque caused by the main rotor is counter-clockwise.
- The direction of the torque caused by the tail rotor is clockwise.

2.3. Stock-Flow Diagram

The stock-flow diagram of the model is given in Figure 1. This diagram represents the physical structure of the described problem. It does not involve decision making heuristics.

[2] Torque is the tendency to rotate things. Angular acceleration is the result of the net torque applied on a mass and the moment of inertia of that mass.

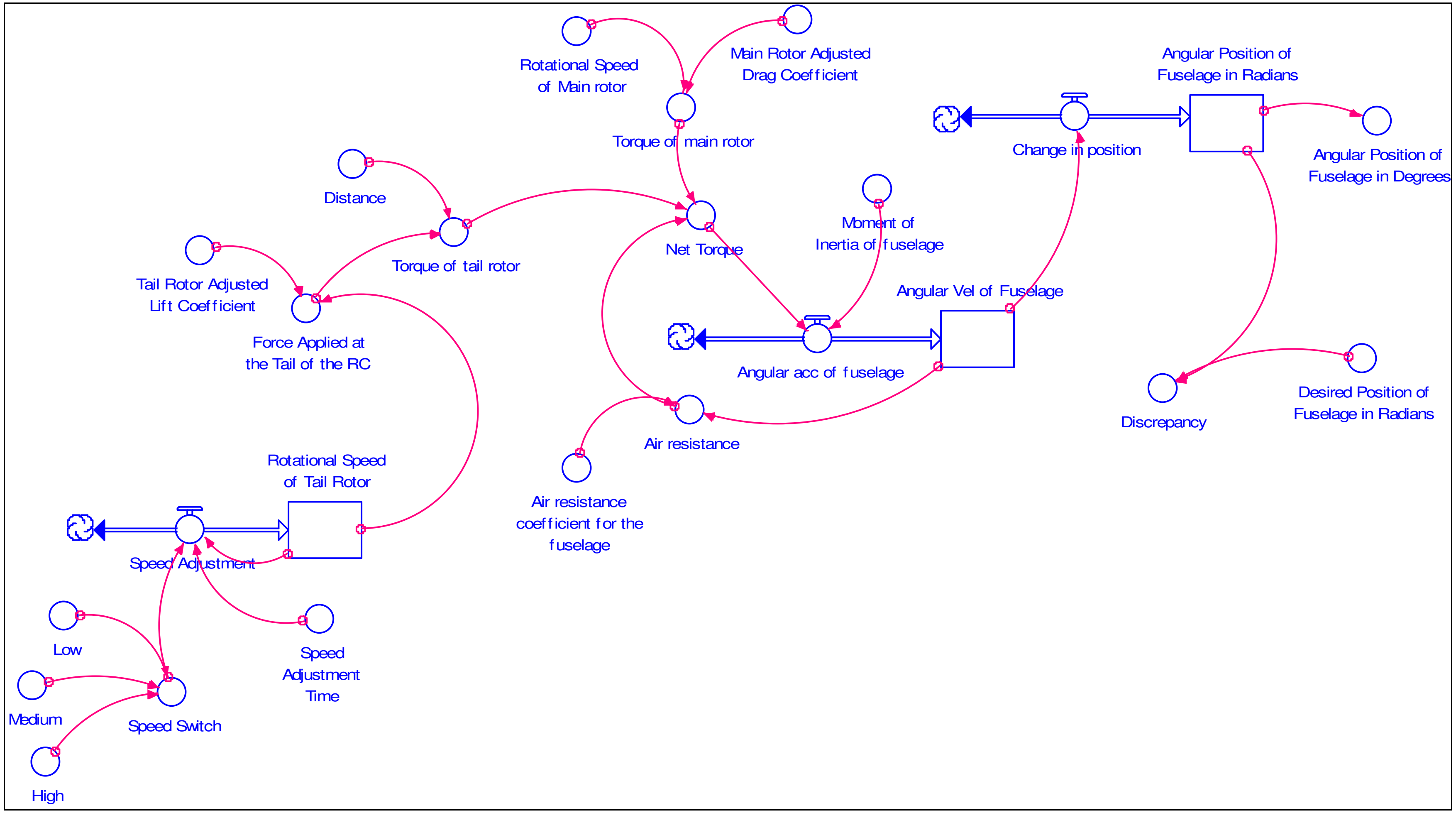

Figure 1 : Stock-flow diagram of the model

2.4. Model Equations

- Initial values and approximate integral equations for the stock variables:

$$Angular\ Position\ of\ Fuselage\ in\ Radians_0 = 0\ [rad] \tag{1}$$

$$\begin{pmatrix} Angular\ Position\ of \\ Fuselage\ in\ Radians \end{pmatrix}_{t+DT} = \begin{pmatrix} Angular\ Position\ of \\ Fuselage\ in\ Radians \end{pmatrix}_t + \begin{pmatrix} Change\ in \\ position \end{pmatrix} \bullet DT\ [rad] \tag{2}$$

$$Angular\ Vel\ of\ Fuselage_0 = 0\ [rad/s] \tag{3}$$

$$\begin{pmatrix} Angular\ Vel \\ of\ Fuselage \end{pmatrix}_{t+DT} = \begin{pmatrix} Angular\ Vel \\ of\ Fuselage \end{pmatrix}_t + \begin{pmatrix} Angular\ acc \\ of\ fuselage \end{pmatrix} \bullet DT \left[\frac{rad}{s}\right] \tag{4}$$

$$Rotational\ Speed\ of\ Tail\ Rotor_0 = 500\ [rad/s] \tag{5}$$

$$\begin{pmatrix} Rotational\ Speed \\ of\ Tail\ Rotor \end{pmatrix}_{t+DT} = \begin{pmatrix} Rotational\ Speed \\ of\ Tail\ Rotor \end{pmatrix}_t + \begin{pmatrix} Speed \\ Adjustment \end{pmatrix} \bullet DT \left[\frac{rad}{s}\right] \tag{6}$$

- Flow variables:

$$Change\ in\ Position = Angular\ Vel\ of\ Fuselage\ [rad/s] \tag{7}$$

$$Angular\ acc\ of\ fuselage = Net\ Torque / Moment\ of\ Inertia\ of\ fuselage\ [rad/s^2] \tag{8}$$

$$Speed\ Adjustment = \frac{Speed\ Switch - Rotational\ Speed\ of\ Tail\ Rotor}{Speed\ Adjustement\ Time} \left[\frac{rad}{s^2}\right] \tag{9}$$

- Other variables:

$$Air\ resistance = \begin{pmatrix} Air\ resistance\ coefficient \\ for\ the\ fuselage \end{pmatrix} \bullet \begin{pmatrix} Angular\ Vel \\ of\ Fuselage \end{pmatrix} \left[\frac{kg \cdot m^2}{s^2}\right] \tag{10}$$

$$\begin{pmatrix} Angular\ Position\ of \\ Fuselage\ in\ Degrees \end{pmatrix} = \begin{pmatrix} Angular\ Position\ of \\ Fuselage\ in\ Radians \end{pmatrix} \bullet \frac{180}{PI}\ [^\circ] \tag{11}$$

$$Discrepancy = \begin{pmatrix} Desired\ Position\ of \\ Fuselage\ in\ Radians \end{pmatrix} - \begin{pmatrix} Angular\ Position\ of \\ Fuselage\ in\ Radians \end{pmatrix} [rad] \tag{12}$$

$$\begin{pmatrix} Force\ Applied\ at \\ the\ Tail\ of\ the\ RC \end{pmatrix} = \begin{pmatrix} Tail\ Rotor\ Adjusted \\ Lift\ Coefficient \end{pmatrix} \bullet \begin{pmatrix} Rotational\ Speed \\ of\ Tail\ Rotor \end{pmatrix}^2 \left[\frac{kg \cdot m}{s^2}\right] \tag{13}$$

$$Net\,Torque = \begin{pmatrix} Torque\ of \\ Main\ Rotor \end{pmatrix} - \begin{pmatrix} Torque\ of \\ Tail\ Rotor \end{pmatrix} - Air\ Resistance \left[\frac{kg \cdot m^2}{s^2} \right] \tag{14}$$

$$Speed\ Switch = \left\{ \begin{array}{l} \text{IF}\{\text{DECISION}\} = \text{-1 THEN } Low \\ \text{ELSE} \left\{ \begin{array}{l} \text{IF}\{\text{DECISION}\} = 1\,\text{THEN } High \\ \text{ELSE } Medium \end{array} \right\} \end{array} \right\} \left[\frac{rad}{s} \right] \tag{15}$$

$$\begin{pmatrix} Torque\ of \\ main\ rotor \end{pmatrix} = \begin{pmatrix} Main\ Rotor\ Adjusted \\ Drag\ Coefficient \end{pmatrix} \bullet \begin{pmatrix} Rotational\ Speed \\ of\,Main\ rotor \end{pmatrix}^2 \left[\frac{kg \cdot m^2}{s^2} \right] \tag{16}$$

$$\begin{pmatrix} Torque\ of \\ tail\ rotor \end{pmatrix} = \begin{pmatrix} Force\ Applied\ at \\ the\,Tail\ of\ the\ RC \end{pmatrix} \bullet Distance \left[\frac{kg \cdot m^2}{s^2} \right] \tag{17}$$

The output of the decision heuristic utilized in controlling the direction becomes the input for the variable *Speed Switch* (see {DECISION} in Equation 15). Note that two decision heuristics will be given in section 3. When *Rotational Speed of Tail Rotor* is equal to *Medium*, *Torque of tail rotor* becomes equal to *Torque of main rotor*. When *Rotational Speed of Tail Rotor* is equal to *Low*, *Net Torque* will rapidly become positive creating a positive *Angular acc of fuselage*. Conversely, when *Rotational Speed of Tail Rotor* is equal to *High*, *Net Torque* will rapidly become negative creating a negative *Angular acc of fuselage*. (see equations 15, 6, 13, 17, and 14 in the given order).

- Parameters:

$$Air\ resistance\ coefficient\ for\,the\ fuselage = 0.11211\ \left[kg \cdot m^2/s\right] \tag{18}$$

$$Distance = 0.8\ [m] \tag{19}$$

$$High = 615\ [rad\,/\,s] \tag{20}$$

$$Low = 350\ [rad\,/\,s] \tag{21}$$

$$Main\ Rotor\ Adjusted\ Drag\ Coefficient = 4.1202/32400\ \left[kg \cdot m^2\right] \tag{22}$$

$$Medium = 500\ [rad\,/\,s] \tag{23}$$

$$Moment\ of\ Inertia\,of\ \ fuselage = 0.1\ \left[kg \cdot m^2\right] \tag{24}$$

$$Rotational\ Speed\ of\ Main\,Rotor = 180\ [rad\,/\,s] \tag{25}$$

$$Speed\ Adjustment\,Time = 0.2\ [s] \tag{26}$$

$$Tail\ Rotor\ Adjusted\ Lift\ Coefficient = 4.1202/((500\text{^}2)*0.8)\ [kg \cdot m] \qquad (27)$$

The magnitudes *High* (Equation 20) and *Low* (Equation 21) is determined such that the amount of gaps between *Medium* and each of these levels are the same in terms of the force generated by the tail rotor.

- <u>Target Direction:</u>

$$Desired\ Position\ of\ Fuselage\ in\ Radians = \{\text{TARGET DIRECTION}\} \qquad (28)$$

Desired Position of Fuselage in Radians can take any value. Therefore, the experimenter can choose a value as he wishes.

3. Decision Heuristics

The essence of the model is as follows: Initially, the helicopter is positioned at 0°. The aim is to set the angular position of the fuselage at a predetermined desired level. Rotational speed of the helicopter's tail rotor will be changed accordingly to reach the desired level.

In this section, mainly two approaches will be presented. The model will be simulated using both of the approaches under two different settings; in the absence and presence of measurement delays. The first one is the naive approach and the second one is a heuristic called Modified-VSL, which is adapted from two previous heuristics suggested by Yasarcan and Barlas (2005), and Yasarcan (2010). Note that, although radian is used throughout the model, for ease of understanding, the position of the helicopter will be plotted in degrees on the graphs.

3.1. Naive Heuristic

The working principle of naive heuristic is quite simple. It looks at the current discrepancy and decides on the rotational speed level according to the sign of the discrepancy.

$$Naive\ decision = \left\{ \begin{array}{l} \text{IF Discrepancy} < 0 \text{ THEN } 1 \\ \text{ELSE} \left\{ \begin{array}{l} \text{IF Discrepancy} > 0 \text{ THEN -1} \\ \text{ELSE } 0 \end{array} \right\} \end{array} \right\} [dimensionless] \qquad (29)$$

The result of *Naive decision* becomes the input to *Speed Switch* (see Equation 15). The causal loops diagram including *Naive decision* (Equation 29) and the rest of the model (equations 1-28) is given in Figure 2. There are two negative feedback loops in the diagram. One is the decision making feedback loop involving *Naive decision* and the other one is *Air resistance* loop representing the drag force applied on the fuselage by air.

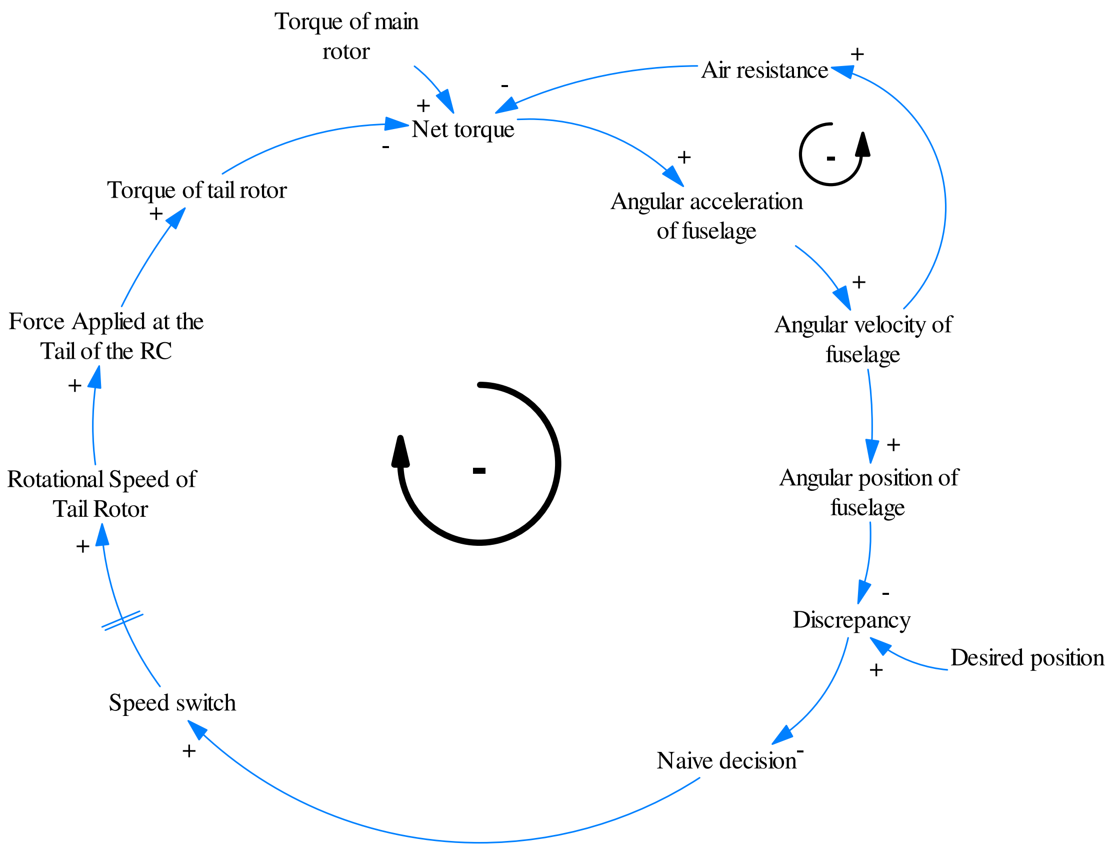

Figure 2 : Causal loop diagram for naive heuristic

3.1.1. Without measurement delay

In this case, it is assumed that there is no delay involved in measuring the angular position of the fuselage. Therefore, the actual value of the angular position of the fuselage can be used in the decision making heuristic. The graph in Figure 3 shows the dynamics of the angular position of the body of the helicopter when naive heuristic is used in controlling the object. We set the desired position to 180°. The naive heuristic cannot stabilize the helicopter at around the target and makes it oscillate with an average peak-to-peak amplitude of 465°. We have also plotted zero and 2×PI (one cycle) levels. As it can be observed, naive heuristic is not successful in controlling the direction of the helicopter because of the difficulties introduced by the indirect control. Therefore, a better heuristic is needed.

In order to compare different heuristics, we have introduced a penalty formulation as given below:

$$Penalty_0 = 0\,[rad \cdot s] \tag{30}$$

$$Penalty_{t+DT} = Penalty_t + Penalty\ formation \bullet DT\ [rad \cdot s] \quad (31)$$

$$Penalty\ formation = |Discrepancy|\ [rad] \quad (32)$$

As it can be seen from the penalty formulation equations 30-32, *Penalty* is the cumulative absolute *Discrepancy* (see also Equation 12). Penalty value resulting from the run given in Figure 3 is 106.

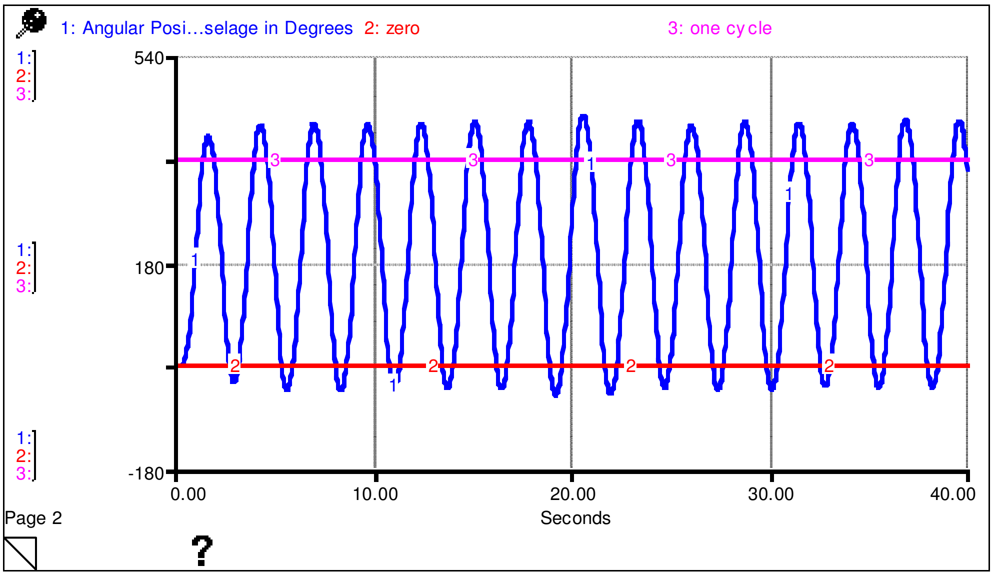

Figure 3 : Angular position of fuselage with naive heuristic under no delay setting

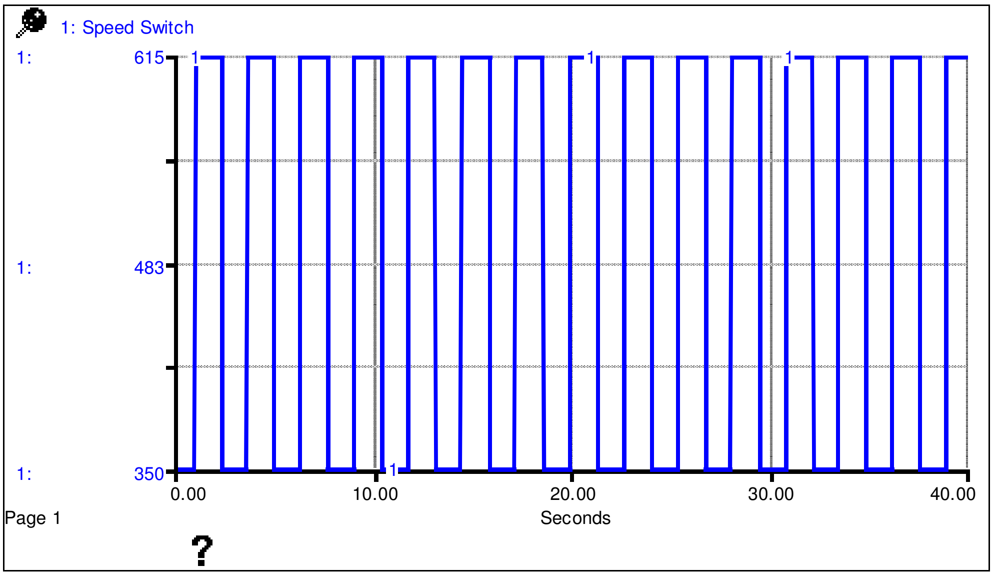

Figure 4 : Rotational speed selections for the tail rotor with naive heuristic under no delay setting

The graph in Figure 4 shows the rotational speed level selections generated by the naive heuristic. It can be seen that the heuristic switches between *High* and *Low* about 7 times in 10 seconds. As long as the discrepancy is not equal to zero, the naive heuristic sticks to the same rotational speed level for the tail rotor. Thus, when the fuselage reaches the target, it has a high angular velocity. Therefore, it cannot stop at the desired level but continues to rotate. Hence, the resulting oscillations observed in Figure 3 becomes inevitable.

3.1.2. With measurement delay

We added the following equations to the model:

$$\begin{pmatrix} Measured\ Angular \\ Position\ of\ Fuselage \\ in\ Radians \end{pmatrix} = \mathrm{SMTH3}\left(\begin{pmatrix} Angular\ Position \\ of\ Fuselage \\ in\ Radians \end{pmatrix}, \begin{pmatrix} Measurement \\ Delay\ Time \end{pmatrix} \right) [rad] \quad (33)^{3}$$

$$Measurement\ Delay\ Time = 0.5\ [s] \quad (34)$$

$$Discrepancy = \begin{pmatrix} Desired\ Position\ of \\ Fuselage\ in\ Radians \end{pmatrix} - \begin{pmatrix} Measured\ Angular\ Position \\ of\ Fuselage\ in\ Radians \end{pmatrix} [rad] \quad (35)$$

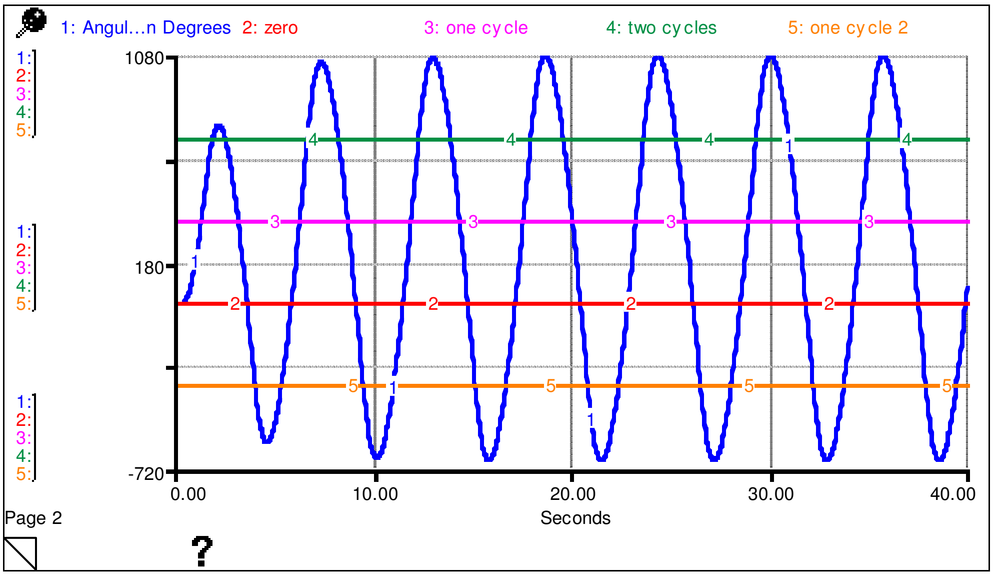

Figure 5 : Angular position of fuselage with naive heuristic under delayed setting

Under the with measurement delay setting, the actual angular position of the fuselage cannot be used in the decision heuristics. In this case, there is a delay of 0.5 seconds involved in

[3] SMTH3(input, delay time) is a function that gives a third order exponentially smoothed version of the input as its output.

measuring the angular position of the fuselage. Therefore, *Measured Angular Position of Fuselage in Radians* (Equation 33) is used in the calculation of *Discrepancy* (Equation 35), which later is used in *Naive decision* (Equation 29). Naturally, this deteriorates the performance obtained from the heuristic as it can be seen in Figure 5; the average peak-to-peak amplitude of the oscillations becomes 1770° and the associated *Penalty* value becomes 349.9.

The graph in Figure 6 shows the rotational speed level selections generated by the naive heuristic under the delayed setting. The heuristic switches between *High* and *Low* about 4 times in 10 seconds.

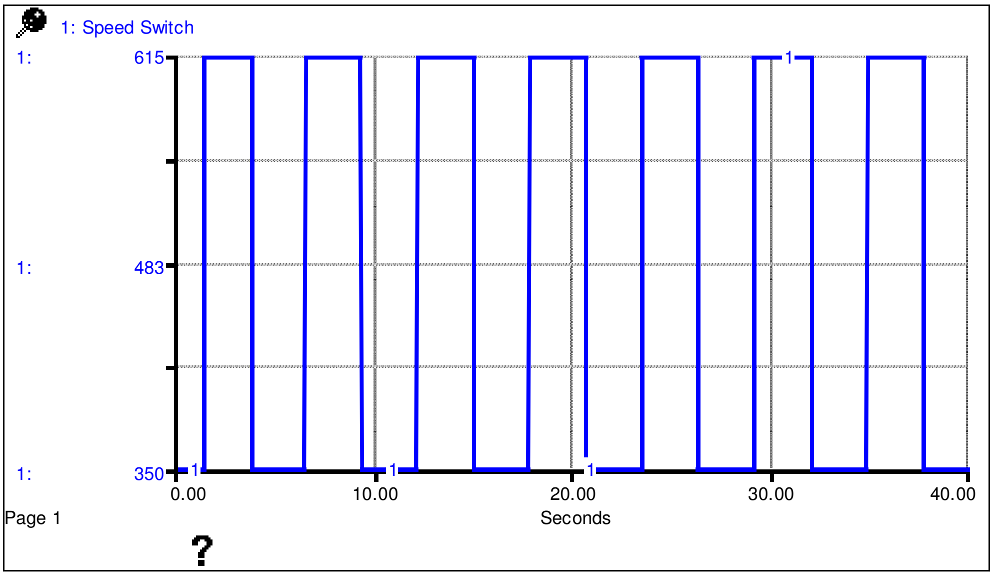

Figure 6 : Rotational speed selections for the tail rotor with naive heuristic under delayed setting

3.2. Modified-VSL

This heuristic is adapted from two other heuristics, Measurement Delay Heuristic (Yasarcan, 2010) and Virtual Supply Line Heuristic (Yasarcan and Barlas, 2005). It makes decisions based on the discrepancy between the current position and the desired (target) position and on the past decisions. As supply line is considered to avoid oscillations in stock control problems, a similar approach should be applied for the direction control problem. However, there is no supply line in this case. In order to overcome the obstacle of having no supply line, we propose taking the past decisions into account. This approach prevents overcorrection. We named the new heuristic as Modified-VSL. Its formulation is given below:

$$Past\ decisions_0 = 0\ [dimensionless] \tag{36}$$

$$Past\ decisions_{t+DT} = Past\ decisions_t + \begin{pmatrix} Current\ decision \\ - Decay \end{pmatrix} \bullet DT\ [dimensionless] \quad (37)$$

$$\begin{pmatrix} Adjusted \\ decision \end{pmatrix} = \left\{ \begin{array}{l} \text{IF} \begin{pmatrix} Naive\ decision \\ - Past\ decisions \end{pmatrix} \leq \text{-0.98 THEN -1} \\ \text{ELSE} \left\{ \begin{array}{l} \text{IF} \begin{pmatrix} Naive\ decision \\ - Past\ decisions \end{pmatrix} \geq \text{0.98 THEN 1} \\ \text{ELSE 0} \end{array} \right\} \end{array} \right\} [dimensionless] \quad (38)$$

$$Current\ decision = Adjusted\ decision / Time\ constant\ [1/s] \quad (39)$$

$$Decay = Past\ decisions / Decay\ time\ [1/s] \quad (40)$$

$$Decay\ time = 0.4\ [s] \quad (41)$$

$$Time\ constant = 1\ [s] \quad (42)$$

Naive decision is an input to Modified-VSL formulation (Equation 38). The output of Modified-VSL is *Adjusted decision*, which is input to *Speed switch* (Equation 15).

Modified-VSL prevents overcorrection by adjusting *Naive decision* with *Past decisions*. However, *Past decisions* should also be controlled. Otherwise, *Past decisions* would dominate the decision making process. Therefore we added a decay flow to *Past decisions* stock and selected a good value for the decay time after a few simulation runs (Equation 41). See also the two additional feedback loops in Figure 7 (compare the figures 2 and 7).

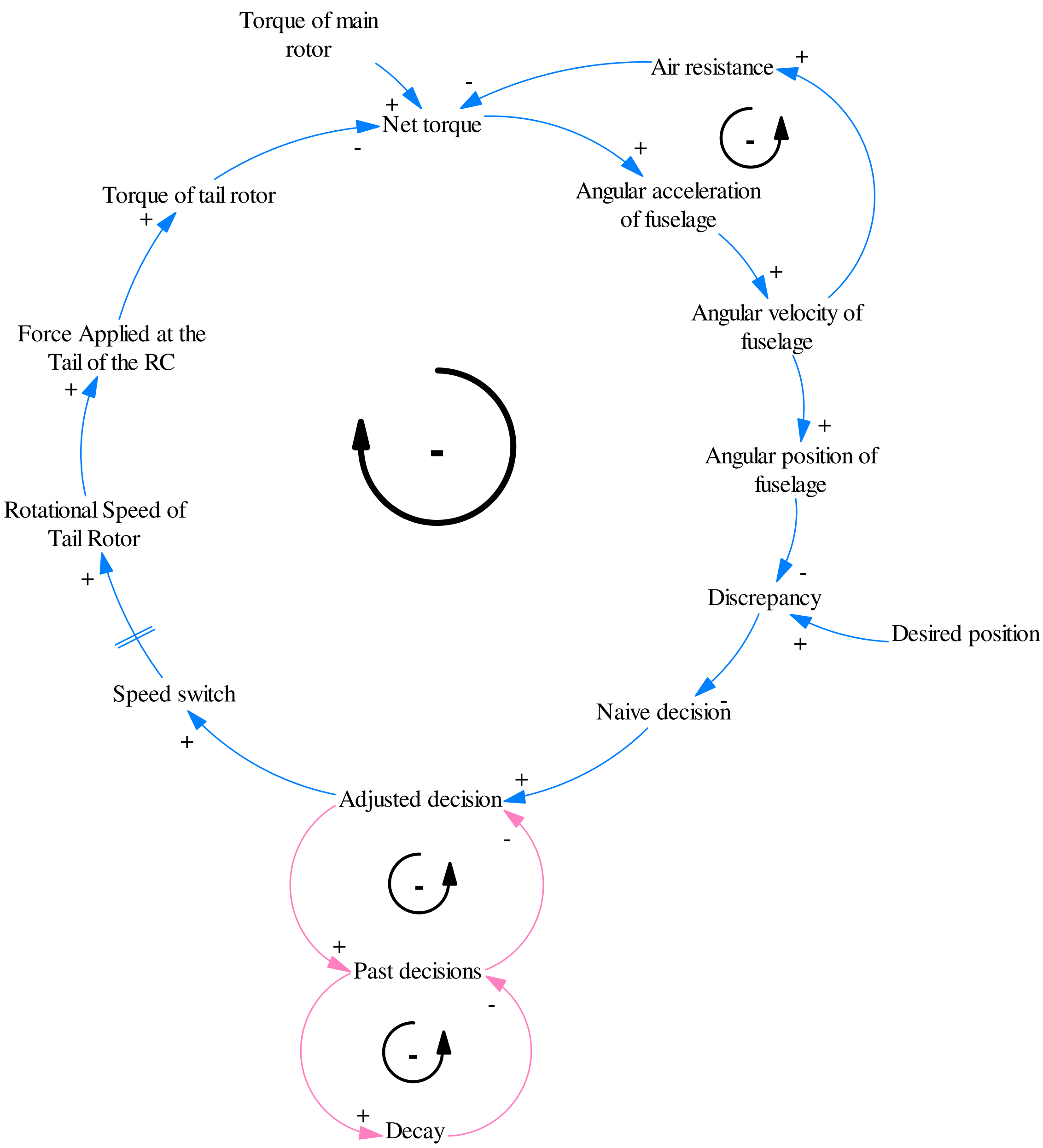

Figure 7 : Causal loop diagram for Modified-VSL

3.2.1. Without measurement delay

In this case, the actual value of the angular position of the fuselage can be used in the decision making heuristic because there is no delay involved in measuring it. The graph in Figure 8 shows the dynamics of the angular position of the body of the helicopter when Modified-VSL heuristic is used in controlling the object. We set the desired position to 180°. The Modified-VSL heuristic can satisfactorily stabilize the helicopter at around the target, preventing large oscillations; The average peak-to-peak amplitude is at around 5°.

The new *Penalty* value is 6.9, which is much less than the penalty value obtained by the naive heuristic. The obvious reduction in the penalty value clearly indicates the improvement in the behavior.

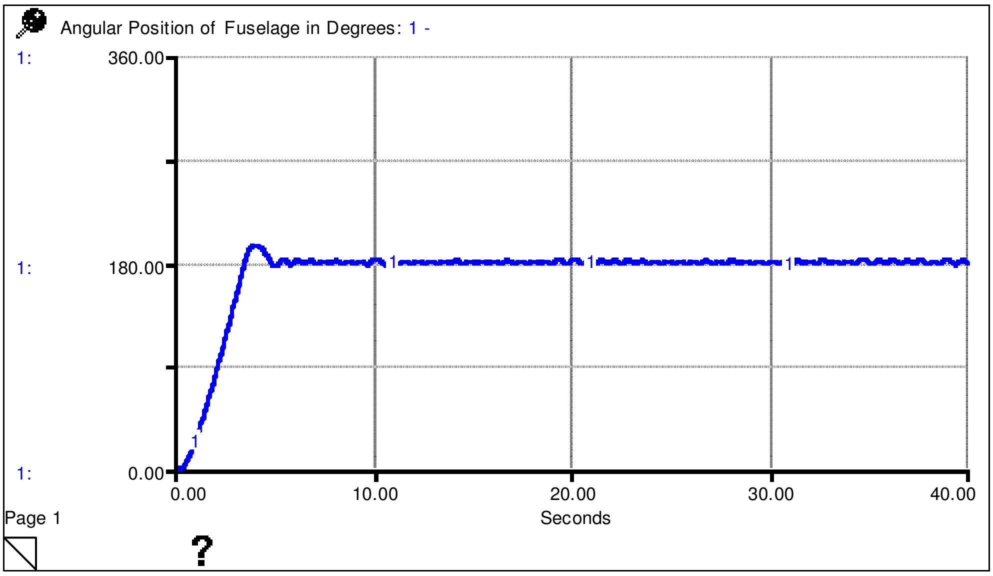

Figure 8 : Angular position of fuselage with Modified-VSL under no delay setting

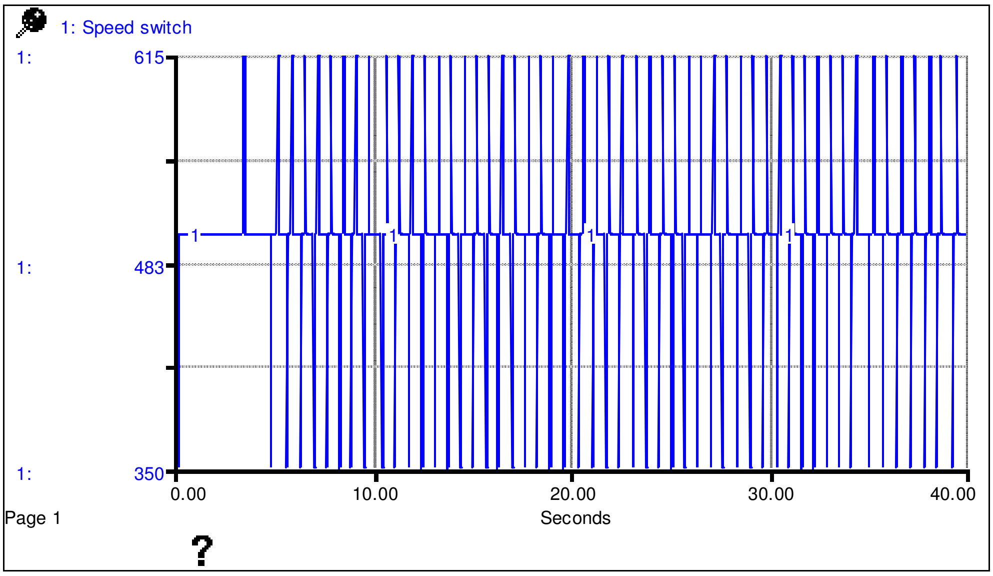

Figure 9 : Rotational speed selections for the tail rotor with Modified-VSL under no delay setting

The graph in Figure 9 shows the rotational speed level selections generated by the Modified-VSL heuristic. It can be seen that the heuristic switches among the three rotational speed levels about 35 times in 10 seconds.

3.2.2. With measurement delay

The measurement delay equations (33-35) given in section 3.1.2 are valid in this section too. Again, *Measured Angular Position of Fuselage in Radians* (Equation 33) is used in the calculation of *Discrepancy* (Equation 35). There is a deterioration in the performance (Figure 10); the average peak-to-peak amplitude of the oscillations becomes 37° and the associated *Penalty* value becomes 16.1. Although there is a significant deterioration in the performance compared to the no delay setting, still the performance obtained by Modified-VSL heuristic is much better than the performance obtained by the naive heuristic. Comparatively, the deterioration in performance under delayed setting is much less for Modified-VSL than the naive heuristic.

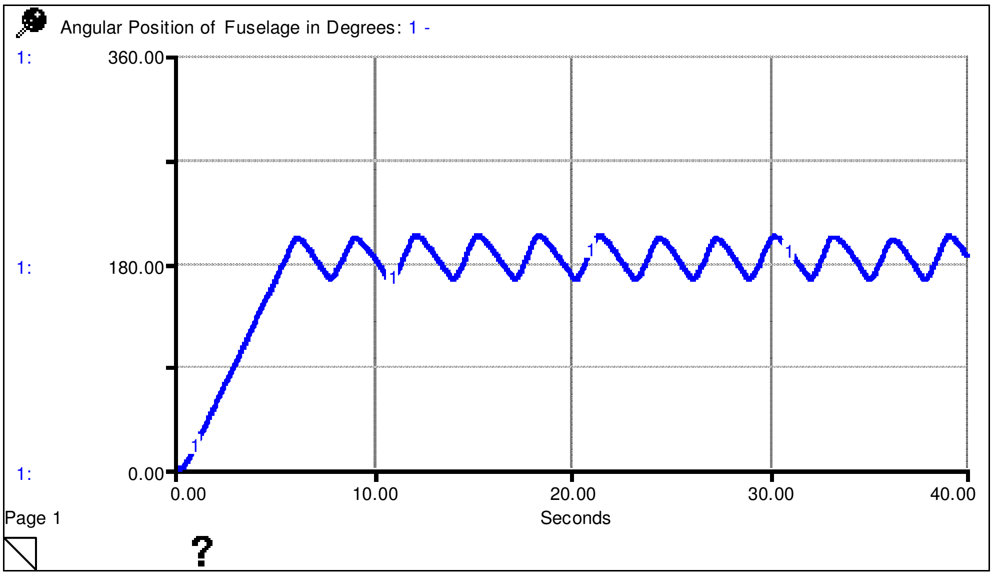

Figure 10 : Angular position of fuselage with Modified-VSL under delayed setting

Note that *Decay time* is increased from 0.4 to 0.9. The amount of increase is equal to the value of *Measurement Delay Time*. The graph in Figure 11 shows the rotational speed level selections generated by Modified-VSL heuristic under the delayed setting. The heuristic switches among the three rotational speed levels about 7 times in 10 seconds.

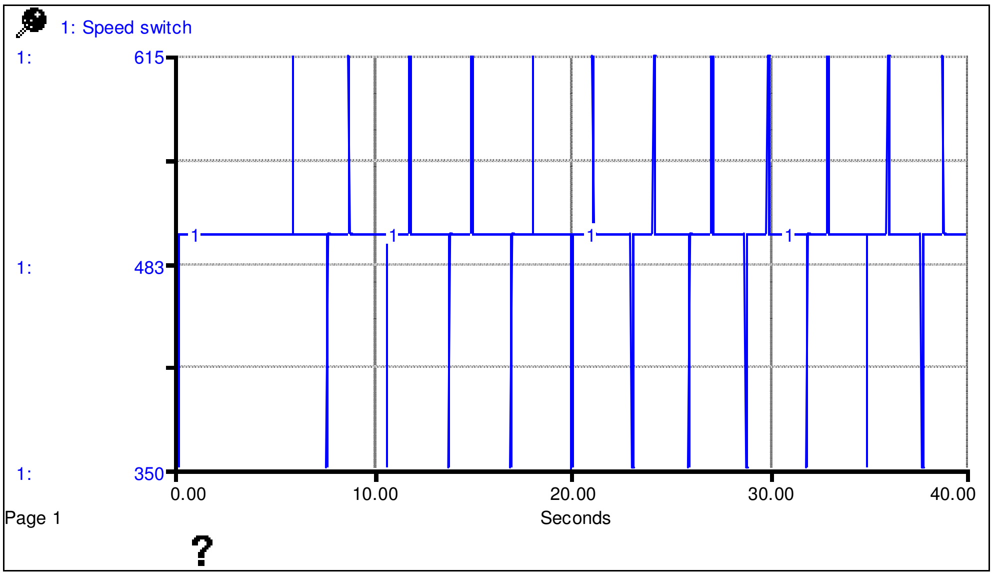

Figure 11 : Rotational speed selections for the tail rotor with Modified-VSL under delayed setting

3.3. Comparison of the Two Approaches

In this section, performances of the two heuristics and effect of measurement delay will be compared in terms of their associated penalty values.

Table 1 : Comparison of the two heuristics

		Delay Existence		
	Penalty	Without measurement delay	With measurement delay	***Effect of delay***
Heuristics	Naive	106	349.9	3.30
	Modified-VSL	6.9	16.1	2.33
	Effect of heuristic	15.36	21.73	

The above table compares the performances of the two heuristics and also shows effect of delay on the performance of each heuristic. Modified-VSL performs about 16-fold better in the absence of measurement delay, about 22-fold better in the presence of measurement delay. The effect of measurement delay on Modified-VSL is not as strong as its effect on the naive heuristic; the deterioration in penalty values is 3.30-fold for the naive case where it is 2.33 for Modified-VSL. Therefore, one can conclude that Modified-VSL is more robust under delayed settings than the naive approach.

3.4. A Variation in Discrepancy Calculation

In previous sections, the discrepancy was calculated as if the helicopter moves on a straight axis. That was necessary in order to obtain comparable penalty values. Once a heuristic is selected to be used in the control, there is no further need for comparison. Hence, it would be wiser if the heuristic approaches to values higher than 180° from above rather than below because the distance from above would be less than the distance from below.

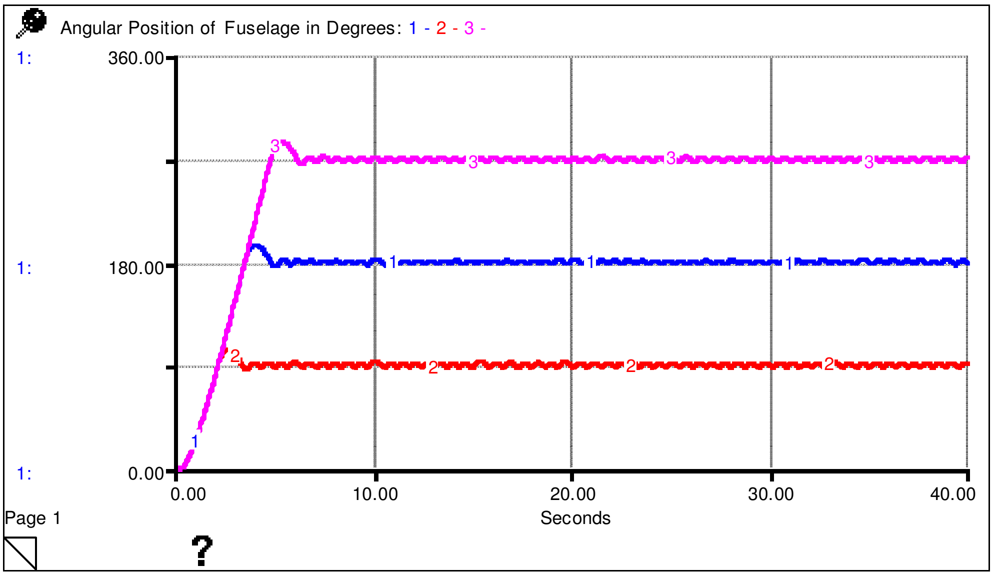

Figure 12 : Angular position of fuselage with Modified-VSL under no delay setting (Equation 12 is used for *Discrepancy*)

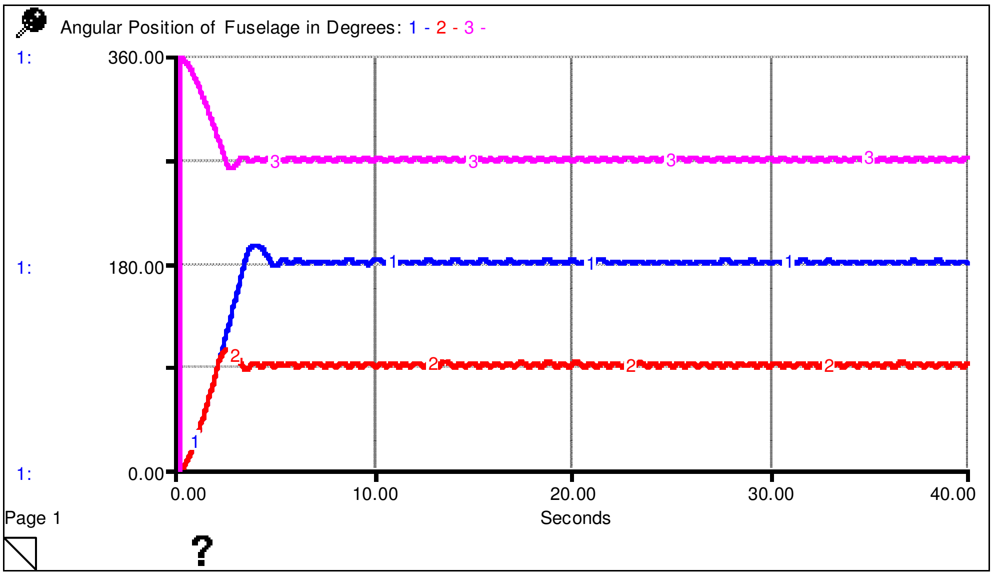

Figure 13 : Angular position of fuselage with Modified-VSL under no delay setting (*Discrepancy* calculations is based on a modular formulation)

Assume we simulate the model with Modified-VSL under no delay setting. We would obtain the dynamics seen in Figure 12. In the first run, the desired position is equal to 180°. In this case, it does not matter if the angular position approaches the desired level from above or from below. In the second run, the desired position is equal to 90°. In this case, it is better to approach the desired level from below as given in the figure. However, in the third run, the desired position is equal to 270°. In this case, we lose time trying to approach the desired level from below. It would be wiser to approach it from above. After a slight adjustment in the discrepancy formulation, we obtained the dynamics given in Figure 13. Only the third run is different in figures 12 and 13.

4. Conclusion

In this study, we first developed a system dynamics model of a model helicopter. We showed that the naive heuristic is unsuccessful in controlling the direction of the helicopter. Later, we introduced a heuristic that we named Modified-VSL. This heuristic was adapted from two other heuristics, Measurement Delay Heuristic (Yasarcan, 2010) and Virtual Supply Line Heuristic (Yasarcan and Barlas, 2005). Modified-VSL was successfully utilized in controlling the direction of a simulated model helicopter.

Naive heuristic and Modified-VSL are tested under two scenarios: no delay setting and delayed setting. The suggested heuristic, Modified-VSL, shows about 16-fold improvement in the absence of measurement delay, and about 22-fold improvement in the presence of measurement delay compared to the performance obtained from the naive heuristic. Thus, Modified-VSL is found to be successful.

Virtual Supply Line, Measurement Delay, and Modified-VSL heuristics can be applied to different types of control problems. We plan to continue this line of research.

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