System Dynamics Applied to Combat Models
(Lanchester Laws)

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Abstract

Different mathematical models explain the main features of combats, among them are models referring to the attrition of the forces involved; numerous battles employing these models have been recreated. As the matter remains active mainly inside a reduced community of operational research defense experts, in this study basic models of Lanchester´s Laws are reviewed, but employing system dynamics concepts and tools, with the objective being to open the subject to a more diversified audience. System dynamics also provide easier ways to account for "soft" variables, normally present in real combat situations. For applying those models a fictitious land combat case between two forces is presented, enriching the basic models with some additions to test commander decisions.

Keywords

System Dynamics, Combat Models, Lanchester Equations, War games

1. Introduction

Over the years some mathematical models have been used to explain the main features of combat, particularly the attrition of the forces involved, from which numerous battles employing these models have been recreated. Nevertheless their usefulness to the general audience seems to be limited, as the subject remains inside a reduced community of operational research experts.

System Dynamics [Forrester, 1961] provides a more understandable and intuitive way to deal with the subject, and allows opening to a more diversified audience. Also employing the concepts and tools of System Dynamics facilitates the possibility of enhancing those equations with "soft" variables: fatigue, diplomacy, information, economics, relative distances, and so on, all of which are extremely difficult to manage only with mathematical models. The following study has not the intention to be a complete description of this matter, and only some basics models will be treated to promote a discussion inside the community interested in developing further studies.

2. Lanchester´s Laws

Back in 1916 F. W. Lanchester [Lanchester, 1916], a British engineer and inventor, formulated two differential equations for attrition, explaining different types of warfare. Also, the Russian M. Osipov [Osipov, 1915], from whom there are no personal information, deserves credits in this type of study. Deliberately avoiding the mathematical description of those equations, that can be founded in many references as the ones indicated; we will concentrate only on their description employing the tools of System Dynamics.

2.1 Lanchester´s Square Law (Aimed Fire):

Figure 1a shows the causal loop diagram of the simplest form of Lanchester´s Law indicating the aggregated causes of attrition of combat forces. Figure 1b is the corresponding stock and flow model. The attrition rate is equal to the number of the forces remaining on the opposing side multiplied by their respective effectiveness.

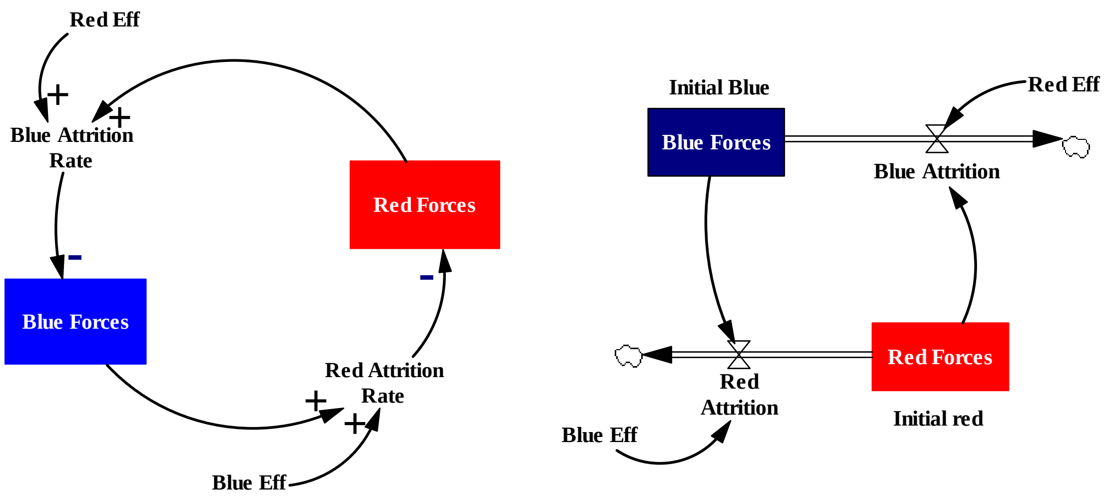

Figure: 1a - Causal Diagram.**1b-** Stock and flow model of Lanchester´s Square Law

Example 1. - Suppose:

Initial Blue = 2000 (men)   Initial Red = 1000 (men)   (Blue has twice initial forces)
Blue Eff = 0.001 (1/Hour)   Red Eff = 0.002 (1/Hour) (Red has Twice Eff)
Find out who wins, how long the battle lasts; Blue and Red survivors

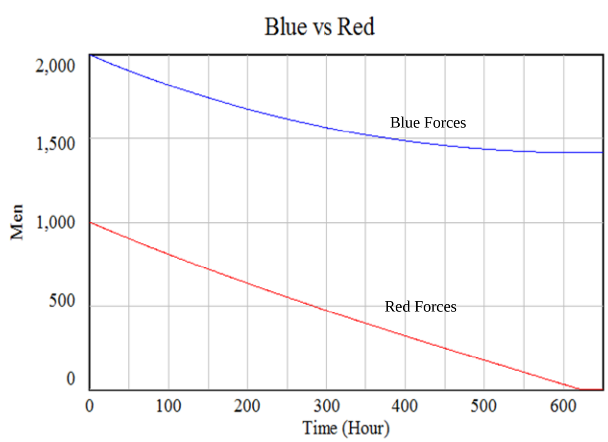

Figure 2. Results of the Example 1

Figure 2 shows the outcome of example 1: Blue wins, the battle lasted 620 hours, with 1414 Blue survivors and the Red force is annihilated. It must be noticed that even if the Blue force has only half of effectiveness, but doubled the quantity of Red soldiers, the

Blue forces will win. From this fact comes the name of Lanchester´s Square. This effect correspond to one of the main principles employed in all wars, having different phrasing such that: “Divide and Conquer”, “God is on the side of the big battalions” (Napoleon), “Superiority in numbers is the most important factor in the result of a combat…the greatest possible number of troops should be brought into action at the decisive point”, and so on**.**

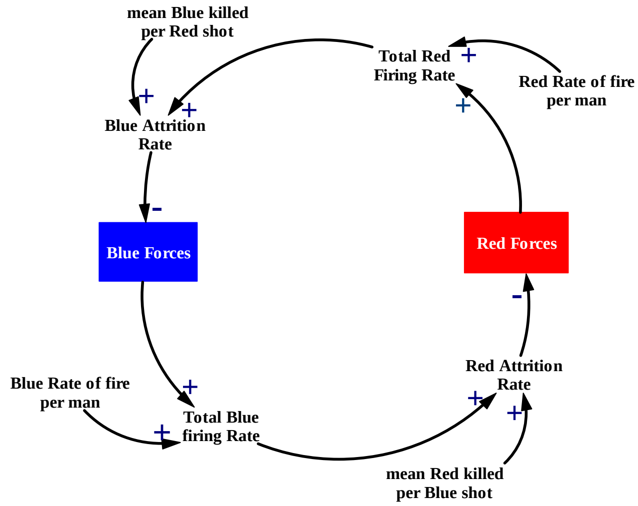

Figure 3. Causal Diagram of Lanchester´s Square Law

Figure 3 corresponds to the causal loop diagram for land combat where the attrition rate in both sides is due to the combined effect of the firing rate per man the remaining forces and the effectiveness of the shots, reflected in the parameter Mean men (Blue or Red, respectively) killed per each shot. This model will be applied in a later example of combat.

2.2 Lanchester´s Linear Law (Unaimed Fire)

In this case both fires are directed into the operating area, rather than being aimed at a specific unit, and then the attrition rate of both forces will be proportional to the other force. If the number of targets is doubled, the number of hits will also be doubled. This case can be explained by a causal diagram as the one indicated in figure 4. It should be noted that this attrition coefficient is not the same as the one employed in the square law, as they have different dimensions, therefore must not be directly compared.

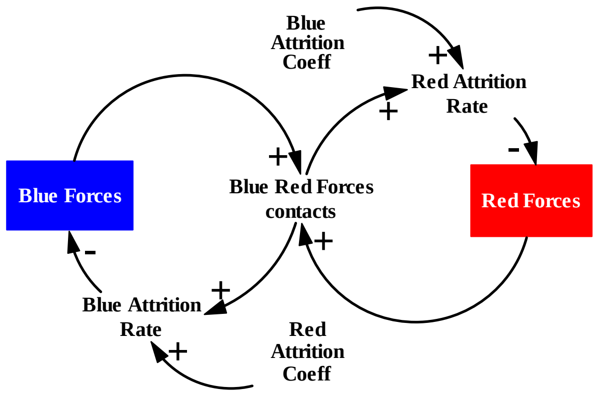

Figure 4. Causal Diagram of Lanchester´s Linear Law

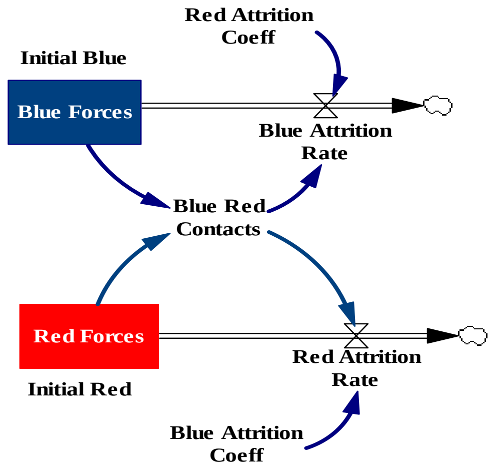

Figure 5. Model of Lanchester´s Linear Law

Figure 5 indicates the stock and flow model for the Linear Law. The main equations are:
Blue Red Forces contacts = Blue Forces * Red Forces
Blue Attrition Rate = Blue Red contacts * Red Attrition Coeff
Red Attrition Rate = Blue Red contacts * Blue Attrition Coeff

Example 2:

Initial Blue: 2000 (men)
Blue Attrition Coeff = 0.0001

Initial Red: 1000 (men)
Red Attrition Coeff (Case a) = 0.0002
Red Attrition Coeff (Case b) = 0.0001

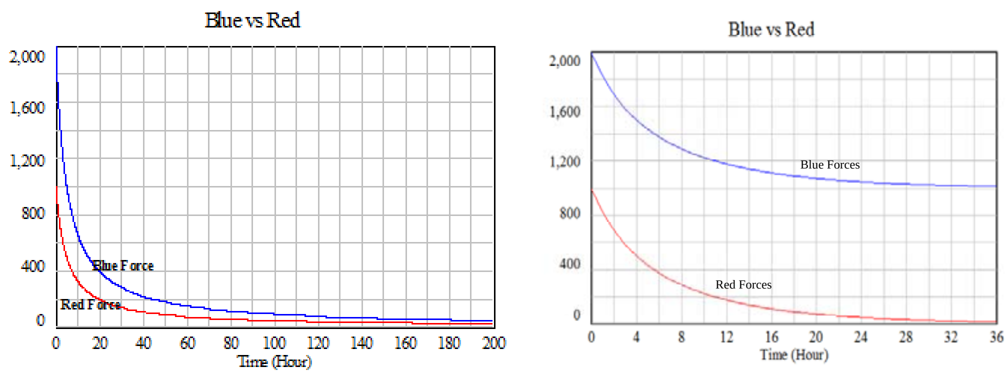

Figure 6. Results of Lanchester´s Linear Law Example 2. Left Case a, right Case b.

The figure 6 shows the outcome: In Case a, both forces annihilate, even if the Blue doubled the Red forces but have only half of Attrition Coeff. In Case b, where both forces have equal Attrition Coeff but the Blue doubles in forces, Blue wins and the Red forces are annihilated. This model is employed to explain ancient wars, where the forces involved are only the ones to have personal contact, but also the model results very useful for modeling guerrilla types of incidents and ambushes. Example 5 deals with this specific and important case.

2.3 Enriching the Lanchester Basic Models

Aggregating some other information the Lanchester´s models may be enriched: own casualties, reinforcements, range dependency.

2.3.1 Own casualties

For example the following chart [Lucas, 2009] shows the combat deaths of different wars referring to troops killed in action, or dead of wounds. Other includes deaths from disease, privation, and accidents, and includes losses among prisoners of war. Wounded excludes those who died of their wounds, who are included under Combat Deaths. It is noted that the percentage of casualties due to non direct combat is far greater than the direct ones caused by enemy fire; therefore it may be interesting to include those casualties in a model. The model can accommodate those own casualties, for example as a percentage of the forces (Figure 7)

	<------------Casualties------->			
	[----Deaths---]			
Conflict	KIA	Other Deaths	Wounded	Total
Mexican War	1,733	11,550	4,152	17,435
Civil War: Union	110,070	249,458	275,175	634,703
Confederate	74,524	124,000	137,000	335,524
Combined	184,594	373,458	412,175	970,227
Spanish-American War	385	2,061	1,662	4,108
World War I	53,513	63,195	204,002	320,710
World War II	292,131	115,185	670,846	1,078,162
Korean War	33,651	NA	103,284	136,935
Vietnam War	47,369	10,799	153,303	211,471
Gulf War	148	145	467	760

2.3.2 Reinforcements

Also the forces may be reinforced during the battle, either continuously during the whole battle, or scattered in groups. Both situations are indicated in the stock and flow model shown in figure 7 and example 3.

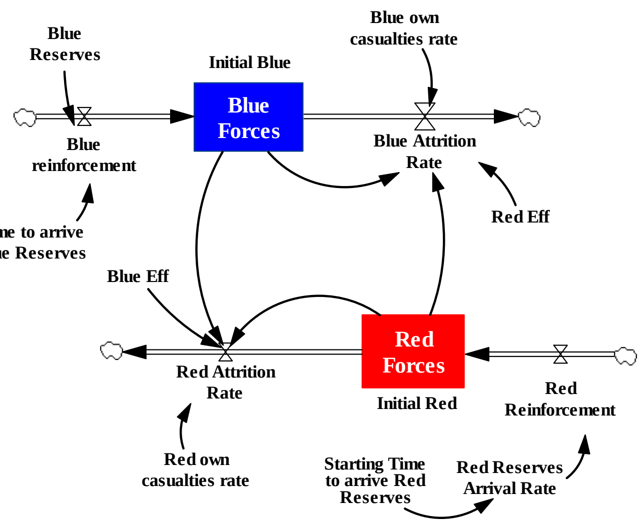

Figure 7. Model including reinforcements and own casualties to the Lanchester´s Square Law

The main equations are:

Blue Attrition Rate = Red Eff * Red Forces + Blue Forces * Blue own casualties rate

Red Attrition Rate = Blue Eff * Blue Forces + Red Forces*Red own casualties rate

Blue reinforcement = Blue Reserves * (PULSE (Time to arrive Blue Reserves, TIME STEP) / TIME STEP+PULSE (Time to arrive Blue Reserves +100, TIME STEP) / TIME STEP)

Red Reinforcement = Red Reserves Arrival Rate

Red Reserves Arrival Rate = 10 * PULSE (Starting Time to arrive Red Reserves, 100)

Example 3: Reinforcements
Initial Blue & Red = 1000
Blue and Red Eff =0.01
Blue and Red Casualty Rate= 0.001
Both sides start to reinforce at time 20.
Blue reinforce with 500 men at time 20 and 500 men at time 120 (Total 1000)
Red reinforce continuously at a rate of 10 men / Hour from time 20 to time 120 (Total 1000)

The results are shown in figure 8.

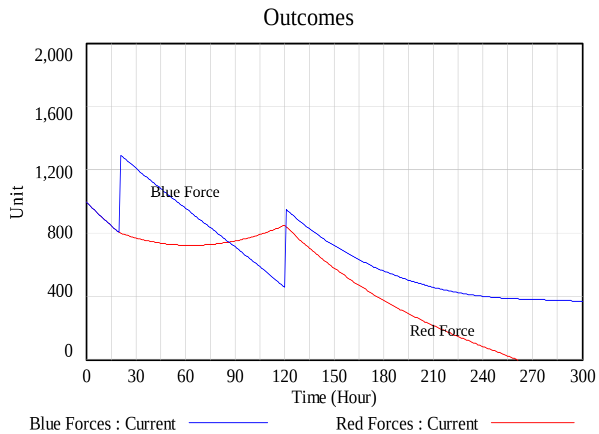

Figure 8. Results of Example 3. Reinforcements to the Lanchester´s Linear Law

In this case both forces start to be reinforced at time 20. The Red one continuously and the Blue ones in two groups of 500 soldiers each, arriving at times 20 and 120. The results, for this event is that the Red forces are defeated. Of course the situation may change if one or more of the parameters changes. The important lesson given in this example is the easiest way to model any of those alternatives, employing system dynamics.

2.3.3 Range dependency

Another consideration that may be included into the models is the difference in efficiency of the shots due to the different maximum effective distances of the guns. In the following example we consider two forces approaching with some relative speed and with difference into the effective distance of their weapons. For this case we are using the Lanchester´s Square model, but this addition can be applicable to any model, useful not only for the maximum distance but also for modeling an increase in the probability of hits with the decreasing distance to the opposing force. The stock and flow model is shown in figure 9.

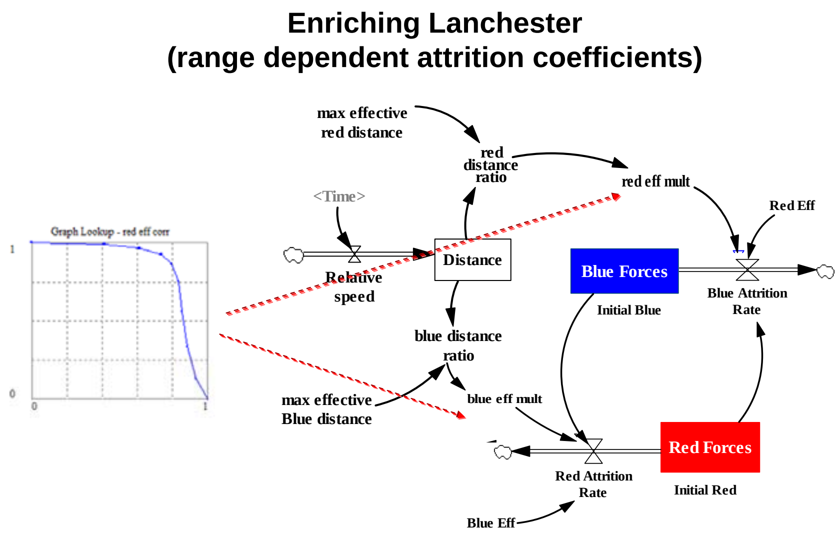

Figure 9. Attrition dependent of the range of the weapons

The main equations are:
Blue Attrition Rate = Red Eff*Red Forces*red eff mult
red eff mult = WITH LOOKUP (red distance ratio)
red distance ratio = Distance /max effective red distance
Red Attrition Rate = Blue Eff*Blue Forces*blue eff mult
blue eff mult = WITH LOOKUP (blue distance ratio)
blue distance ratio = Distance /max effective blue distance
Distance = INTEG (Relative speed)

Example 4: Range dependency

Initial Blue & Red = 10 unit   Blue and Red Eff=0.01
Max. Effective Red Distance = 60   Max. Effective Blue distance = 40

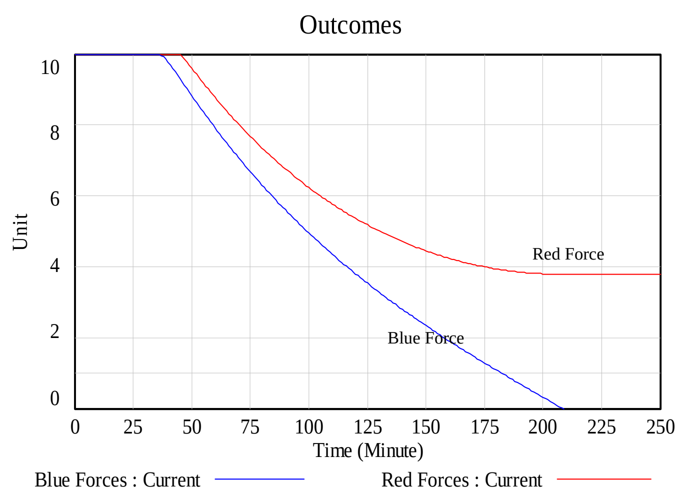

Figure 10. Results of example of attrition dependent on the range of the weapons

The Blue force is annihilated because the superiority in effective distance of the Red weapons allows the Red starting earlier the firing.

2.3.4 Heterogeneous Forces

Another possibility of enriching the model is to have more than two forces. In the example indicated in figure 11 are considered 3 forces: US Tanks, US Helicopters and Iraqis Tanks, with their different efficiencies against each others. In this same way, if required, may be added more forces and/or be combined with the enrichments just described.

Gulf War: Blue: US Tanks + US Helos; Red: Iraquies Tanks

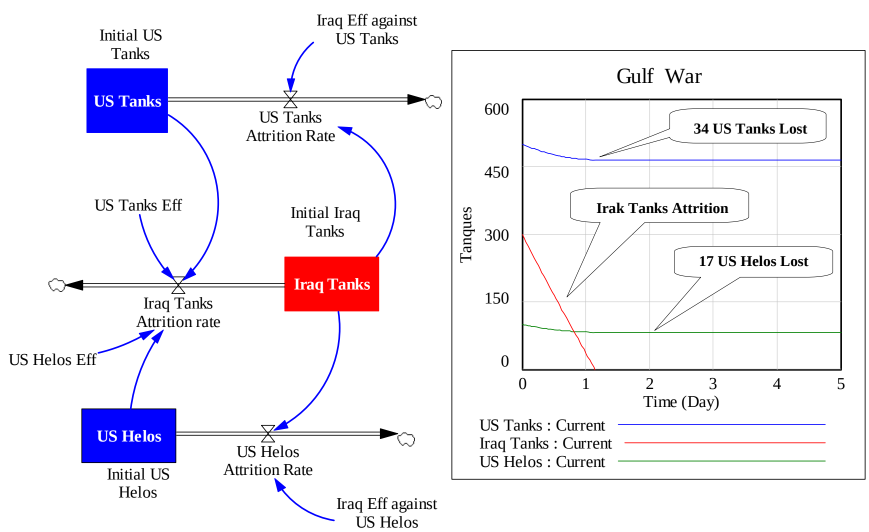

Figure 11. Heterogeneous forces

To obtain those results for this case of heterogeneous forces it was considered the following data:

Iraq Tanks Attrition rate = US Helos Eff * US Helos + US Tanks Eff * US Tanks

Initial US Tanks = 500	Initial US Helos = 100	Initial Iraq Tanks = 300
US Helos Eff = 0.8	Iraq Eff against US Helos = 0.1	
US Tanks Eff = 0.4	Iraq Eff against US Tanks = 0.2	

2.4 Guerrilla Warfare

Also it may be incorporated both linear and square law for modeling guerrilla warfare. The attrition of the conventional forces correspond to the square law, as the guerrilla are able to fire to the conventional forces in full view, therefore the conventional losses are proportional to the number of the guerrilla firing, but the conventional forces are unable to target the guerrillas and must fire blindly into the area the guerrilla occupy. In these circumstances the guerrilla losses will be proportional to the number of ambushed forces and the number of guerrilla occupying the area. The model of this case and results of example 5 are shown in figure 12.

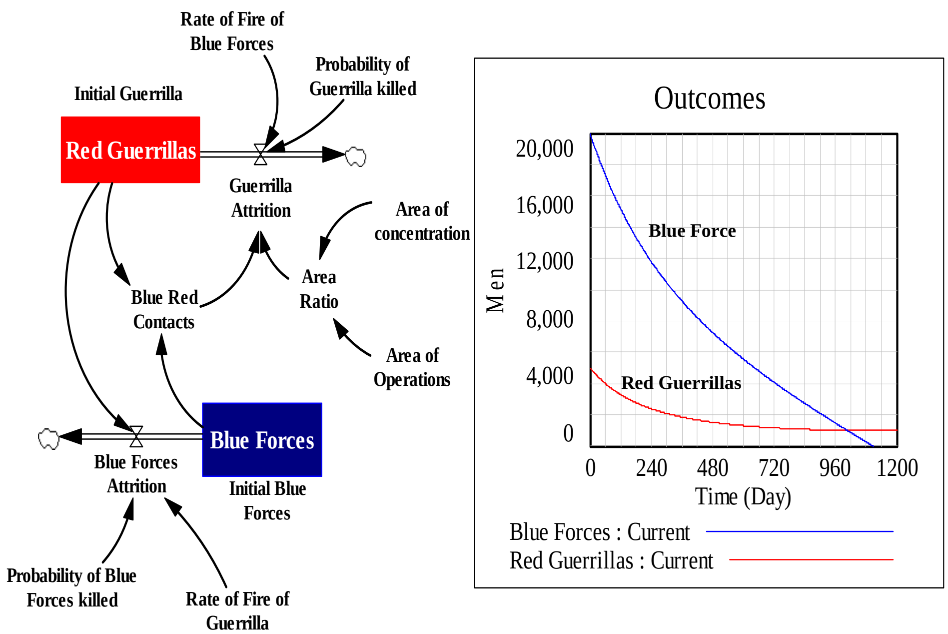

Figure 12. Guerrilla warfare

The main equations are:
Area Ratio = Area of concentration / Area of Operations
Guerrilla Attrition = Area Ratio * Blue Red Contacts * Rate of Fire of Blue Forces * Probability of Guerrilla killed
Blue Forces Attrition = Red Guerrillas * Probability of Blue Forces killed * Rate of Fire of Guerrilla
The Area Ratio has reduced the effectiveness of the conventional troops because the total probability of unaimed fire hitting a lesser ambushing force is much smaller than the probability of aimed fire hitting full exposed conventional troops.

Example 5. Guerrilla warfare

Initial Blue Forces = 20000
Rate of Fire of Blue Forces = 10
Probability of Blue Forces killed = 0.001
Area of concentration= 2

Initial guerrilla = 5000
Rate of Fire of Guerrilla = 10
Probability of Guerrilla killed = 0.001
Area of Operations = 100000

2.5 Application of the Lanchester equations to a simulated combat

A scenario for a simple land combat model

We will simulate the combat between two opposing forces, the Blue and the Red [Coyle, 1996]. The Blue forces consists of 1200 initial men in the front line, with their complete ammunition stock; 2000 soldiers in Reserve, with their ammunition ready to be transported in 30 trucks to be shared between reserve forces and ammunition. It should be noted that the Blue Reserve forces only have a limited transport capability, to share among the reserve soldiers and their ammunition. Also the Blue forces have 20 armored helicopters to be employed mainly to attack the Red reinforcements, in order to deter and to make difficult their advance to the front line.

The Red forces are only 1000 initial men in the front line of the battle (i.e. 200 soldiers less than the Blue combatant), and 2000 soldiers on reserve, to be committed at the request of the Tactical Commander. The Red Reserves have sufficient transport and ammunition for sustaining the whole combat.
We will be testing the following tactical decision:
The Red Tactical Commander will take advantage of his transport facilities and will commit the reserves as soon as the battle starts, accepting the heavy casualties that may be caused by the action of the Blue helicopters.
The Blue Tactical Commander is confident about the 20% difference of the initial forces for providing a winning edge, and the action of the helicopters for difficulty the Red reinforcement. Also been conscious of his limited transport capability will commit his reserves only if the Blue forces are matched by the Red ones, i.e. at a Red/Blue Ratio=1. In this event, he will give the orders to commit the reserves leaving to the Reserves Commander and the Logistic Commander to deal with the Transport Commander for sending to the front line the soldiers and their ammunition.
The following causal diagram shows this scenario. In the center is the Lanchester model previously explained with their corresponding additions to consider the reinforcements, the transport of men and ammunitions in the Blue side (in the Red side the ammunitions transport is omitted as no new information will provide), and the action of the Blue Helicopters against the Red reserves.

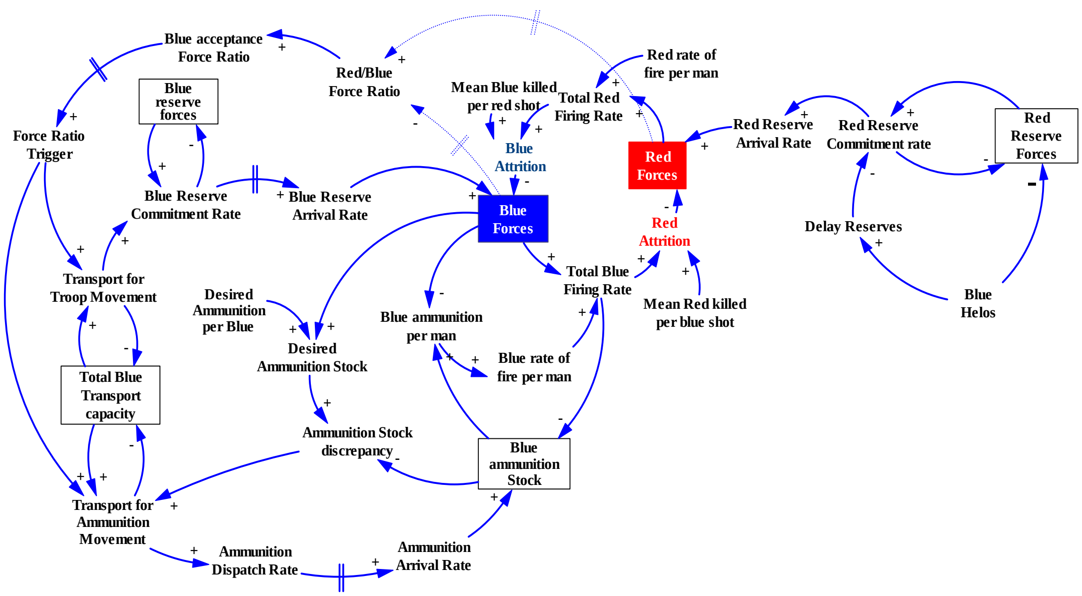

Figure 13. Causal loop diagram of the combat scenario

The CLD is shown in Figure 13 and the stock and flow model in Figure 14.

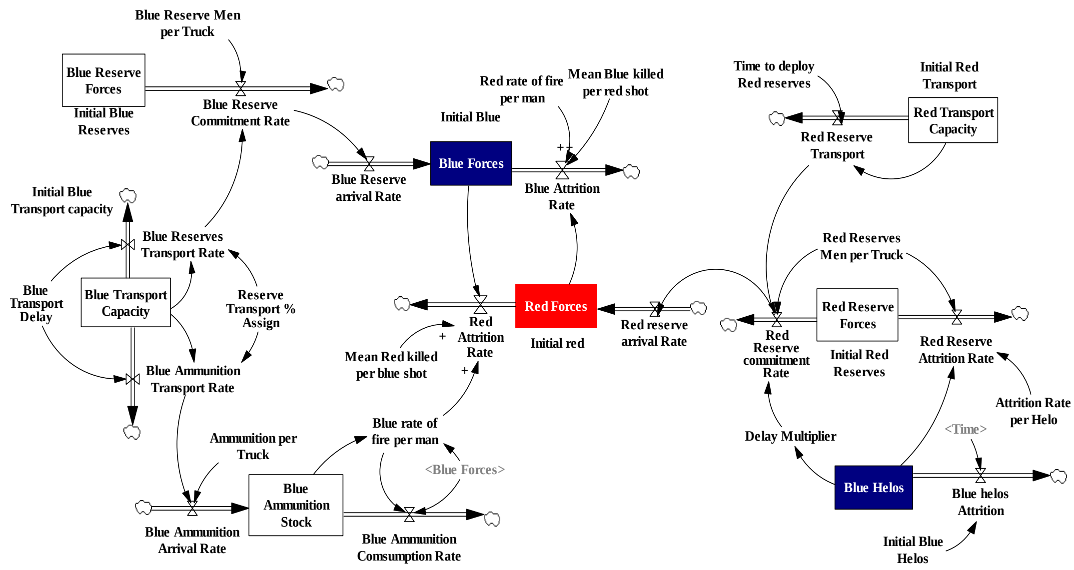

Figure 14. Combat model

After initiating the combat, at less than 4 hours, the Red forces matched the Blue ones. The action of the helicopters were not sufficient to detain the Red reserves incorporation to the front line, and even with the delay, the incorporation of their reserves were enough to counter against the attrition rate caused by the Blue forces.

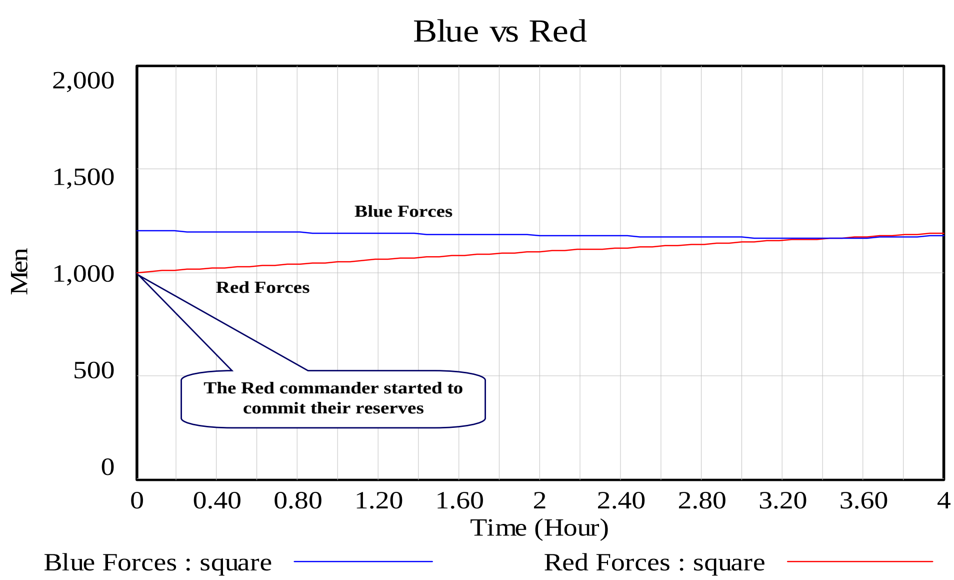

Figure 15. Results at 4 hours of combat

Following the previous planning, the Blue Tactical Commander orders to commit the Reserves, consisting in soldiers and their ammunitions. Their split in the transport is left to the field commanders. Here we may have at least two opposing events:

a. - The Blue Reserve Commander insist to have his forces in the front line at once, requesting the maximum possible transport:

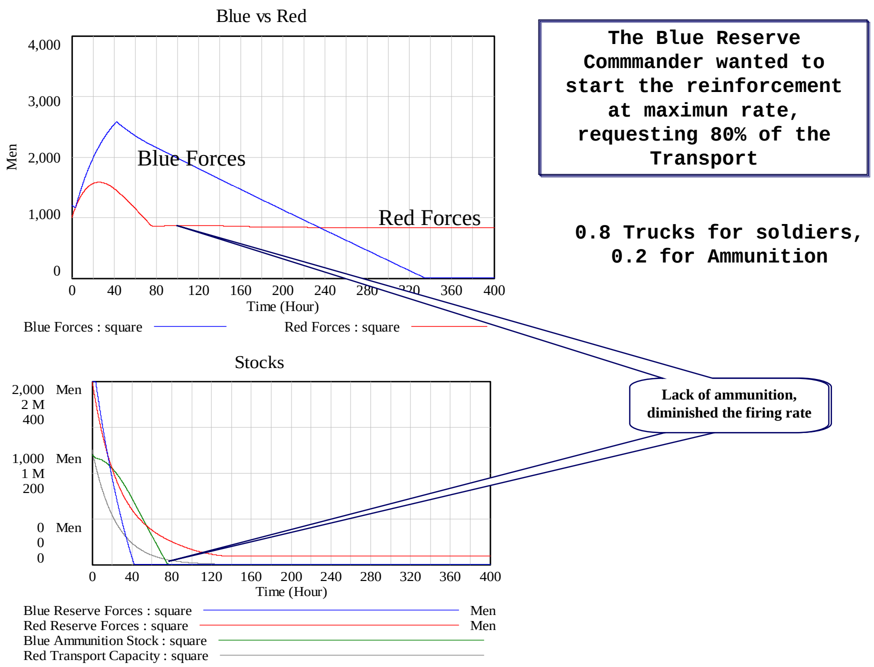

Figure 16. Results with 80% of transport for soldiers

The outcome of this event is shown in the simulation; a complete defeat of the Blue forces, mainly due to the lack of ammunition, which will be diminishing the firing rate of the soldiers.

b. - The Logistic Commander obtains the maximum transport:

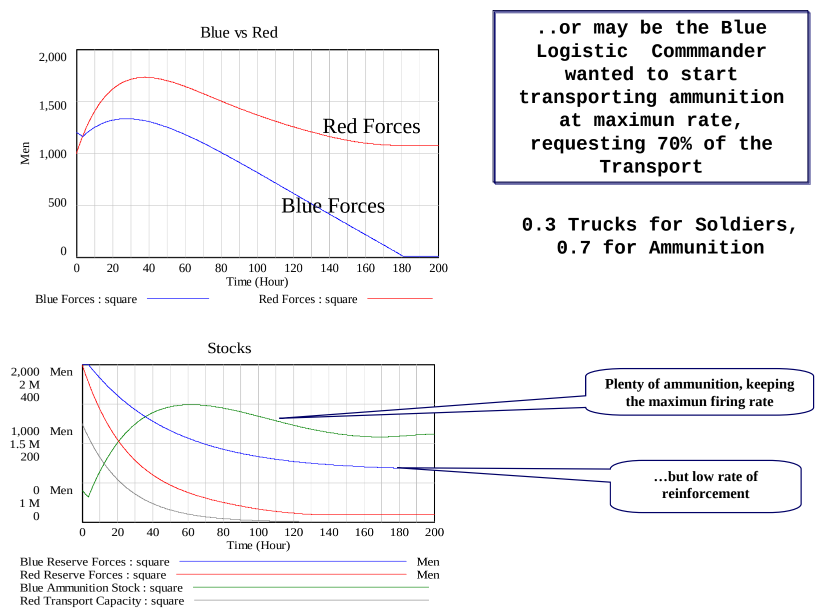

Figure 17. Results for 70% of trucks for ammunitions

In this case the Blue also will be defeated; now the Blue forces will have plenty of ammunition in the front line but not enough soldiers.
As we see from the above figures 16 and 17, both commanders are doing their best efforts to comply with the orders, for the benefits of the war, and their efforts are plausible, but we see that any of those events causes a complete defeat of the Blue forces, with very heavy casualties.
Looking more closely to this part of the causal diagram (Figure 18) it is shown that the split of the Blue transport between Men and Ammunition is a high leverage point, as it affects the complete outcome of the combat, therefore the Tactical Commander should not delegate the authority to any other field commander, as he is the only one empowered to take decisions at "system level".
This is one of the central aspects of System Dynamics referring to the system view that should be taken in the decisions, and where the proper tools employed in this discipline helps in understanding the important aspects of the systems.

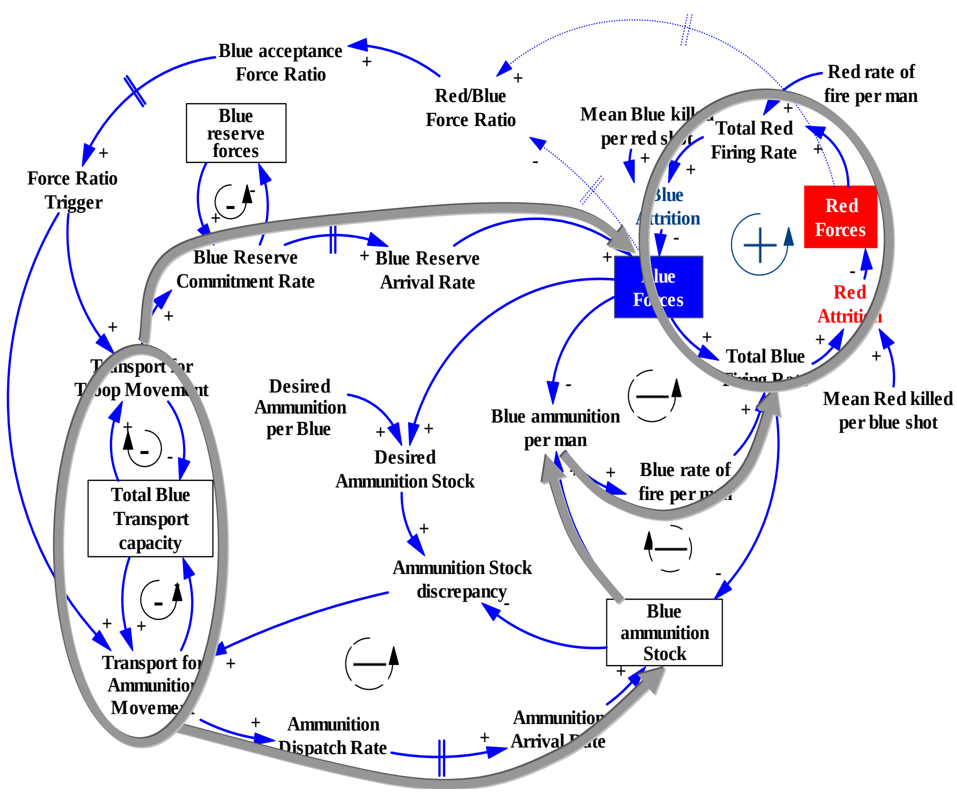

Figure 18. Causal diagram of the combat scenario indicating the leverage of the decision of transport splitting

c.- The Tactical Commander understands the importance of the issue and makes his own decision, not delegating to their subordinates commanders, as shown below:

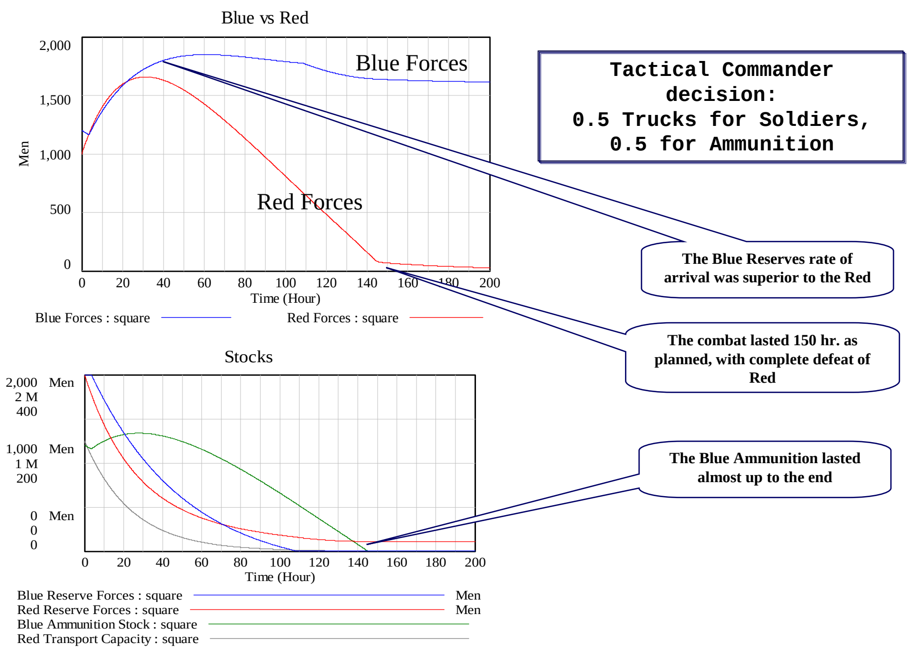

Figure 19. Results of the combat after Tactical Commander decision

In this case the outcome of the combat is a Blue success. System Dynamics have shown that the application of their concepts applied to a combat situation as shown in this simple scenario, may help to the commanders in devising sound tactical concepts and procedures.

To end up the case, we can adapt from 'Policies, Decisions and Information Sources for Modeling', Jay Forrester page 52 in *Modeling for Learning Organizations*, the following lesson that applies to the example just described:

"A Commander sets the stage for action by choosing which information sources to take seriously and which to ignore. A Commander's success depends on both, selecting the most relevant information and on using that information effectively. How quickly or slowly is information converted into action? What is the relative weight given to different information sources in light of desired objectives? How are these desired objectives created from available information?"

3. Conclusions

We had reviewed several combat models, coming from different sources, but adapting to employ system dynamics concepts and tools. The objectives of the models are a recurrent subject in the system dynamist world and had been treated in different studies. The combat models objectives are not as different as the general models of any science, but it would not be a complete treatment of this subject if we not recall again. A combat model is a fair representation of reality observed in each battle. It helps in tactical planning, even having a limited power to foresee battle details. To be useful must be

able to implement and produce credible results. It must be built upon assumption grounded in sound tactical theory. The model should allow to the user the opportunity to vary inputs concerning the allocation and deployment of the platforms. One of the normal weaknesses of modeling is trying to include as much details as possible, sometimes absolutely unnecessary, which only obscure the main issues. In a real battle there are so many unforeseen factors that probably are useless trying to include in a model many details, and loosing the main aspects. The concept of "bounded rationality" [Morecroft, 2007] in the decisions is applicable as well in a battle environment, as the commander takes his decision based upon in few pieces of information and normally with time pressures. McGunnigle and Lucas [McGunnigle John, 1999] addressed the military value of information in conflict, developing some experiments, one of them resulting that many military decision makers do not always use information optimally.
Ghaffarzadegan [Ghaffarzadegan, Lynies, Richardson, 2009] review the benefits of using small system dynamics models to address public policy questions and obtain insightful and important lessons. These same concepts are applicable to combats models. All those characteristics made System Dynamics very suitable for modeling combat situations, unfortunately not widely employed, at least in the open literature. This study tries to overcome these failures and hopes futures works in this important field could make be available to the general audience.

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