"DYNAMICA" ISSN 0306-7564 Volume 2, Part 3

Technical Note : The Aggregate Ordering Equation for a Class of Inventory Control Systems

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In System Dynamics work concerned with modelling whole industries it is frequently necessary to model the aggregate ordering policy of a group of companies. The purpose of this note is to derive suitable rules for the case of an industry in which all the companies operate a Cyclic Review or (s,t) system.

Assumptions

It is assumed that the industry is made up of a large number of identical companies (N) each of which operates a cyclic review system with cycle time (c). It is assumed that the ordering times for the individual companies are uniformly distributed so that $\frac{N}{c}$ companies place an order per unit time.

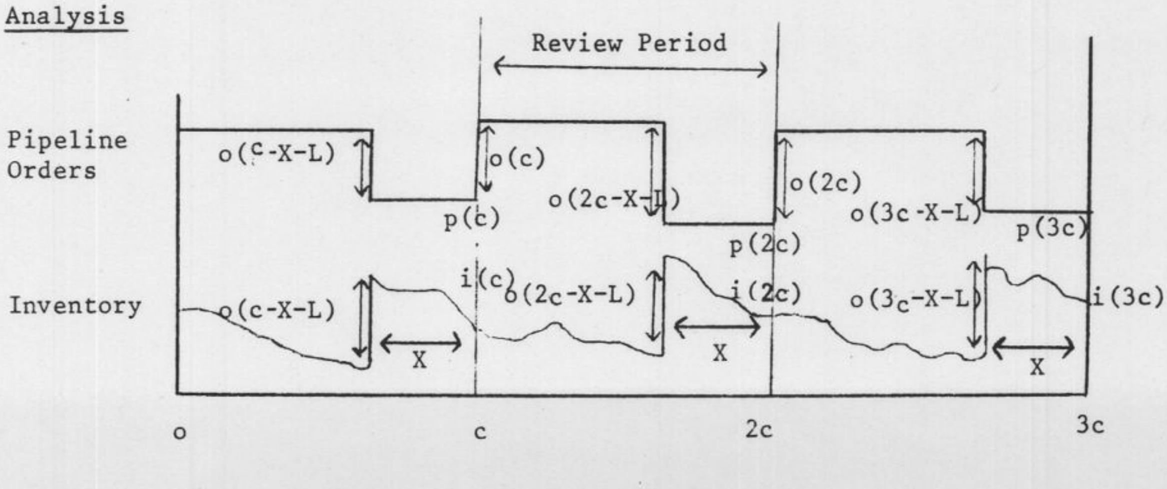
An individual company which reaches the end of a review period at time t will place an order o(t) given by

$$o(t) = r(t) - i(t) + La(t) - p(t) \qquad (1)$$

where r(t) is the current value of target inventory
i(t) is the current value of actual inventory
a(t) is the current average usage rate
p(t) is the current value of pipeline orders
L is the lead time between order and delivery assumed <u>fixed</u>.

These values will of course be identical for all companies ordering at time t. Note however that the identity of the companies will change with t.

Analysis



The characteristic of a cyclic review system with fixed lead time is that for an individual company it operates as shown in the figure. Since in the further analysis we wish to use Laplace Transforms we assume for simplicity - without serious loss of generality - that prior to t=0 the system is in equilibrium and that all the values concerned are measured relative to this equilibrium state.

We shall also need the following variables:

$O(t)$	total <u>industry</u> order rate at time t
$I(t)$	total <u>industry</u> inventory at time t
$P(t)$	total <u>industry</u> pipeline orders at time t
$N\tau(t) = T(t)$	total <u>industry</u> target stocks at time t
$Na(t) = A(t)$	total <u>industry</u> average usage rate at time t
$Nu(t) = U(t)$	total <u>industry</u> usage rate at time t
$u(t)$	individual company usage rate at time t

On the basis of the assumptions made and the diagram the following equations are easily derived.

$$i(t) = i(t-c) - \int_{t-c}^{t} u(t) + o(t-X-L) \qquad (2)$$

$$p(t) = p(t-c) + o(t-c) - o(t-X-L) \qquad (3)$$

$$I(t) = O(t-L) - U(t) \qquad (4)$$

$$P(t) = O(t) - O(t-L) \qquad (5)$$

$$O(t) = \frac{N}{c}\, o(t) \qquad (6)$$

$$U(t) = \frac{N}{c}\, u(t) \qquad (7)$$

Application of standard Laplace Transform methods to equations (1) - (7) leads to the following transforms:

$$i(s) = \frac{e^{-Xs}\, o(s)}{(1 - e^{-cs})} - \frac{u(s)}{s} \qquad (8)$$

$$p(s) = \frac{(e^{-cs} - e^{-(L+X)s})\, o(s)}{(1 - e^{-cs})} \qquad (9)$$

$$o(s) = (1 - e^{-cs})\left(\tau(s) + La(s) + \frac{u(s)}{s}\right) \qquad (10)$$

$$I(s) = \frac{N}{c}\left\{\frac{e^{-Ls}\, o(s)}{s} - \frac{u(s)}{s}\right\} \qquad (11)$$

$$P(s) = \frac{N}{c}\left\{\frac{(1 - e^{-Ls})\, o(s)}{s}\right\} \qquad (12)$$

$$O(s) = \frac{N}{c}\, o(s) \qquad (13)$$

It follows from equations (10) and (13) that a suitable equation for industry order rate is

$$O(t) = \left\{T(t) + LA(t) + \int_0^t U(t)\right\} - \left\{T(t-c) + LA(t-c) + \int_0^{t-c} U(t)\right\} \qquad (14)$$

It is easily verified that O(t) cannot be expressed exactly in terms of I(t) and P(t) in any convenient form. The equation

$$O(t) = \frac{\left\{T(t) - I(t)\right\}}{c} + \frac{\left\{LA(t) - P(t)\right\}}{c} + \frac{O(t-L)}{2} \qquad (15)$$

can be shown by the methods of (Sharp, 1976) to have a frequency response that differs from that of (13) by terms of order $s^3 = (j\omega)^3$ only and thus provides an acceptable approximation at the lower frequencies that are usually of interest in an S.D. study.

Adopting the usual third order delay approximation to a pipeline delay we have on the basis of (14) and (15) that suitable DYNAMO equations for industry order rate are <u>either</u>

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R  O.KL = (T.K-I.K+L*A.K-P.K)/C+AVO.K/2
A  AVO.K = DLINF3(O.JK,C)                        (16)
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<u>or</u>

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L   CUMU.K = CUMU.J+DT*U.JK
A   Y.K    = T.K+L*A.K+CUMU.K
A   DY.K   = DLINF3(Y.K,C)                       (17)
R   O.KL   = Y.KL-DY.K
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where T.K = T(t),etc.

It will be noted

a) that neither (16) or (17) correspond to the equations usually used (c.f. Forrester, 1961) and there is therefore a possibility of incorrect model dynamics with the usual equations.
b) that in the constant usage rate case Industry inventory is not equal to Target Inventory as indeed is obvious if a diagram is drawn showing the operation of such a system.
c) that in both (16) and (17) the cyclic review constant appears naturally as an equation parameter.
d) since production capacity merely represents a different type of inventory and capacity requirements are commonly reviewed in connection with the budgetting cycle, the equations given above might also serve for modelling capacity acquisition by an industry.

<u>References</u>

Forrester J.W., 'Industrial Dynamics', MIT, 1961

Sharp J.A., 'The Role of Forecasts in System Dynamics', Dynamica, VolII,2,1976.