CONTINUOUS NON-LINEAR FUNCTIONS FOR USE IN SYSTEM DYNAMICS MODELLING

Khalid Saeed, Arit Arayman Irdamidris, Asian Institute of Technology, Bangkok, Thailand

Khalid Saeed is Associate Professor of System Science and Management at AIT. Arit Arayman Irdamidris is a Research Associate in the Division of Industrial Engineering and Management at AIT.

ABSTRACT

This paper suggests continuous mathematical functions that can be used to replace DYNAMO supplied TABLE functions for most modelling applications where non-linear functions are needed. The proposed functions are selected for the ease of control of their slopes and limit parameters. Their use may help to increase the size of the model that can be handled on small systems while also avoiding the discontinuities of the TABLE functions they replace.

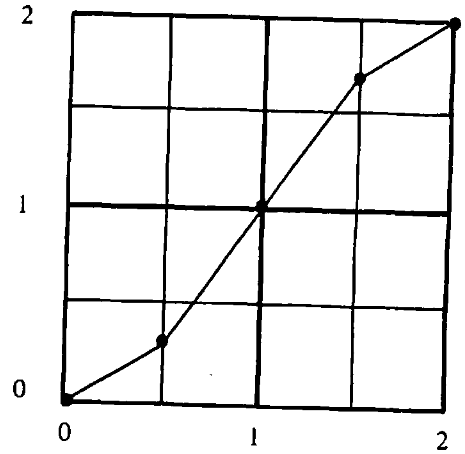
1. RATIONALE

Many types of non-linear functions are employed in Systems Dynamics models to represent cause and effect relationships between pairs of variables. These relationships often abstract complex real world processes which do not involve significant delay as compared with the other delays in the conceptual systems being modelled.

The computer code used for simulating system dynamics models, DYNAMO, offers a rather sophisticated method for representing the non-linear relationships of the model through the use of TABLE functions. These functions allow the user to construct a non-linear relationship graphically. The co-ordinates of the turning points of the function are, then, stated in the arguments of the TABLE function macro which computes a value of the dependent variable for any value of the independent variable within the specified range. This is done by interpolating over the segment of the function that contains the value of the independent variable. Figure 1 shows a DYNAMO representation of a typical non-linear function.

Later versions of DYNAMO offer several variations of the basic function described above. For example, there is TABHL which uses the extreme values when the specified range is exceeded. There is TABXT, which extrapolates the last two extreme values when the range is exceeded, and TABBL which passes a polynomial through the specified points and interpolates on this polynomial instead of on the segments of a discontinuous function as in the previous cases. The properties of the polynomial beyond the specified range are, however, uncertain[1].

The advantages of the DYNAMO supplied TABLE functions include the ease with which they can be developed, the facility for changing their parameters and shapes in the rerun mode, and the possibility of representing a wide variety of shapes.



A Y.K=TABLE (TY, X.K, 0, 2,.5)
T TY=0/.3/1/1.7/2.0

Figure 1: Table function in DYNAMO.

The DYNAMO supplied TABLE functions also have a few disadvantages. Firstly, they are discontinuous which may lead to strange behaviour when their derivatives are computed in a model. Secondly, it is quite cumbersome to conduct parameter sensitivity tests for a TABLE function. All co-ordinates of the function must be supplied every time a change in its slope is made. Thus, a slight misspecification may often cause a change in shape that violates the original assumptions made for the function. Thirdly, the interpolation procedure involved incorporates several procedural steps which take much computer time. Finally, many versions of DYNAMO, especially those for small computing systems can handle only a limited number of TABLE function values which places an additional limit on the size of the model that can be simulated.

This paper suggests several continuous functions that can be used in System Dynamics models to represent non-linear relationships and that have some advantages over the DYNAMO supplied TABLE functions. Also when DYNAMO is not available and the user cannot write an interpolation routine, these relationships might be the only type of functions available for representing the non-linear relationships.

2. GENERAL CHARACTERISTICS OF NON-LINEAR RELATIONSHIPS USED IN SYSTEM DYNAMICS

The non-linear relationships used in system dynamics models are often normalised with respect to a measurable set of "normal" operating conditions. A typical equation involving TABLE functions may appear as follows:

RV.KL=LV.K*FRN*M1.K*M2.K*M3.K

Where RV is the rate of change. LV, the level, FRN the normal fractional change, M1, M2 M3 are the various multipliers represented by TABLE functions. Normal values of these multipliers are invariably equal to one, although sometimes (as for example when a rate represents the net flow into a level) zero might be used as the normal value and FRN omitted: However this would call for modifying the dimensions of at least one of the multipliers.

As far as the slope of these functions is concerned, the non-linear functions most commonly used can be divided into 3 main categories each of which has 2 sub-categories. The three main categories are: convex functions, concave functions and S-shaped functions. Each of these can either be positively sloping or negatively sloping. Furthermore, the S-shaped functions may have either (1,1) or (0,0) as normal value.

Besides the normal value, the other points of interest in a TABLE function are its extreme values. The lowest extreme value of a concave function with a positive slope is usually zero, while that for one with a negative slope is usually a finite, but relatively large, number. The upper extreme values of such functions are infinity and zero respectively, but the values of the independent variables at which these extremes are approached are often not precisely known. These extremes are, however, determined by the slope of the concave function.

It is customary to use zero or near zero as the lowest extreme value in the case of a simple convex function with positive slope and a reasonable upper saturation limit which is often not precisely known. A convex function with a negative slope may sometimes be used when representing the effect of availability of fixed resources. In such a case, both extreme conditions are known and are usually one and zero respectively.

S-shaped functions are widely used to represent behavioural responses based on experience. The positively sloping S-shaped functions often have a near zero lower limit and an upper limit which is also not strictly specified. In case of a negatively sloping function, the extreme value corresponding to the lowest value of the independent variable must be specified while the other extreme value is near zero. The slope of this curve would determine when the later extreme value approaches zero. In both positively and negatively sloping cases, the steepness of the function is often a relatively more important parameter than its extreme values, although a system may be insensitive to both.

The S-shaped functions with a (0,0) normal value are usually symmetrical around this value. When these functions represent a policy, the modeller may wish to change their slope with, or without, changing the specified extreme values.

Many other irregularly shaped functions may also be used in System Dynamics models. But the more complex the shape of a function the greater the number of assumptions that it incorporates and the longer the rope to hang the modeller with. The use of irregularly shaped functions, therefore, is not favored by the authors. In the following sections well behaved mathematical functions, which meet the criteria of shape and extreme value for the right types of non-linear functions discussed above, will be proposed.

3. CONCAVE FUNCTIONS

These functions may be positively or negatively sloping. They should pass through the point where both dependent and independent variables assume a value of unity, and the user should be able to specify their lowest extreme value and slope. The following formula is recommended for a positively sloping concave function:

$$Y=N+(1-N)*(X**SLP)$$

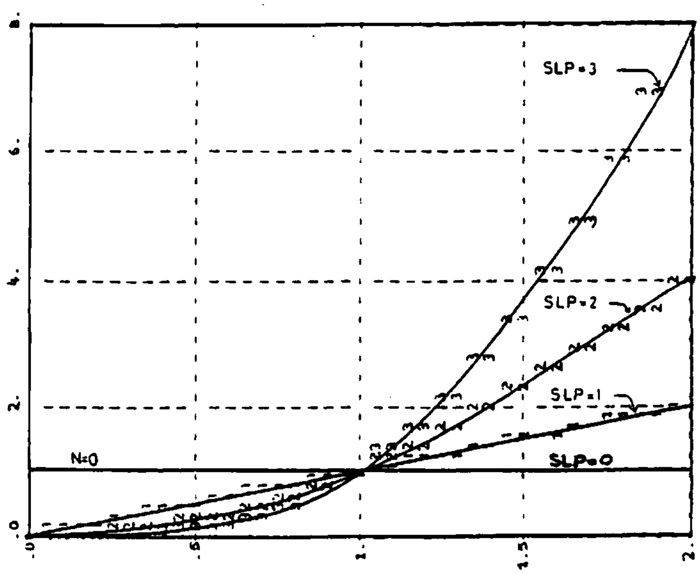
where, Y = Dependent variable

X = Independent variable

N = Value of Y at X=0

SLP = Slope parameter (SLP > 1)

Figure 2 shows the behaviour of this function for N=0 and for different values of SLP. The function reduces to a straight line when SLP=1 and to a horizontal line passing through the (1,1) point when SLP=0. The steepness of the function rises when SLP is increased.



X	=	.000	Y	=	.000	N	=	0
X	=	.500	Y	=	.500	SLP	=	1
X	=	1.000	Y	=	1.000			
X	=	1.500	Y	=	1.500			
X	=	2.000	Y	=	2.000			

****

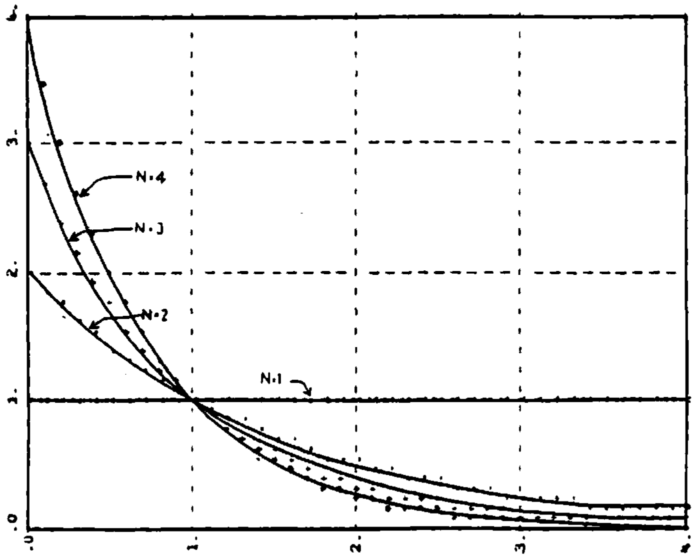
X	=	.000	Y	=	.000	N	=	0
X	=	.500	Y	=	.250	SLP	=	2
X	=	1.000	Y	=	1.000			
X	=	1.500	Y	=	2.250			
X	=	2.000	Y	=	4.000			

****

X	=	.000	Y	=	.000	N	=	0
X	=	.500	Y	=	.125	SLP	=	3
X	=	1.000	Y	=	1.000			
X	=	1.500	Y	=	3.375			
X	=	2.000	Y	=	8.000			

****

Figure 2: Concave Function with a Positive Slope
*Y = N + (1-N) (X**SLP)*




```
X = .000     Y = 2.000     N = 2
X = .500     Y = 1.414
X = 1.000    Y = 1.000
X = 1.500    Y = .707
X = 2.000    Y = .500
X = 2.500    Y = .354
X = 3.000    Y = .250
X = 3.500    Y = .177
X = 4.000    Y = .125
**********************************************
X = .000     Y = 3.000     N = 3
X = .500     Y = 1.732
X = 1.000    Y = 1.000
X = 1.500    Y = .577
X = 2.000    Y = .333
X = 2.500    Y = .192
X = 3.000    Y = .111
X = 3.500    Y = .064
X = 4.000    Y = .037
**********************************************
X = .000     Y = 4.000     N = 4
X = .500     Y = 2.000
X = 1.000    Y = 1.000
X = 1.500    Y = .500
X = 2.000    Y = .250
X = 2.500    Y = .125
X = 3.000    Y = .063
X = 3.500    Y = .031
X = 4.000    Y = .016
**********************************************
```

Figure 3: Concave Function with a Negative Slope
*Y = N * EXP ((-LOGN(N))(X))*

Negatively sloping concave functions are more commonly used than positvely sloping ones. These can be represented as follows:

Y=(N)*(EXP((-LOGN(N))(X))

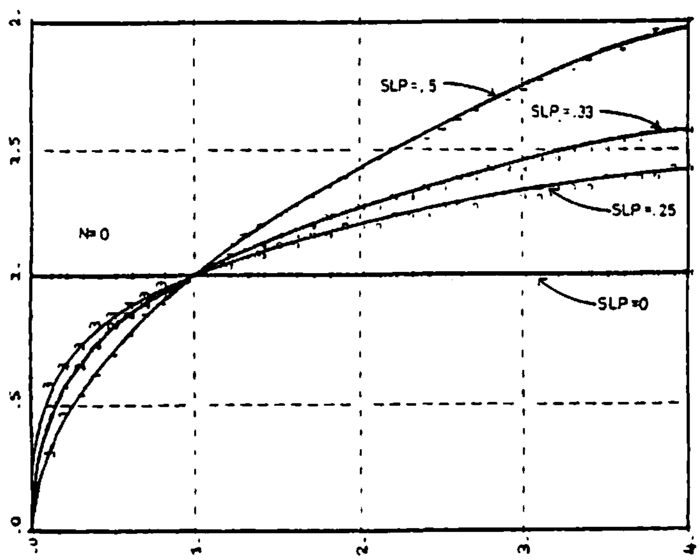
where, Y = Dependent variable
X = Independent variable
N = Value of Y at X=0

The slope of this function is controlled by the lowest extreme value N. The function becomes a horizontal straight line which passes through (1,1) when N=1. The characteristics of this function are illustrated in Figure 3.

4. CONVEX FUNCTIONS

These functions may also be positively or negatively sloping. They should pass through the point (1,1) and their slope should be controllable. The formula found appropriate for the positively sloping convex function is the same as that for the positively sloping concave function, although the slope parameter should always be < 1.

Y=N+(1-N)*(X**SLP)



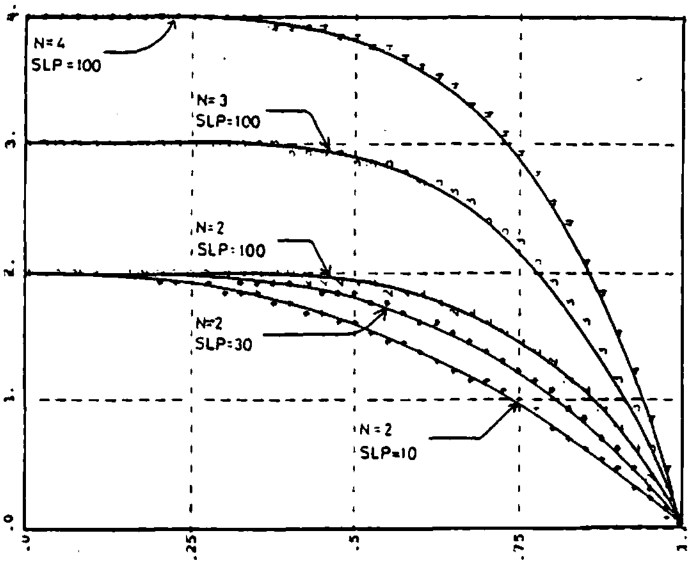

```
X = .000     Y = .000      N   = 0
X = .500     Y = .707      SLP = .5
X = 1.000    Y = 1.000
X = 1.500    Y = 1.225
X = 2.000    Y = 1.414
X = 2.500    Y = 1.581
X = 3.000    Y = 1.732
X = 3.500    Y = 1.871
X = 4.000    Y = 2.000
**********************************************
X = .000     Y = .000      N   = 0
X = .500     Y = .794      SLP = .3333
X = 1.000    Y = 1.000
X = 1.500    Y = 1.145
X = 2.000    Y = 1.260
X = 2.500    Y = 1.357
X = 3.000    Y = 1.442
X = 3.500    Y = 1.518
X = 4.000    Y = 1.587
**********************************************
X = .000     Y = .000      N   = 0
X = .500     Y = .841      SLP = .25
X = 1.000    Y = 1.000
X = 1.500    Y = 1.107
X = 2.000    Y = 1.189
X = 2.500    Y = 1.257
X = 3.000    Y = 1.316
X = 3.500    Y = 1.368
X = 4.000    Y = 1.414
**********************************************
```

Figure 4: Convex Function with a Positive Slope
*Y = N + (1-N) (X ** SLP)*

where, Y = Dependent variable
N = Value of Y at X=0 and
SLP = Slope parameter (SLP < 1)

Figure 4 shows the characteristics of this function for various values of SLP. The function is neutralised into a horizontal line when SLP=0.

The negatively sloping convex function can be represented by the following formula:



X	=	.000	Y	=	2.000	N	= 2
X	=	.250	Y	=	1.997	SLP	= 100
X	=	.500	Y	=	1.916		
X	=	.750	Y	=	1.468		
X	=	1.000	Y	=	.000		

****

X	=	.000	Y	=	3.000	N	= 3
X	=	.250	Y	=	2.995	SLP	= 100
X	=	.500	Y	=	2.677		
X	=	.750	Y	=	2.202		
X	=	1.000	Y	=	.000		

****

X	=	.000	Y	=	4.000	N	= 4
X	=	.250	Y	=	3.993	SLP	= 100
X	=	.500	Y	=	3.836		
X	=	.750	Y	=	2.937		
X	=	1.000	Y	=	.000		

****

X	=	.000	Y	=	2.000	N	= 2
X	=	.250	Y	=	1.918	SLP	= 10
X	=	.500	Y	=	1.595		
X	=	.750	Y	=	.969		
X	=	1.000	Y	=	.000		

****

X	=	.000	Y	=	2.000	N	= 2
X	=	.250	Y	=	1.982	SLP	= 30
X	=	.500	Y	=	1.811		
X	=	.750	Y	=	1.246		
X	=	1.000	Y	=	.000		

****

Figure 5: Convex Function with a Negative Slope
Y = N-(N) (X ** (LOGN(SLP)))

Y=N-(N)*(X**(LOGN(SLP))) SLP > 0
0 < X < 1

where, Y = Dependent variable
N = Value of Y at X=0
SLP = Slope parameter

This function will not pass through (1,1) point but through (1,0). It is useful for representing capacity constraints such as the effect of land fraction occupied on housing construction rate (2). Figure 5 shows the characteristic behaviour of this function for various values of N and SLP. It is, however, cumbersome to neutralise this function using the initial value and the slope parameter. When N=1 and a very large value is used for SLP, the function will be nearly 1.0 for all values of X < 1, but this value will suddenly drop to zero when X=1. The use of a switching function, instead, is recommended. The value of the function becomes negative when X > 1.0, but when the capacity constraint is represented by a fraction, the maximum value of X for this function will never exceed one.

5. S-SHAPED FUNCTIONS

S-shaped functions are perhaps the most widely used non-linear functions in System Dynamics modelling. The S-shaped curve inflexes around a normal condition and attempts to translate the behavioural responses of a decision-maker as the condition deviates from normal. The normal condition often represents unity value for both dependent and independent variables when the rates of inflow to, and outflow from, a level are separated and zero when they are not. These functions can be positively as well as negatively sloping. Mathematical formulations for each of these cases are discussed below:

5.1 Positively Sloping S-shaped Functions

The following function is suggested when the normal condition occurs at X=1, Y=1:

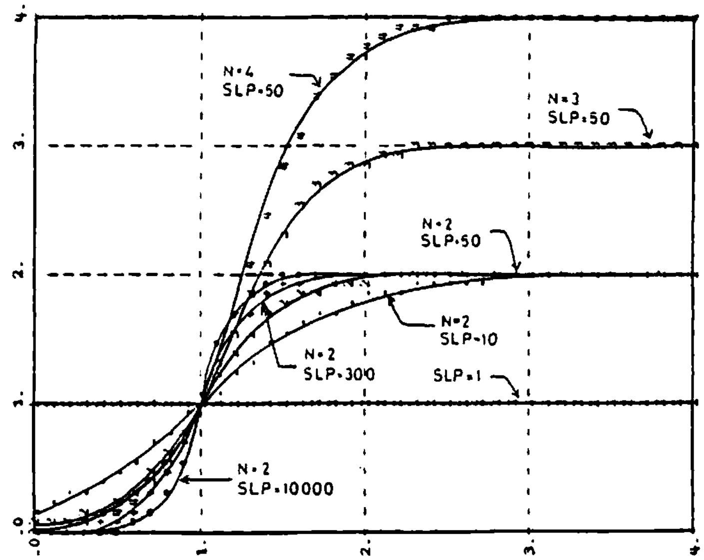
Y=N/(1+(N-1)(SLP)(EXP(-LOGN(SLP)(X)))) SLP > 1

where, Y = Dependent variable
X = Independent variable
N = Upper limit for Y
SLP = Steepness parameter when N is fixed

Figure 6 shows the behaviour of this function for various values of N and SLP. It should be noticed that Y always has a small positive value when X=0, which is a useful property as this would be needed for most applications. The function would give a value of Y=1 for all values of X when SLP=1. Thus, it can be easily neutralised when a model is being tested.

A positively sloping S-shaped function can be constructed as follows when the normal condition occurs at X=0, Y=0:

Y=((2*N)/(1+EXP((-LOGN(SLP))(X))))-N



X = .000	Y = .162	N = 2
X = .500	Y = .481	SLP = 10
X = 1.000	Y = 1.000	
X = 1.500	Y = 1.519	
X = 2.000	Y = 1.818	
X = 2.500	Y = 1.939	
X = 3.000	Y = 1.980	
X = 3.500	Y = 1.994	
X = 4.000	Y = 1.998	

****

X = .000	Y = .007	N = 2
X = .500	Y = .109	SLP = 300
X = 1.000	Y = 1.000	
X = 1.500	Y = 1.891	
X = 2.000	Y = 1.993	
X = 2.500	Y = 2.000	
X = 3.000	Y = 2.000	
X = 3.500	Y = 2.000	
X = 4.000	Y = 2.000	

****

X = .000	Y = .000	N = 2
X = .500	Y = .020	SLP = 10000
X = 1.000	Y = 1.000	
X = 1.500	Y = 1.980	
X = 2.000	Y = 2.000	
X = 2.500	Y = 2.000	
X = 3.000	Y = 2.000	
X = 3.500	Y = 2.000	
X = 4.000	Y = 2.000	

****

X = .000	Y = .039	N = 2
X = .500	Y = .248	SLP = 50
X = 1.000	Y = 1.000	
X = 1.500	Y = 1.752	
X = 2.000	Y = 1.961	
X = 2.500	Y = 1.994	
X = 3.000	Y = 1.999	
X = 3.500	Y = 2.000	
X = 4.000	Y = 2.000	

****

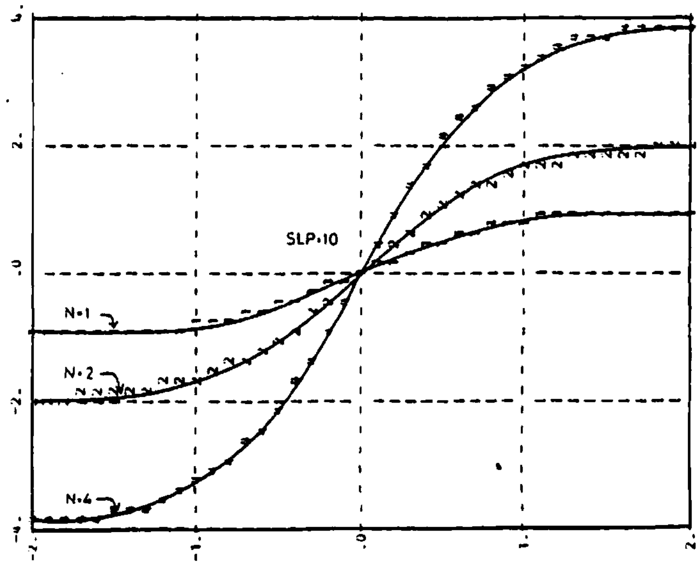
X = .000	Y = .030	N = 3
X = .500	Y = .198	SLP = 50
X = 1.000	Y = 1.000	
X = 1.500	Y = 2.339	
X = 2.000	Y = 2.885	
X = 2.500	Y = 2.983	
X = 3.000	Y = 2.998	
X = 3.500	Y = 3.000	
X = 4.000	Y = 3.000	

****

X = .000	Y = .026	N = 4
X = .500	Y = .180	SLP = 50
X = 1.000	Y = 1.000	
X = 1.500	Y = 2.806	
X = 2.000	Y = 3.774	
X = 2.500	Y = 3.966	
X = 3.000	Y = 3.995	
X = 3.500	Y = 3.999	
X = 4.000	Y = 4.000	

****

Figure 6: S-Shaped Function with a Positive Slope and Normal at (1,1)
Y = N/(1 + (N-1) (SLP) (EXP-LOGN(SLP) (X))))



X = -2.000	Y = -.9802	N = 1
X = -1.500	Y = -.9387	SLP = 10
X = -1.000	Y = -.8182	
X = -.500	Y = -.5195	
X = .000	Y = .0000	
X = .500	Y = .5195	
X = 1.000	Y = .8182	
X = 1.500	Y = .9387	
X = 2.000	Y = .9802	

****

X = -2.000	Y = -1.960	N = 2
X = -1.500	Y = -1.877	SLP = 10
X = -1.000	Y = -1.636	
X = -.500	Y = -1.039	
X = .000	Y = .000	
X = .500	Y = 1.039	
X = 1.000	Y = 1.636	
X = 1.500	Y = 1.877	
X = 2.000	Y = 1.960	

****

X = -2.000	Y = -3.921	N = 4
X = -1.500	Y = -3.755	SLP = 10
X = -1.000	Y = -3.273	
X = -.500	Y = -2.078	
X = .000	Y = .000	
X = .500	Y = 2.078	
X = 1.000	Y = 3.273	
X = 1.500	Y = 3.755	
X = 2.000	Y = 3.921	

****

Figure 7: S-Shaped Function with a Positive Slope and Normal at (0,0)
*Y = ((2*N)/(1 + EXP((-LOGN(SLP)) (X))))-N*
Fixed SLP; N is varied

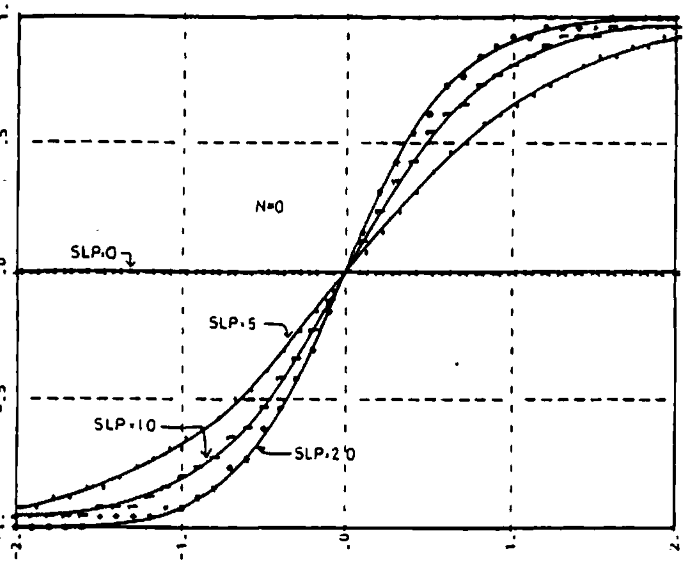
where, Y = Dependent variable
X = Independent variable
N = Upper and lower extreme values
SLP = Slope parameter when N is fixed

Figure 8 illustrates the behaviour of this function for a given value of N and for different values of SLP. Figure 7 shows its behaviour when SLP is fixed and the value of N is varied. The function is neutralised when SLP=0.

5.2 Negatively Sloping S-shaped Functions

A negatively sloping S-shaped function can be constructed as follows when the normal condition occurs at X=1, Y=1:

Y=(N/(N-1))/((X**SLP)+(1/(N-1)))



X	=	-2.000	Y	=	-.9231	N = 1	
X	=	-1.500	Y	=	-.8358	SLP = 5	
X	=	-1.000	Y	=	-.6667		
X	=	-.500	Y	=	-.3820		
X	=	.000	Y	=	.0000		
X	=	.500	Y	=	.3820		
X	=	1.000	Y	=	.6667		
X	=	1.500	Y	=	.8358		
X	=	2.000	Y	=	.9231		

****

X	=	-2.000	Y	=	-.9802	N = 1	
X	=	-1.500	Y	=	-.9387	SLP = 10	
X	=	-1.000	Y	=	-.8182		
X	=	-.500	Y	=	-.5195		
X	=	.000	Y	=	.0000		
X	=	.500	Y	=	.5195		
X	=	1.000	Y	=	.8182		
X	=	1.500	Y	=	.9387		
X	=	2.000	Y	=	.9802		

****

X	=	-2.000	Y	=	-.9950	N = 1	
X	=	-1.500	Y	=	-.9779	SLP = 20	
X	=	-1.000	Y	=	-.9048		
X	=	-.500	Y	=	-.6345		
X	=	.000	Y	=	.0000		
X	=	.500	Y	=	.6345		
X	=	1.000	Y	=	.9048		
X	=	1.500	Y	=	.9779		
X	=	2.000	Y	=	.9950		

****

Figure 8: S-Shaped Function with a Positive Slope and Normal at (0,0)
Y = ((2)/(1 + EXP((-LOGN(SLP)) (X))))-N*
Fixed N; SLP is varied

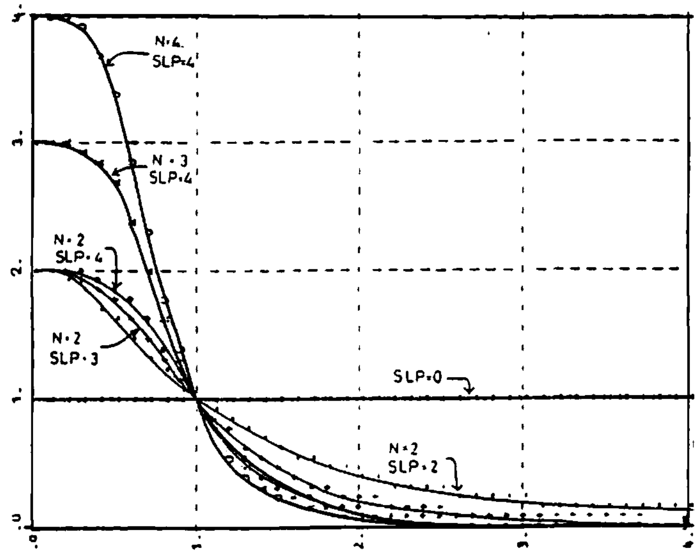
where, Y = Dependent variable

X = Independent variable

N = The value of Y when X=0

SLP = Slope parameter when N is fixed

Figure 9 illustrates the behaviour of this function for different values of N and SLP. The function can be neutralised when SLP=0.



X	=	.000	Y	=	2.000	N = 2	
X	=	.500	Y	=	1.600	SLP = 2	
X	=	1.000	Y	=	1.000		
X	=	1.500	Y	=	.615		
X	=	2.000	Y	=	.400		
X	=	2.500	Y	=	.276		
X	=	3.000	Y	=	.200		
X	=	3.500	Y	=	.151		
X	=	4.000	Y	=	.118		

****

X	=	.000	Y	=	2.000	N = 2	
X	=	.500	Y	=	1.778	SLP = 3	
X	=	1.000	Y	=	1.000		
X	=	1.500	Y	=	.457		
X	=	2.000	Y	=	.222		
X	=	2.500	Y	=	.120		
X	=	3.000	Y	=	.071		
X	=	3.500	Y	=	.046		
X	=	4.000	Y	=	.031		

****

X	=	.000	Y	=	2.000	N = 2	
X	=	.500	Y	=	1.882	SLP = 4	
X	=	1.000	Y	=	1.000		
X	=	1.500	Y	=	.330		
X	=	2.000	Y	=	.118		
X	=	2.500	Y	=	.050		
X	=	3.000	Y	=	.024		
X	=	3.500	Y	=	.013		
X	=	4.000	Y	=	.008		

****

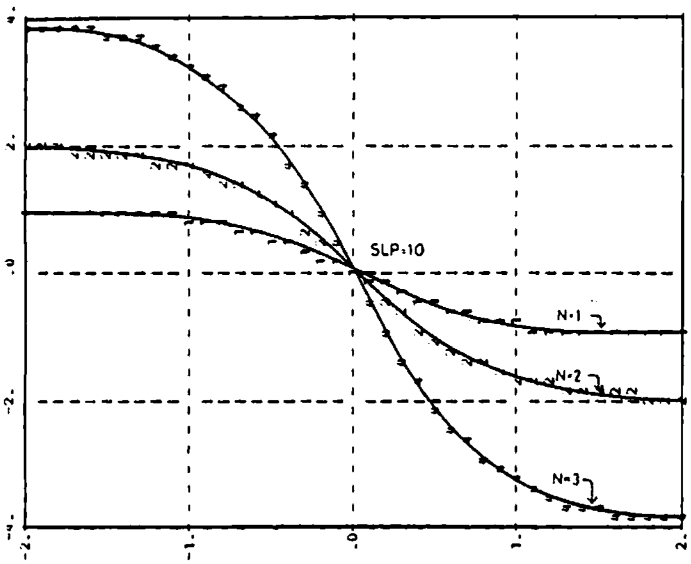
X	=	.000	Y	=	3.000	N = 3	
X	=	.500	Y	=	2.667	SLP = 4	
X	=	1.000	Y	=	1.000		
X	=	1.500	Y	=	.270		
X	=	2.000	Y	=	.091		
X	=	2.500	Y	=	.038		
X	=	3.000	Y	=	.016		
X	=	3.500	Y	=	.010		
X	=	4.000	Y	=	.006		

****

X	=	.000	Y	=	4.000	N = 4	
X	=	.500	Y	=	3.368	SLP = 4	
X	=	1.000	Y	=	1.000		
X	=	1.500	Y	=	.247		
X	=	2.000	Y	=	.062		
X	=	2.500	Y	=	.034		
X	=	3.000	Y	=	.016		
X	=	3.500	Y	=	.009		
X	=	4.000	Y	=	.005		

****

Figure 9: S-Shaped Function with a Negative Slope and Normal at (1,1)
*Y = (N/(N-1))/((X ** SLP) + (1/(N-1)))*



X = -2.000	Y = .9802	N = 1
X = -1.500	Y = .9387	SLP = 10
X = -1.000	Y = .8162	
X = .500	Y = .5195	
X = .000	Y = .0000	
X = .500	Y = -.5195	
X = 1.000	Y = -.8162	
X = 1.500	Y = -.9387	
X = 2.000	Y = -.9802	

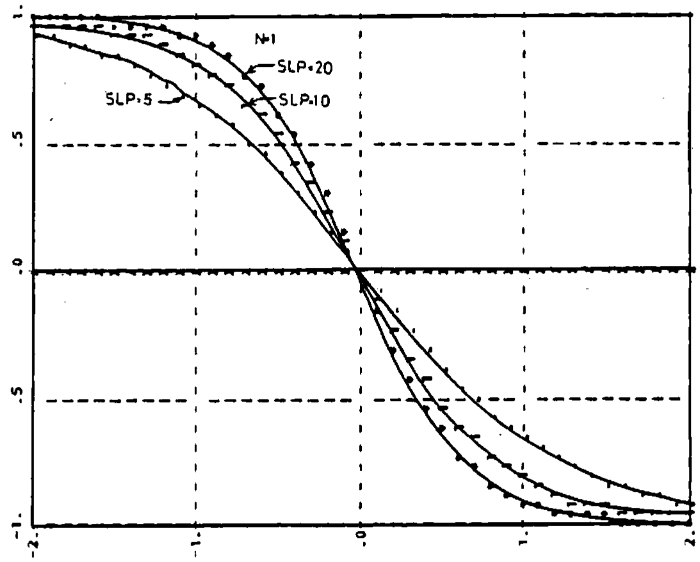
****

X = -2.000	Y = 1.960	N = 2
X = -1.500	Y = 1.877	SLP = 10
X = -1.000	Y = 1.636	
X = .500	Y = 1.039	
X = .000	Y = .000	
X = .500	Y = -1.039	
X = 1.000	Y = -1.636	
X = 1.500	Y = -1.877	
X = 2.000	Y = -1.960	

****

X = -2.000	Y = 3.921	N = 4
X = -1.500	Y = 3.755	SLP = 10
X = -1.000	Y = 3.273	
X = .500	Y = 2.078	
X = .000	Y = .000	
X = .500	Y = -2.078	
X = 1.000	Y = -3.273	
X = 1.500	Y = -3.755	
X = 2.000	Y = -3.921	

Figure 10: S-Shaped Function with a Negative Slope and Normal at (0,0)
Y = -((2*N)/(1 + EXP ((-LOGN(SLP))(X))))+N
Fixed SLP; N is varied



X = -2.000	Y = .9231	N = 1
X = -1.500	Y = .8358	SLP = 5
X = -1.000	Y = .6667	
X = .500	Y = .3820	
X = .000	Y = .0000	
X = .500	Y = -.3820	
X = 1.000	Y = -.6667	
X = 1.500	Y = -.8358	
X = 2.000	Y = -.9231	

****

X = -2.000	Y = .9802	N = 1
X = -1.500	Y = .9387	SLP = 10
X = -1.000	Y = .8162	
X = .500	Y = .5195	
X = .000	Y = .0000	
X = .500	Y = -.5195	
X = 1.000	Y = -.8182	
X = 1.500	Y = -.9387	
X = 2.000	Y = -.9802	

****

X = -2.000	Y = .9950	N = 1
X = -1.500	Y = .9779	SLP = 20
X = -1.000	Y = .9048	
X = .500	Y = .6345	
X = .000	Y = .0000	
X = .500	Y = -.6345	
X = 1.000	Y = -.9048	
X = 1.500	Y = -.9779	
X = 2.000	Y = -.9950	

Figure 11: S-Shaped Function with a Negative Slope and Normal at (0,0)
Y = -((2*N)/(1 + EXP ((-LOGN(SLP))(X))))+N
Fixed N; SLP is varied

A negatively sloping S-shaped function can be represented as follows when the normal condition occurs at X=0, Y=0:

$$Y=-((2*N)/(1+EXP((-LOGN(SLP))(X))))+N$$

where, Y = Dependent variable
X = Independent variable
N = Upper and lower extreme values
SLP = Slope parameter when N is fixed

The behaviour of this function for a fixed N and for different values of SLP is shown in figure 10. That for fixed SLP and for various values of N is shown in Figure 11. The function can be neutralized when SLP is made equal to zero.

6. CONCLUSION

The range of functions discussed in this paper is not exhaustive, although they will be useful for representing most non-linear processes. The formulae proposed for these functions were selected for their simplicity and the ease with which the extreme values and slopes of the functions could be changed. These functions are of special interest when hardware and software facilities are limited. They improve the behaviour of a model by eliminating discontinuities and make parameter sensitivity analysis easier. It is, however, difficult to relate the mathematical form and the graphical shape of a function, unless the user is familiar with mathematics.

When DYNAMO is available, it should be possible to construct user-defined macros for these functions that incorporate nemonics for their shape. A program containing such macros is placed in Appendix. This program can be appended before any model, and non-linear functions constructed by it can be called in the model when needed. The DYNAMO supplied TABLE functions are, however, superior in terms of their transparency and ease of control or their shape.

REFERENCE

1. PUGH III, Alexander L., (1976), DYNAMO Users Manual, 5th edition, MIT Press.

2. FORRESTER, Jay W., (1969), Urban Dynamics, MIT Press.

APPENDIX

```
NOTE MACROS FOR CONTINUOUS NON-LINEAR
FUNCTIONS
NOTE
NOTE KHALID SAEED, ARIF A IRDAMIDRIS, AIT,
APRIL 1983
NOTE
NOTE ***CONCAVE WITH POSITIVE SLOPE CNCVP,
NORMAL AT (1,1) MACRO CNCVP (X, N, SLP)
A CNCVP.K=N+(1-N) (X.K**SLP)
MEND
NOTE ***CONCAVE WITH NEGATIVE SLOPE CNCVN,
NORMAL AT (1,1) MACRO CNCVN (X,N)
A CNCVN.K=N*EXP((-LOGN(N)) (X.R))
MEND
NOTE ***CONVEX WITH POSITIVE SLOPE CNVXP,
NORMAL AT (1,1) MACRO CNVXP (X, N, SLP)
A CNVXP.K=N+(1-N) (X.R**SLP)
MEND
NOTE ***CONVEX WITH NEGATIVE SLOPE CNVXN,
EXTREMES AT (O,N), AND (1,0) MACRO CNVXN
(X, N, SLP)
A CNVXN.K=N-(N) (X.R**(LOGN (SLP)))
MEND
NOTE ***S-SHAPED WITH POSITIVE SLOPE SP11,
NORMAL AT (1,1) MACRO SP11 (X, N, SLP)
A SP11.K=N/(1+(N-1) (SLP) (EXP(-LOGN(SLP) (X.R))))
MEND
NOTE ***S-SHAPED WITH POSITIVE SLOPE SPOO,
NORMAL AT (0,0) MACRO SPOO (X, N, SLP)
A SPOO.K=((2*N)/(1+EXP((-LOGN (SLP)) (X.R))))-N
MEND
NOTE ***S-SHAPED WITH NEGATIVE SLOPE SN11,
NORMAL AT (1,1) MACRO SN11 (X, N, SLP)
A SN11.K=(N/(N-1))/((X.K**SLP)+(1/(N-1)))
MEND
NOTE ***S-SHAPED WITH NEGATIVE SLOPE SNOO,
NORMAL AT (0,0) MACRO SNOO (X, N, SLP)
A SNOO.K=-((2*N)/(1+EXP((-LOGN (SLP)) (X.R))))+N
MEND
```