Endogenous Industrial Cycles in a Reshaped “Neoclassical” Model

©Alexander V. RYZHENKOV
Ec. Faculty of Novosibirsk State University, 1 Pirogov street, Novosibirsk 630090 Russia;
IEIE SB RAS, 17 Acad. Lavrentiev Avenue Novosibirsk 630090 Russia

To Sesquicentennial (2017) of K. Marx’
Capital: Critique of Political Economy, vol. I.

Contents

Odysseus [a system dynamicist-alike. – A.R.] and the Sirens [have much in common with “neoclassical” creatures appealing to the spirit, not to the flesh. – A.R.] by H.J. Draper
https://upload.wikimedia.org/wikipedia/commons/7/72/Ulysses_and_the_Sirens_by_H.J._Draper.jpg

Abstract. Two closely related “neoclassical” models of economic growth (1st with hidden, 2nd, more general, with intended) economies of scale are considered. The main variables are relative wage and employment ratio, whereas a ratio of investment to profit is constant. The spurious efficiency wage hypothesis supports equations for a growth rate of output per worker. Workers’ competition for jobs is stabilizing and their fight for increased wages is destabilizing as revealed. In each model, a stationary state is locally asymptotically stable in a system of two ODEs. Deceptively, there is no possibility for endogenous industrial cycle.

A 3rd extended model, containing the greed feedback loops, reflects the destabilizing cooperation and stabilizing competition of investors. In a system of three ODEs, rate of capital accumulation becomes the new phase variable. Its targeted long-term decrease raises profit rate together with reducing relative wage and capital-output ratio. Oscillations imitating industrial cycles are endogenous. Crisis is a manifestation of relative and absolute over-accumulation of capital. Limit cycle with a period of about 7 years results from supercritical Andronov – Hopf bifurcation.

Goodwin's M-1

Three intensive FB loops:
1st order – 2 *alternating*,
2nd order – 1 *negative*

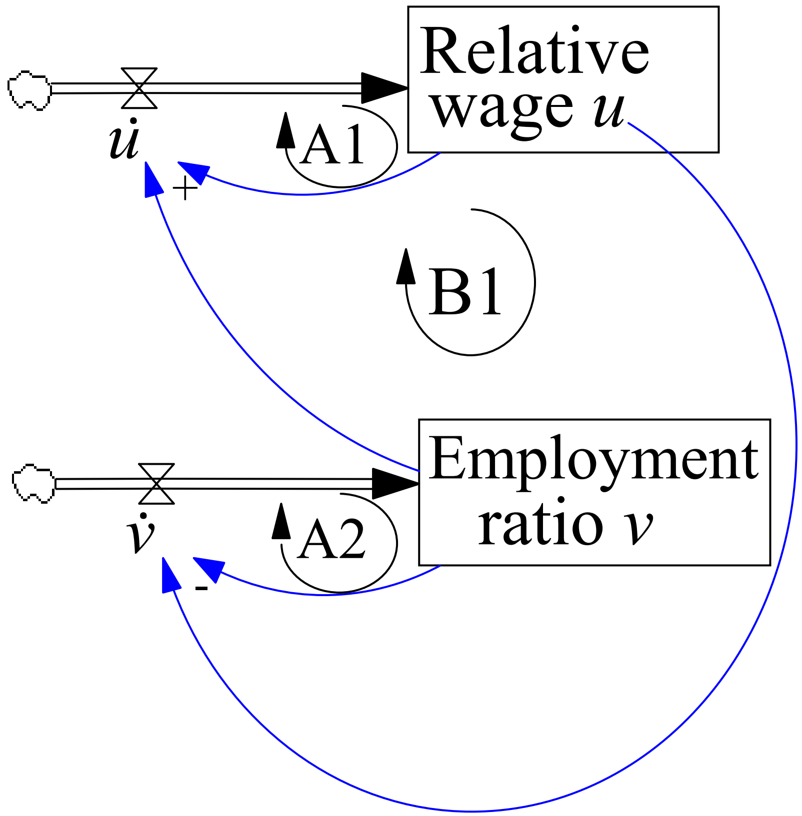


Figure 1 – structure of M-1

Table 1. The main extensive negative FB loop B1 of length 8

Relative wage u $\xrightarrow{-}$ Profit rate
Growth rate of fixed capital
Growth rate of employment ratio
Net change of v
Employment ratio v
Growth rate of wage
Growth rate of relative wage
Net change of u

Table 2. Main variables in P-1 with constant elasticity of substitution (CES)

Variable	Expression
Net product	q
Fixed production assets	k
Capital-output ratio	$s = k/q$
Employment	l
Employment (in efficiency units)	$l_e = le^{\alpha t}(k/k_0)^{\gamma}$, $\alpha > 0, \gamma = 0$
Output per worker	$a = q/l$
Labour force	$n = n_0 e^{\beta t}$, $\beta \geq 0$
Wage	w
Total wage	wl
Relative wage (unit value of labour power)	$u = w/a = wl/q$
Profit	$M = q - wl = (1-u)q$
Profit rate	$R = (1-u)/s$

In CES production function $F(k,l_e) = c\left[\mu k^{-\delta} + (1-\mu)l_e^{-\delta}\right]^{-1/\delta}$, $c > 0$ is efficiency parameter, $0 < \mu < 1$ is distribution parameter and $0 < \delta$ is substitution parameter (Arrow et al. 1961), CES $\varepsilon = 1/(1+\delta)$.

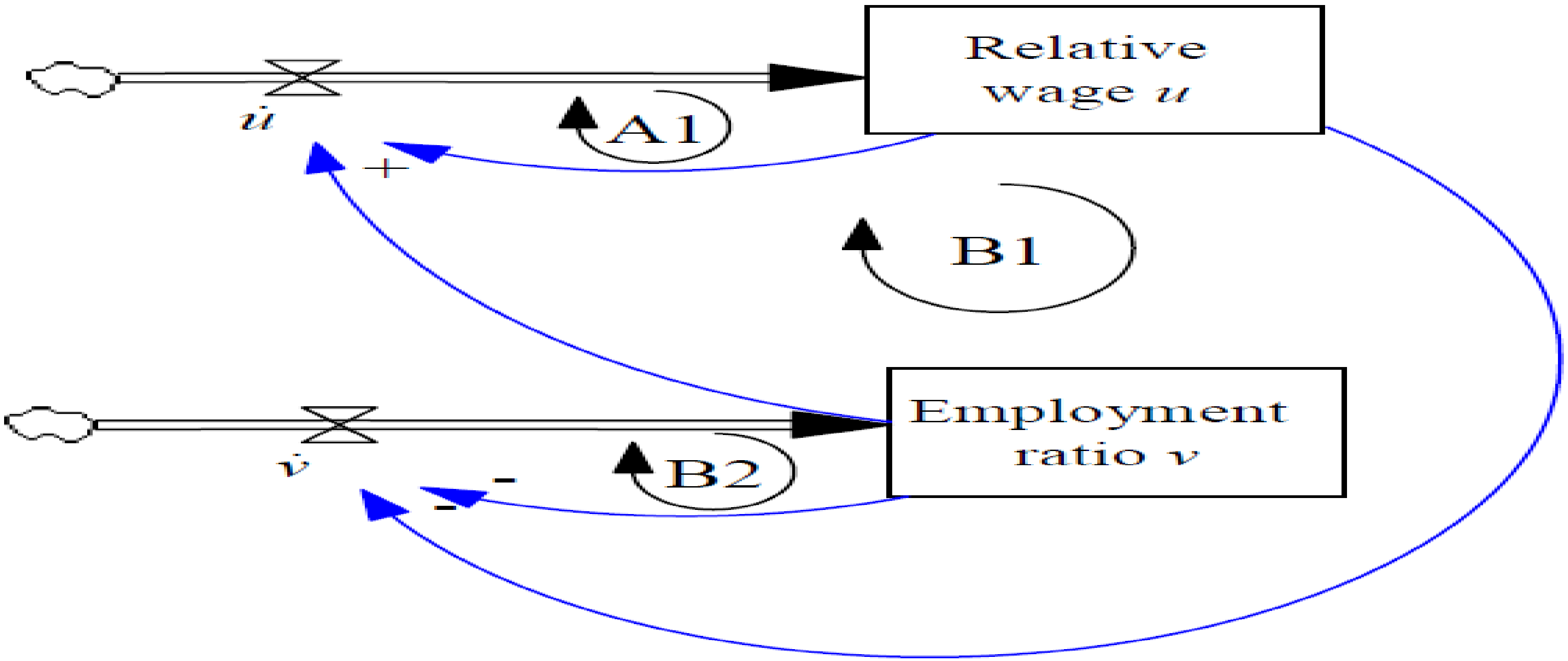


Figure 2 – A condensed causal structure of P-1; three FB loops:
1st order – 2 (1 – *negative*, 1 – *alternating*), 2nd order – 1 *negative*

Two loops are descendant from M-1: A1 of length 1 & B1 of length 3. Former A2 is transformed in B2: Employment ratio $v \xrightarrow{-}$ Net change of v.

The CES production function has a constant return to scale according to the standard "neoclassical" definition. It ignores feedback loops involving growth rate of output per worker and other variables. Taking these FB loops into account results in the deeper definition of economy of scale.

Table 3. Efficiency wage and reinforcing roundabout economies of scale

No.	Order and polarity	Extensive feedback loop
1	1, +	$\hat{a} \xrightarrow{-} \hat{u} \rightarrow \widehat{k/l} \xrightarrow{-} \hat{v} \rightarrow \dot{v} \rightarrow v \rightarrow \hat{w}$
2	1, +	$\hat{a} \xrightarrow{-} \hat{u} \rightarrow \dot{u} \rightarrow u \xrightarrow{-} \hat{k} \rightarrow \hat{v} \rightarrow \dot{v} \rightarrow v \rightarrow \hat{w}$
3	2, +	$\hat{a} \xrightarrow{-} \hat{u} \rightarrow \dot{u} \rightarrow u \rightarrow s \xrightarrow{-} \hat{k} \rightarrow \hat{v} \rightarrow \dot{v} \rightarrow v \rightarrow \hat{w}$

A Phillips eq. is $f(v) = -g + r/(1-v)^2$. The initial state: $u_0 = 0.702$, $v_0 = 0.937$, $z = 0.156$; common parameters: $g = 0.04$, $\alpha = 0.012$, $\beta = 0.015$, $r = 0.0002$, $d = \alpha + \beta = 0.027$, additionally in M-1: stationary $u_G \approx 0.666$, $v_G \approx 0.938$, initial $s_0 = 1.93$; additionally in P-1 $\delta = 0.33$, $\varepsilon = 0.752$, $\mu = 0.3$, $\rho = 0.1$, stationary $s_p = 1.512$, $u_p \approx 0.738 > u_G$, $v_p \approx 0.937 < v_G$.

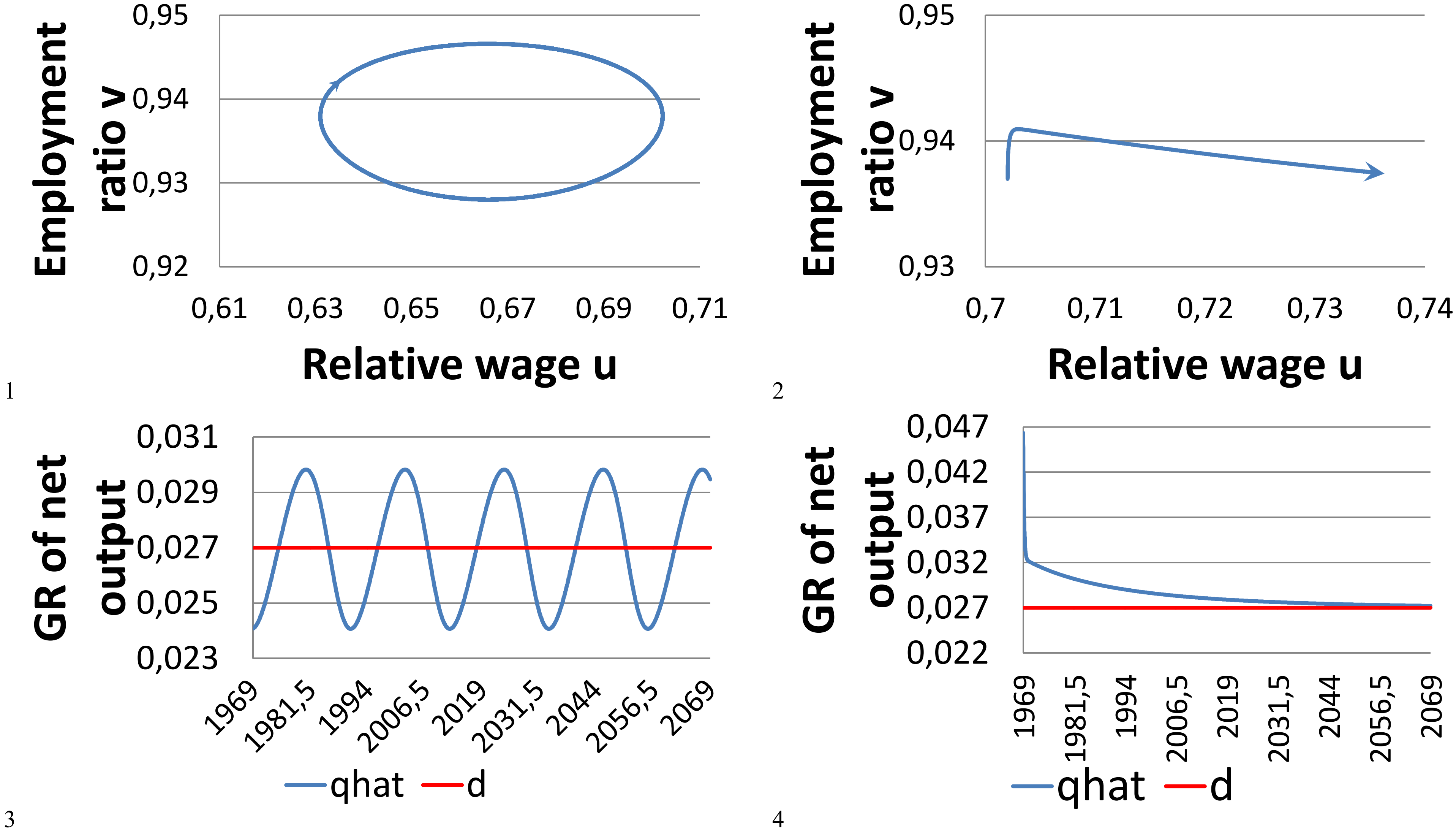


Figure 3 – A centre in M-1 (panels 1, 3), stable node in P-1 (panels 2, 4)

Checking P-1 structural stability in P-2 with new scale effects ($1 > \gamma > 0$)

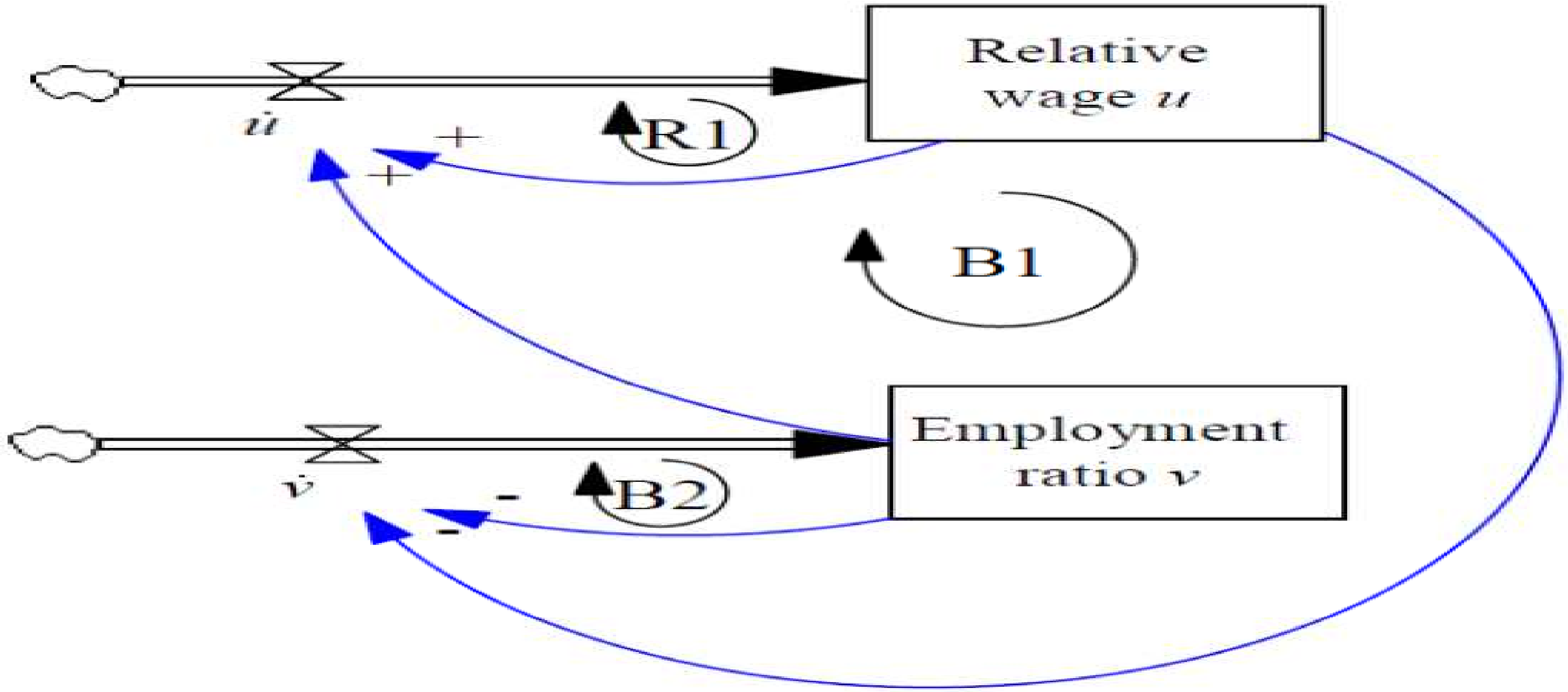


Figure 4 – A condensed causal structure of P-2; three FB loops:
1st order – 2 (1 – *positive*, 1 – *negative*), 2nd order – 1 *negative*

Loops descendant from P-1: B2 of length 1 & B1 of length 3. Former A1 is transformed in R1 of length 1: Relative wage *u* →Net change of *u*.

Conceptual weakness of P-2 rooted in “neoclassical” beliefs

There is VES production function in terms of k and l for $1 > \gamma > 0$. The equation for growth rate of output per worker (Aguiar-Conraria 2008):

$$\hat{a} = \frac{\alpha\delta + \hat{w} + \delta\gamma\hat{k}}{1+\delta} = \frac{1}{1+\delta}[\alpha\delta + f(v) + \delta\gamma\hat{k}].$$

Table 4. Efficiency wage and additional scale effects in P-2

No.	Order and sign	Extensive feedback loop
4	1, +	$\hat{a} \xrightarrow{-} \hat{u} \to \dot{u} \to u \xrightarrow{-} \hat{k}$
5	1, +	$\hat{a} \xrightarrow{-} \hat{u} \to \dot{u} \to u \to s \xrightarrow{-} \hat{k}$
6	2, -	$\hat{a} \xrightarrow{-} \hat{u} \to \dot{u} \to u \xrightarrow{-} \hat{k} \to \widehat{k/l} \xrightarrow{-} \hat{v} \to \dot{v} \to v \to \hat{w}$
7	2, -	$\hat{a} \xrightarrow{-} \hat{u} \to \dot{u} \to u \to s \xrightarrow{-} \hat{k} \to \widehat{k/l} \xrightarrow{-} \hat{v} \to \dot{v} \to v \to \hat{w}$

Starting with a rather problematic quasi-empirical relation between net output per worker and real wage only, a CES production function was derived in Arrow et al. 1961. CES production function $F(k, l_e)$ becomes VES production function $\Phi(k, l)$ in P-2 and Z-1. It is not homogenous in terms of l and k, except Cobb – Douglas case with degree of homogeneity $1 + \gamma(1–\mu)$.

Proposition 1. The logically contradictory principle of distribution of national income (net product) is introduced in P-2: on the one hand, wage is determined there by labour "marginal product" $w = \Phi_l$, and total wage $lw = l\Phi_l$, on the other hand, profit rate is lower than the "marginal product" of capital $R = M/k = (1–u)/s = F_k < \Phi_k$ if $1 > \gamma > 0$.

Corollary. (a) Profit is lower than imputed profit. (b) Net product is lower than imputed total income. (c) The “factor price ratio” exceeds “marginal rate of technical substitution”: $w/R > \Phi_l / \Phi_k$.

The relative wage u expresses the value of labour power; the profit of capitalists M is a transformed form of surplus value S created by the working class: $S = (1 – u)L = M/a$.

Proposition 4. (a) Stationary relative wage u_a and capital-output ratio s_a, being the higher, the higher is rate of accumulation z, achieve maximum when $z = 1$; (b) stationary employment ratio v_a is independent of z; (c) stationary growth rates $\hat{a}_a = (\hat{k/l})_a = \hat{w}_a$, $\hat{k}_a = \hat{q}_a$ do not depend on z too; (d) stationary profit rate d / z, being the lower, the higher is rate of accumulation z, achieves minimum when $z = 1$ (cf. "golden rule" of accumulation).

Proposition 5. The stronger is the solidarity of workers in struggle for the relative wage, the higher is δ, and therefore strengthening this solidarity is a means of enhancing stationary relative wage (value of labour power) u_a. This increase has no influence on a stationary rate of return $(1 - u_a)/s_a$, as stationary capital-output ratio s_a declines along with accrual in δ.

Proposition 6. Local asymptotic stability (LAS) of hyperbolic E_a takes place according to Routh – Hurwitz criterion, if *Trace* $(J_a) < 0$.

P-2, similar to P-1, can produce converging fluctuations only. Thus for keeping them in life exogenous shocks are necessary. "Neoclassical" Idyll, reigning in P-1 and P-2, is deceptive as sirens' songs.

Model Z-1 of endogenous capital accumulation cycles

Capital accumulation cycles manifest themselves in statistics (update of Ryzhenkov 2010 on Figures 5, 9 and 10).

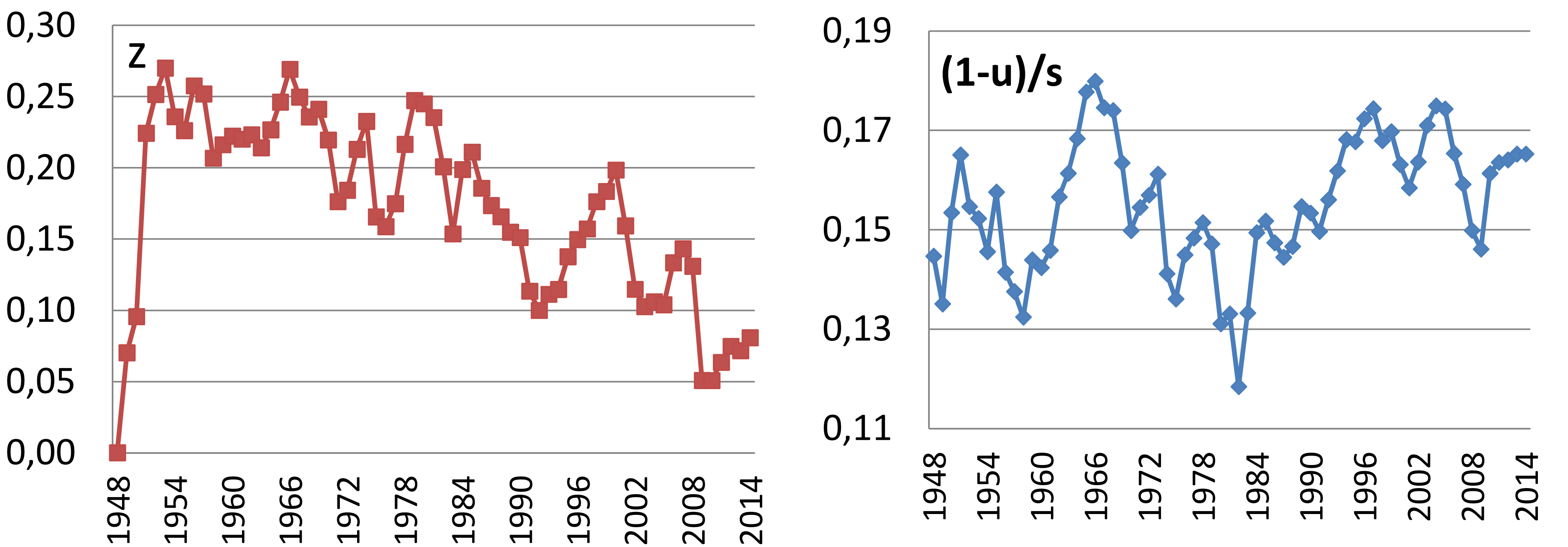


Figure 5 – Rates of capital accumulation and profit in the USA, 1948–2014

The intensive form of Z-1 with rate of capital accumulation

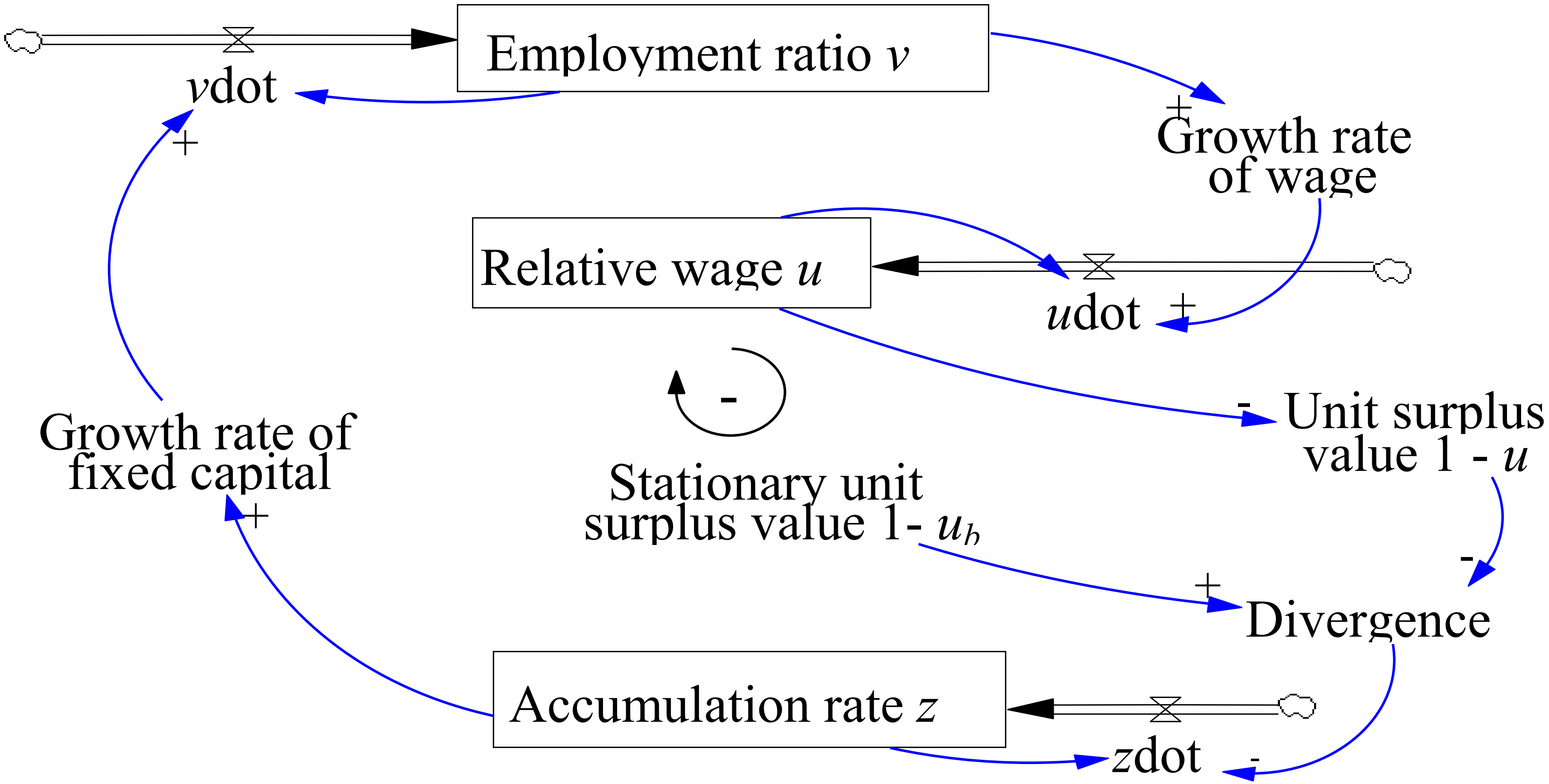


Figure 6 – The negative 3rd order FB loop for rate of accumulation, employment ratio and value of labour power implicit in K. Marx (1867)

The original equation takes into account, first – in agreement with the views of K. Marx (1867: 634) – that net change of the share of investment in surplus product has an opposite sign in response to relative wage gains:

$$\dot{z} = -b\frac{\dot{u}}{1-u}z(Z-z) + p(z_b - z),$$

where $z = \dfrac{\dot{k}}{(1-u)q}$, $b \geq 0$, $p > 0$, $z_b < z_0 \leq 1 < Z$.

This equation reflects, second, objective interest of capitalists in the long-term increase of the rate of profit; restrictions $p > 0$ and $z_b < z_0$ serve a long run increasing profit rate. Product $z(Z - z)$ takes into account, third, logistical dependence of $\dot{z}$ on z that bounds trajectories in the phase space while a magnitude of Z codetermines amplitude of fluctuations.

Z-1 includes the other equations of P-2 without z unchanged. Z-1 also includes the other equations of P-2 with z – yet now this is phase (level) variable instead of being constant in P-2 (as in M-1 and in P-1).

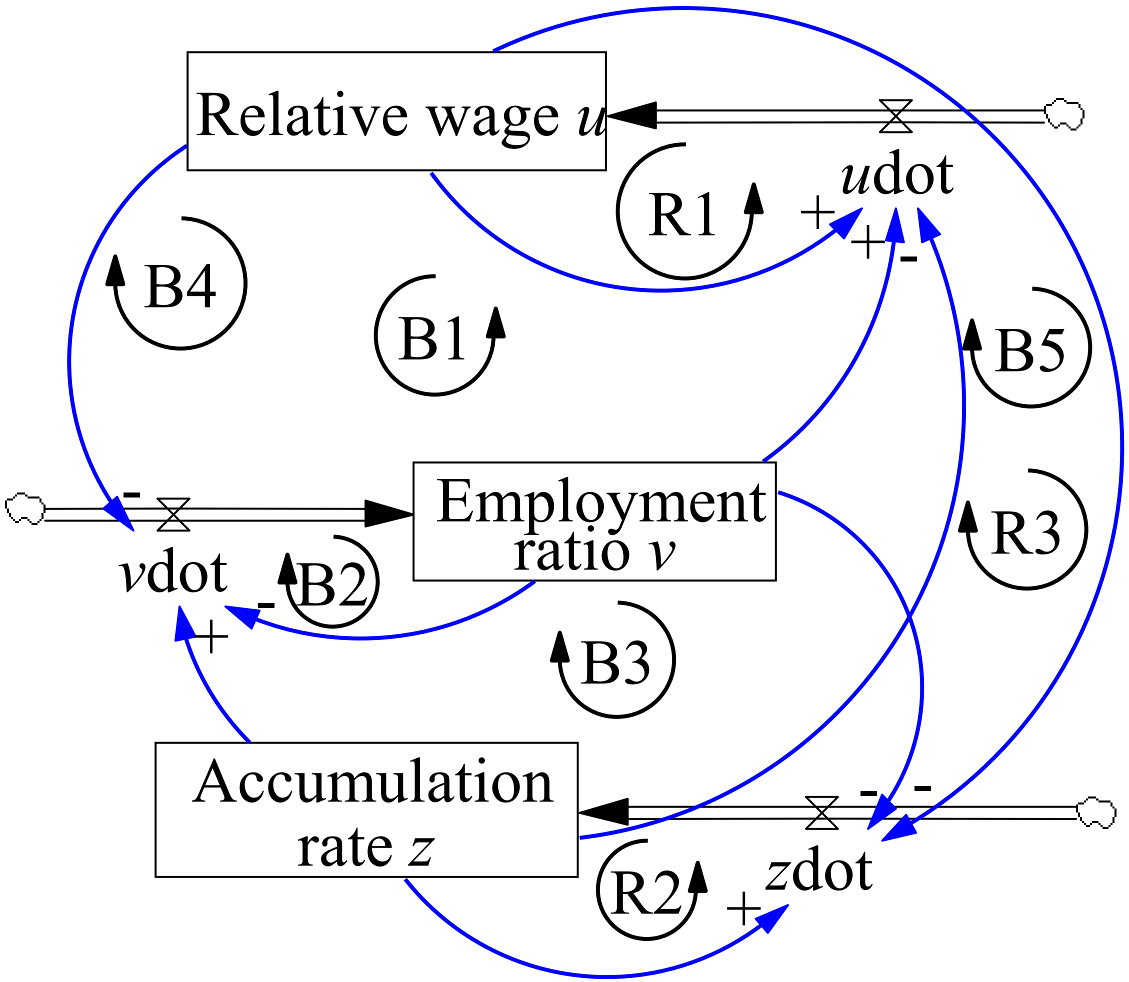


Figure 7 – Eight feedback loops in Z-1: 1st order – 3 (1 – negative, 2 – *positive*), 2nd order – 3 (2 – negative, 1 – *positive*), 3rd order – 2 (2 – negative)

Table 5. The eight intensive FB loops in Z-1

No.	Order	Positive	Negative
3	1st	R1 of length 1 $u \rightarrow \dot{u}$ R2 of length 1 $z \rightarrow \dot{z}$	B2 of length 1 $v \xrightarrow{-} \dot{v}$
3	2nd	R3 of length 3 $u \xrightarrow{-} \dot{z} \rightarrow z \xrightarrow{-} \dot{u}$	B1 of length 3 $u \xrightarrow{-} \dot{v} \rightarrow v \rightarrow \dot{u}$ B3 of length 3 $v \xrightarrow{-} \dot{z} \rightarrow z \rightarrow \dot{v}$
2	3rd		B4 of length 5 $u \xrightarrow{-} \dot{z} \rightarrow z \rightarrow \dot{v} \rightarrow v \rightarrow \dot{u}$ B5 of length 5 $v \xrightarrow{-} \dot{z} \rightarrow z \xrightarrow{-} \dot{u} \rightarrow u \xrightarrow{-} \dot{v}$

Note. After inspecting my poster in Delft on July 18, 2016, M. Happach has perfectly labelled R2 and R3 as greed FB loops. Then I've gratefully acknowledged his valuable and generous insight.

Table 6. Efficiency wage and additional scale effects in Z-1

No.	Order and sign	Extensive FB loop
9	1, +	$\hat{a} \xrightarrow{-} \hat{u} \rightarrow \dot{u} \xrightarrow{-} \hat{z} \rightarrow \dot{z} \rightarrow z \rightarrow \hat{k}$
10	2, +	$\hat{a} \xrightarrow{-} \hat{u} \rightarrow \dot{u} \xrightarrow{-} \hat{z} \rightarrow \dot{z} \rightarrow z \rightarrow \hat{k} \rightarrow \hat{v} \rightarrow \dot{v} \rightarrow v \rightarrow \hat{w}$
11	2, –	$\hat{a} \xrightarrow{-} \hat{u} \rightarrow \dot{u} \xrightarrow{-} \hat{z} \rightarrow \dot{z} \rightarrow z \rightarrow \hat{k} \rightarrow$ $\widehat{k/l} \xrightarrow{-} \hat{v} \rightarrow \dot{v} \rightarrow v \rightarrow \hat{w}$

Table 7. Properties of stationary state E_b in Z-1

Growth rates of net output $\hat{q}_b$, fixed capital $\hat{k}_b$	$d = (\alpha + \beta)/(1 - \gamma)$
Growth rates of output per worker $\hat{a}_b$, capital intensity $(k\hat{/}l)_b$, wage $\hat{w}_b$	$(\alpha + \gamma\beta)/(1 - \gamma)$
Relative wage u_b	$1 - \left(\frac{d}{z_b} \frac{\mu^{1/\delta}}{c} \right)^{\delta/(1+\delta)}$
Capital-output ratio s_b	$\frac{1}{c} \left(\frac{z_b}{d} c\mu \right)^{1/(1+\delta)}$
Employment ratio v_b	$f^{-1}[(1 - \rho)(\alpha + \gamma\beta)/(1 - \gamma)]$
Profit rate $\frac{1 - u_b}{s_b}$	$\frac{d}{z_b}$, $0 < z_b \leq 1$

Proposition 7. The dynamics of the system of three ODEs linearized in the neighbourhood of its hyperbolic stationary state E_b are LAS provided that $0 \leq b < b_0 < b_2$. Then stationary state E_b is also LAS in the non-linear system.

Corollary. (1) If the stationary state E_b is LAS, it saves this property if b becomes lower than its initial magnitude b_i. If the stationary state E_b is not, it gets this property if b becomes sufficiently lower than its initial magnitude b_i. If the stationary state E_b is LAS, it loses this property if b becomes sufficiently higher than its initial magnitude b_i. (2) The stationary state E_b is LAS for $b = 0$ and $p > 0$. (3) The stationary state E_b is LAS for a special case of Z-1 with $b = 0$, $p = 0$ as in M-1, P-1 and P-2.

In a particular simulation run, stationary state E_b is not stable in linearized Z-1: $a_0 \approx 0.0028$, $a_1 \approx 0.8932$, $a_2 \approx 0.0032 > 0$, $a_1 a_2 - a_0 \approx 0$; correspondingly, $b_1 = -37.7085 < b_3 = -37.6209 < b_0 = 54.3987 < b_2 = 54.4863$. Stationary state E_b is stable in nonlinear Z-1 up to $b_{critical} = b_0 + 3 = 57.3987$.

Parameters a_0, a_1, a_2, b_0, b_1, b_2 and b_3 are explicitly defined in this paper presented. They are maintained by a corresponding characteristic equation.

Super-critical Andronov – Hopf bifurcation and self-sustained industrial cycles

Proposition 8. In the non-linear system, stationary state E_b is locally asymptotically stable for $b \le b_0$; E_b loses its stability and the Andronov – Hopf bifurcation does take place at $b_{critical} > b_0$.

According to simulations, a supercritical bifurcation occurs. The period of oscillations is about $2\pi / \sqrt{a_1(b_0)} \approx 6.648$ (years).

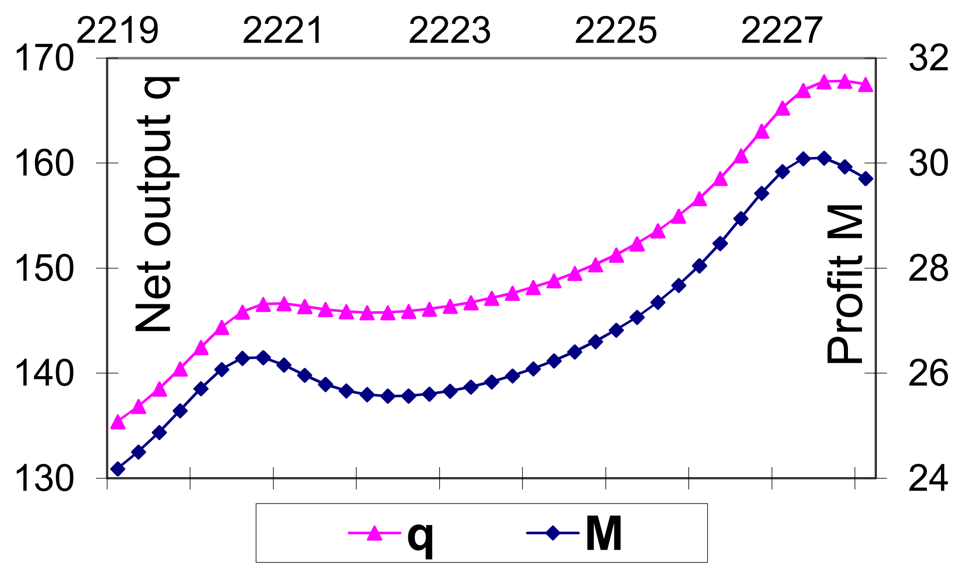
For $\gamma = 0.75$ and $b = b_{critical} = 57.3987 > b_0 = 54.3987$, there is a transition to a limit cycle vicinity (up to years 2200–2230) from the initial phase vector x for 1958. The parameters magnitudes are illustrative – they do not result from some econometric estimation.

Table 8. Cycle of capital accumulation close to limit cycle, 2221–2227.75

Phase of industrial cycle	Phase period	Quantity of quarters	Beginning	End
Crisis – the 1st phase of the cycle	2221.25–2222	4	1	4
Depression	2222.25–2222.75	3	5	7
Recovery	2223–2223.25	2	8	9
Boom	2223.5–2227.75	18	10	27
Complete cycle	2221.25–2227.75	27 quarters or 6.75 years		

Capital as the driver and barrier of capitalist production in Z-1

Excess of capital arises from the same causes as those which call forth unemployment – complementary phenomena footing at the opposite poles.



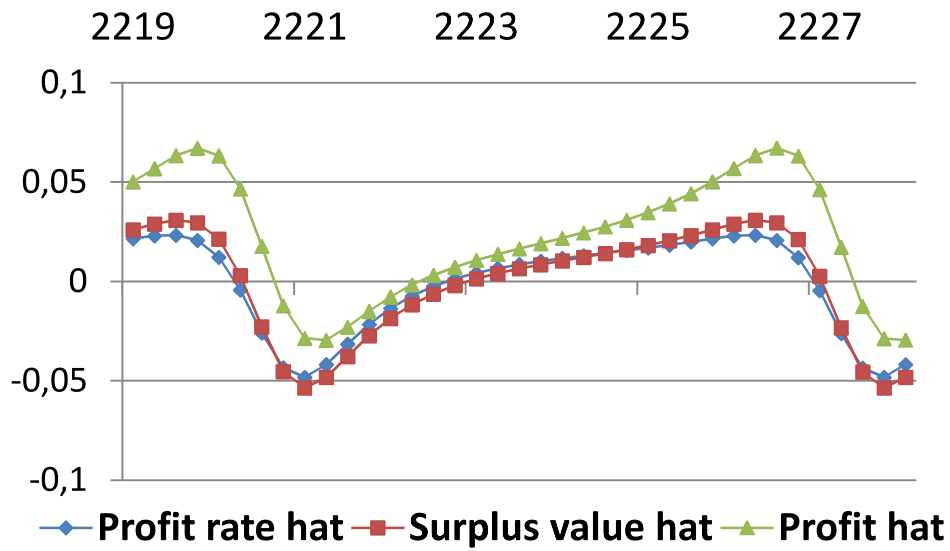


Figure 8 – Lead of highs of profit *M* against highs of net product *q*, 2219–2228 (l.); occurrence of relative and absolute over-accumulation of capital of types I and II: growth rates of profit rate, surplus value and profit, 2219–2228 (r.)

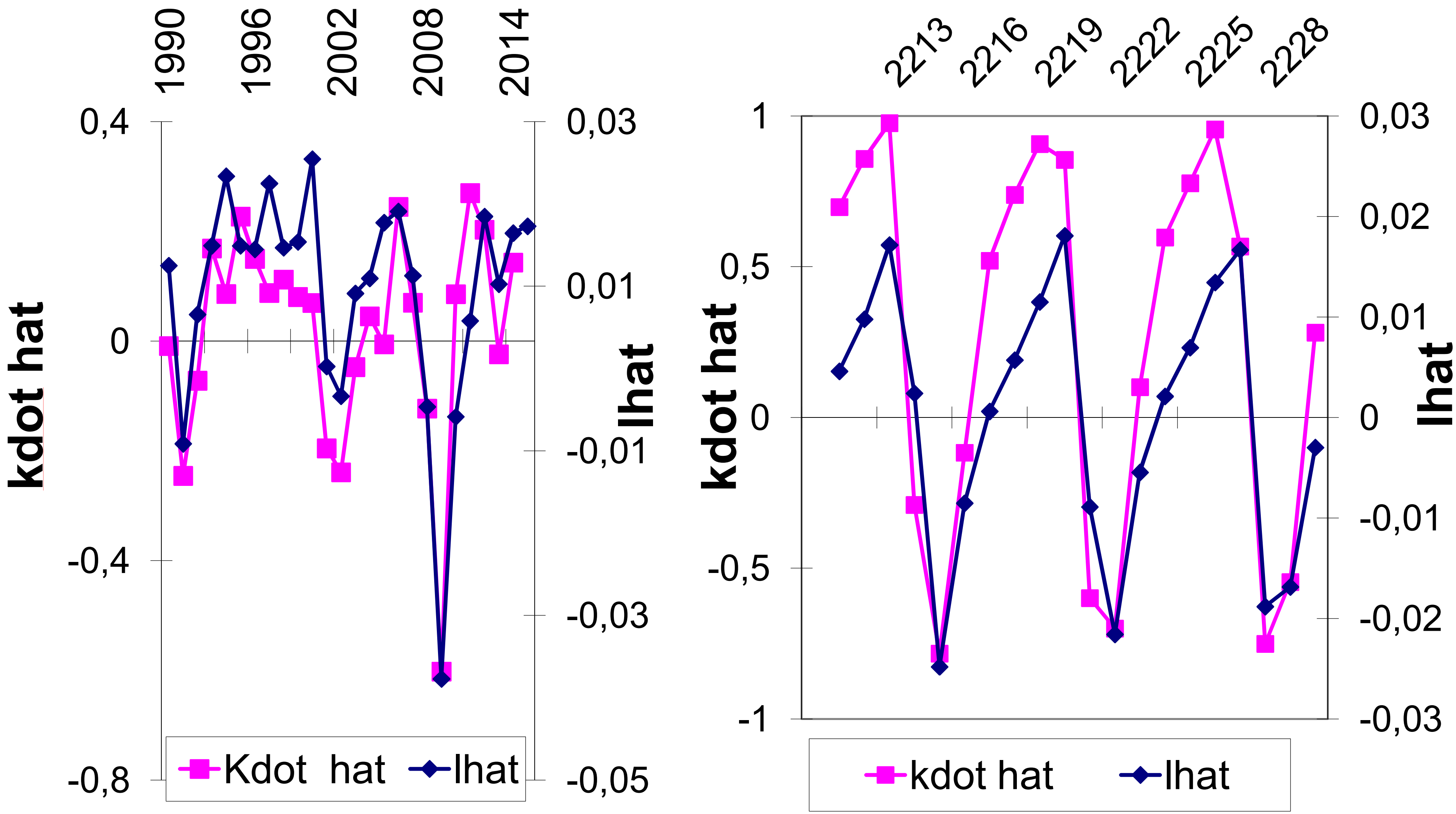


Figure 9 – Growth rates of nonresidential fixed investment and employment in the USA, 1948–2015 (l.) versus those in Z-1, 2213–2228 (r.)

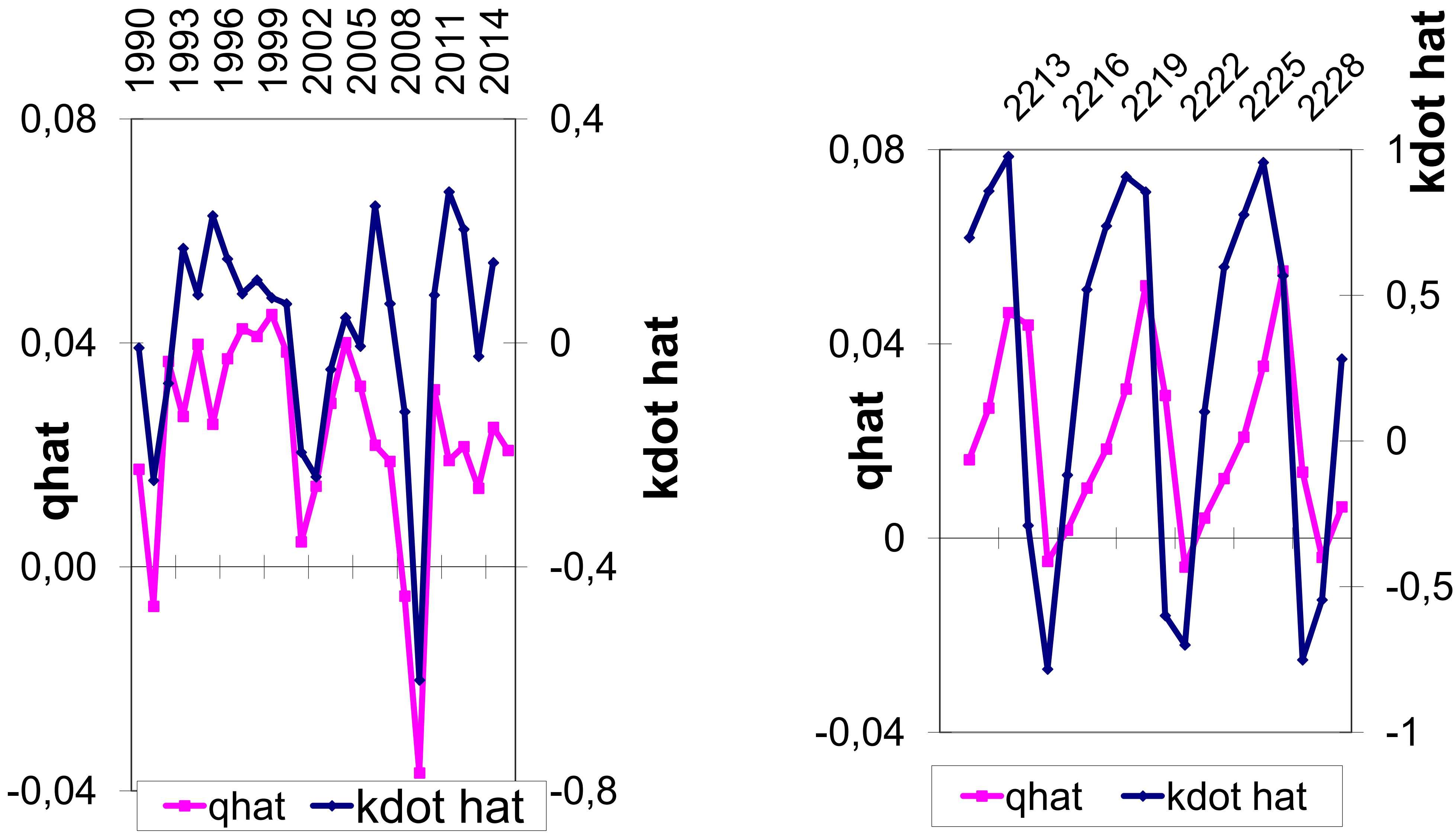


Figure 10 – Growth rates of net output and nonresidential fixed investment in the USA, 1948–2015 (l.) versus those in Z-1, 2213–2228 (r.)

Conclusion

A super-critical Andronov – Hopf bifurcation happens and a closed orbit (limit cycle) is generated in Z-1 with a sufficiently shorter period (6.75 y.) than a period of conservative closed orbits in Goodwin's M-1 (about 22 y.).

Z-1 reflects periodic recurrence of relative and absolute over-accumulation of capital as well as general over-production, as immanent characteristics of industrial cycle. It is shown that capitalists' investment cooperation weakens (competition strengthens) stability of stationary state in Z-1. Targeted reduction of the rate of accumulation increases profit rate and reduces value of labour power contrary to the working class interests.

The "neoclassical" marginal productivity principle of income distribution is broken in Ploeg's P-1, Aguiar-Conraria's P-2 and Z-1. The unreliable efficiency wage hypothesis (implicit in Arrow et al. 1961 and explicit in Solow 1979), inherited from P-1 and P-2, remains as structural element within Z-1. Therefore Z-1 is to be liberated from this and other fallacies.

The flaws in the indigenous "neoclassical" models are mostly due to mixing notions of concrete labour and abstract labour, as well as because of general negation of the K. Marx theory of commodity and surplus value.

J. Forrester (2007: 370) wrote: “System dynamicists must go behind the symptoms of trouble and identify the basic causes.” The “neoclassical” school falls short of these requirements. Dialectic (historic) materialism is deeper than subjectivism (alleged objectivism) of the “neoclassical” school.

The revealed structural specification errors identified in the main "neoclassical" equations violate one of the fundamental prerequisites for the intelligent application of regression and other econometric methods. Thus, a necessary preliminary step for subsequent serious statistical investigations has been accomplished.

Marx’ methodology is congruent with the methodology of vanguard system dynamics. By learning the predecessor’s work seriously and without prejudice, the system dynamics field can flourish further faster and better. This will be a stronger factor of progressive change in the social production relations.

References

Marx K. Capital. A Critique of Political Economy. Vol. I. Book One: The Process of Production of Capital. First published in German in 1867.

Arrow K., Chenery H., Minhas B., and Solow R. 1961. Capital-labor substitution and economic efficiency // *Rev. of Ec. and Stat.*43 (3): 225–250.

Goodwin R.1972. A growth cycle, in: C. Feinstein (ed.). Socialism, Capitalism and Economic Growth, Cambridge University Press, Cambridge.

Solow R. 1979. Another possible source of wage stickiness // *J. of Macroeconomics* 1: 79–82.

Ploeg F. Van der. 1985. Classical growth cycles / *Metroeconomica* 37 (2): 221–230.

Forrester J. 2007. System dynamics – the next fifty years / *System Dynamics Rev.* 23 (2/3): 359–370.

Aguiar-Conraria L. 2008. A note on the stability properties of Goodwin's predator-prey model / *Rev. of Radical Political Economics* 40 (4): 518–523.

Ryzhenkov A. 2010. The structural crisis of capital accumulation in the USA and its *causa prima* / Proc. of the 28th ISDC, Seoul, Korea.

Misprints in the previous conference paper

Ryzhenkov A. 2013. Employment-Centred Stabilisation Policy Propelling the Economy to "Escape Velocity" // Creating the Future from Within : The 31st International Conference of the System Dynamics Society, Cambridge, Massachusetts USA. July 21 – July 25, 2013.
URL: http://www.systemdynamics.org/conferences/2013/proceed/papers/P1170.pdf

Table 2.5. Concretisation of Liénard – Chipart criterion for E_X in AM (page 20)

Last line before correction	$\Delta_3 = a_1 a_2 a_3 - a_0 a_3^2 + a_1^2 a_4 = db^3(2b^2 - db - 2c_2 dX) > 0$, if inequality (2.11) is satisfied
Last line after correction	$\Delta_3 = a_1 a_2 a_3 - a_0 a_3^2 - a_1^2 a_4 = db^3(2b^2 - db - 2c_2 dX) > 0$, if inequality (2.11) is satisfied

Table 3.1. Concretisation of Liénard – Chipart criterion for E_X in modified AM (page 25)

Last line before correction	$\Delta_3 = a_1 a_2 a_3 - a_0 a_3^2 + a_1^2 a_4 = qdb^3(2b^2 - qdb - 2c_2 dX) > 0$, if inequality (3-2.11) is satisfied
Last line after correction	$\Delta_3 = a_1 a_2 a_3 - a_0 a_3^2 - a_1^2 a_4 = qdb^3(2b^2 - qdb - 2c_2 dX) > 0$, if inequality (3-2.11) is satisfied