An Enhancement for the Textbook's Models of Natural Resources and Economic Growth

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Abstract. The state variables of the original theoretical law of capital accumulation are relative labour compensation, employment ratio, gross unit resource rent, produced capital-output ratio, proved non-renewable reserves-output ratio, desired proved non-renewable reserves-output ratio, and depletion of proved non-renewable reserves per unit of net output. This theoretical law is a subject of multiple tests. Taking it as a starting point this paper establishes that the basic "neoclassical" ecological-economic models in the Professor D. Romer textbook can be refined and generalized.

Their premises on infinite growth of output-natural resources ratio and on reducing unit resources depletion almost to zero even along with declining net output are not practically feasible. The long-term environmental policies recommended by this textbook could be damaging or inefficient. The proposed deep alterations of the basic "neoclassical" models shed light on balanced and aggravated regimes of capital accumulation. The advantages of system dynamics approach over single equation technique are vividly revealed with clear implications for environmental policy as well as for higher education.

JEL classification: A2; B16; C6; E10; E27; N5; Q32

Keywords: capital accumulation; material interests; Hotelling rule; proved non-renewable reserves; absolute and differential rent; fragile sustainable development; endemic disequilibrium; mode with aggravation; social closed-loop control

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Introduction

Economists and statisticians continue to unjustifiably reject the theoretical results with low statistical significance. As already explained in the literature on system dynamics, formalist addiction to these ongoing statistical tests is closely related to classical regression model with one equation – the heritage common to all social sciences (Senge 1977, Mass and Senge 1980, Peterson 1980, Barlas 1989, Ryzhenkov 2013).

To the extent that researchers are building models with one equation, a profound testing of behaviour models is not possible.[1] The reader will soon see the reasons why the "neoclassical" school prefers a single logistic equation or its close approximation in macroeconomics.

On the contrary, one of the greatest strengths of the system dynamics method resides in testing the impact of alternative hypotheses using systems of integral and (or) differential equations on the endogenous behaviour of the complete system (Saeed 2013). Causal loop diagrams are a powerful instrument of the system dynamics method that together with other instruments deepens substantially the superfluous socio-economic macro analysis based on one equation. This paper provides new experimental and analytical support for these advantageous and progressive properties.

Strengthening the system dynamics paradigm necessitates building its solid core (Forrester 1983). Adherence to this core building motivated the earlier books and papers that introduced and refined the first hypothetical law (HL-1) of capital accumulation constrained by natural capital (Ryzhenkov 2000, 2007). Its state variables are the relative labour compensation, employment ratio, gross unit rent, produced capital-output ratio, natural capital-output ratio, desired natural capital-output ratio, and depletion of the natural capital per unit of net output.

An application of HL-1 in its probabilistic form together with extended Kalman filtering to the U.S. macroeconomic data 1958–1991 identified unobservable components of this law. In this empirical application the notion of natural capital was narrowed to proved non-renewable (briefly – mineral) reserves since the available statistical sources did not provide consistent data on other kinds of natural capital.[2] Despite these limitations, it was shown that long wave was a dominant non-equilibrium quasi-periodic behavioural pattern of the U.S. capital accumulation dependent on investment policy in proved non-renewable reserves.

Shifts in distribution of net output between the two main social classes under an explicit constraint of proved mineral reserves were considered in (Ryzhenkov 2007). The focus of research is on pro-growth stabilization policy that brings about shifts in primary income distribution between

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[1] A typical argument in favour of models with one equation stresses the advantage of simplification (Boggio 2006: 237): "... we obtain a remarkable simplification of the analysis, which can now proceed with a single equation in one state variable!" Still simplification of this kind should be a warning that the analysis does not lay bare the concealed structure of the subject of scientific inquiry.

Another side of the same coin is populated by "neoclassical" general equilibrium models that apply essentially incoherent systems of multiple microeconomics equations. These models are cooked for maintaining the preferred macroeconomic equations and conclusions on mostly harmonic and efficient nature of private market economy. The intelligent and well-paid creators of these models strengthen illusory consciousness and inhibit scientific socio-economic consciousness of the toiling masses thus facilitating to some extent capitalist production relations.

Yet illusory consciousness cannot help in solving the acute problem of *reinventing life on a shrinking earth* that is the paramount subject of the present conference. Moreover, illusory consciousness enhances excessive depletion of the resource base of humankind as the reader will soon see.

[2] The terms reserve(s), proved (or proven) reserve(s), total resource(s) are in agreement with the UN Framework Classification for mineral reserves/resources. See ENERGY/WP.1/R.70 ECE-UN document [17 February 1997]: United Nations International Framework Classification for Reserves/Resources – Solid Fuels and Mineral Commodities: Geneva.

two main social classes.[3] This research envisages in particular reshaping primary distribution of income that may enhance sustainable development.

The restricted presentation of U.S. natural capital by proved mineral reserves up to the year 1991 only is due to well-known limitations of statistics. Although these data are somehow aged in the mean time, their current usage is acceptable for achieving exclusively methodological purposes of this paper.[4]

The system dynamics approach to proved mineral reserves in this and previous papers substantially differs from the traditional decline curve analysis that applies exponential, hyperbolic and harmonic rate-time equations. The paper (Arps 1945) was a forerunner for the first strand in decline curve analysis within resource economics that relies on the same singled out equations or their combinations (see, for example, Höök 2010). Besides that the second strand in decline curve analysis deserves mentioning: this time as a reservoir engineering empirical technique that extrapolates trends in the production data from oil and gas wells or other deposits.

These strands, surprisingly, are extremely scarce even in mentioning aggravation mode: the parameters of a hyperbolic equation are chosen from segments that guarantee monotonous decline that is not fundamentally dissimilar to exponential decline.[5] The unconventional understanding of excessively strong decline as a manifestation of aggravation mode of capital accumulation in this paper radically differs from the interpretation by the traditional decline curve analysis.

The author agrees with other experts that informed planning and decisions concerning sustainability and resource development require a long-term perspective and an integrated approach to land-use, resource, and environmental management worldwide. This paper offers important generic building blocks for implementing this integrated approach that can be realized by international co-operative efforts.

The original theoretical law (HL-1) is a subject of multiple tests. Taking it as a starting point this paper establishes that the basic “neoclassical” ecological-economic models in the Professor D. Romer textbook can be refined and generalized.

[3] Different types of property income payable out of the value added created by production (profit before taxes, interest, resource rent) belong to primary income of owners of capital (capitalists). Primary income of labour consists mainly of wages and salaries of employees, employers’ social contribution and imputed labour income of self-employed. Only net value added created by production is considered in this paper. The primary income of government is not treated separately, having been included in property income on this stage of research.

[4] The restricted presentation of U.S. natural capital by proved mineral reserves up to the year 1991 only is due to well-known limitations of the BEA statistics (Survey of Current Business, April 1994). The BEA estimated value of resource reserves and changes in reserves for the period 1958–1991 for major mineral resources. The minerals valued include the fuels (petroleum, natural gas, coal, and uranium), the metals (iron ore, copper, lead, zinc, gold, silver, and molybdenum), and other minerals (phosphate rock, sulfur, boron, diatomite, gypsum, and potash). Petroleum and gas account for the lion's share of mineral production in the USA. Observations used in calculations in (Ryzhenkov 2007) and in this paper relate to BEA current rent method I.

[5] One proponent of this approach perceives hyperbolic decline as an ensemble average of exponential declines of different rates, assuming maximum entropy in the distribution in rates (posted on September 27, 2012; URL: http://www.theoildrum.com/node/9495 accessed on June 9, 2015). This paper will not discuss accuracy and validity of this interesting statement.

Their premises on infinite growth of output-natural resources ratio and on reducing unit resources depletion almost to zero even along with declining net output are not practically feasible. The long-term environmental policies recommended by this textbook could be damaging or inefficient. The proposed deep alterations of the basic "neoclassical" models shed light on balanced and aggravated regimes of capital accumulation.

The study applies mathematical theory of nonlinear differential equations (Gandolfo 2010). Multiple simulation experiments in Vensim have been carried out based on the models presented in the next sections. They maintain the theoretical propositions and visualize them. Results of multiple simulations are presented in the supporting file on the conference site.

This paper is written in agreement with Leonardo da Vinci's dictum stating that a challenger, revealing your mistakes, is more useful than a friend who wants to hide them. So the present author would appreciate a critique deserved by this paper and by the previous work.

1. An essence of the original theoretical approach

It is supposed for simplicity throughout the remaining exposition that proved reserves are privately owned by capitalists. The absolute rent is outcome of monopoly of private property on proved reserves whereas differential rent results from monopoly on proved reserves as distinct production objects. Total rent as the sum of differential and absolute rent is treated as a potential source for state royalties and severance taxes up to amount of total rent. Still when punitive royalties and severance taxes equal or exceed total rent, the private property on proved reserves becomes purely formal or turns into subject of confiscation. Therefore total royalty (including severance taxes) is typically lower than total rent. Similarly, royalty per unit of resource extracted does not, as a rule, exceed rent per same unit. Royalties, severance taxes, depletion allowances and price controls are not considered explicitly in HL-1 or NM-1.

1.1. The main assumptions of HL-1

Consider the American economy as restricted to a certain degree by proved mineral reserves while total (non-renewable) mineral resources are not explicit constraints. Import from the rest of the world can substitute extracting non-renewable resources in the USA at least partially. It is assumed additionally that renewable and environmental resources do not constrain production explicitly. Fixed assets and proved mineral reserves are essentially complementary to each other and are also substitutes to some degree depending on the factors distributive shares and employment ratio. These two different kinds of use values are measured in the same monetary units as the net output.

The other important premises are such:

(1) two social classes (capitalists and labourers); the state enforces property rights, yet the cost of such an enforcement is not treated explicitly;

(1.2) three factors of production – labour force, produced capital (fixed assets), proved mineral reserves – are homogenous and non-specific;

(1.3) only one aggregated good is produced for consumption, investment and circulation, its price is identically one;

(1.4) production (supply) equals effective demand;

(1.5) all labour compensation consumed; the gross mineral rent and a part of profit saved and invested;

(1.6) stationary growth in the labour force that is necessarily not fully employed;

(1.7) a growth rate of real labour compensation rises in the neighbourhood of full employment;

(1.8) a change in capital intensity and technical progress are not separable due to a flow of invention and innovation over time;

(1.9) a qualification of the labour force mostly corresponds to technological requirements although the tension between the first and the second is a factor decelerating the long-term growth of labour force.

The model abstracts from over-production of commodities inherent in over-accumulation of capital during certain phases of industrial cycles. The assumption (1.5) corresponds to the immediate aim of capitalist production.

The assumption (1.6) means that the labour force grows exponentially over time. This assumption may be substituted by an assumption of an endogenous growth of labour force (Ryzhenkov 2010). The latter source also contains upgraded equations for endogenous rate of capital accumulation and for income distribution under business-as-usual or after onset of employment-centred stabilization policy.

1.2. The detailed assumptions and model equations

The model is formulated in continuous time. A dot (two dots) above the symbol designates (designate) the first (second) derivative of a variable with respect to $t \geq 0$, the growth rate of the variable is marked by a circumflex ^ directly above it. Subscript 0 refers to the value of the variable at $t = 0$. The model of the (implicitly open) U.S. economy is presented below without description of external economic relations that suffices for the intended comparison of this model with the "neoclassical" models suggested for a closed capitalist economy.

The model (HL-1) consists of the following equations:

$P = K/s$; (1.1)

$a = P/L$; (1.2)

$u = w/a$; (1.3)

$\hat{a} = m_1 + m_2 \widehat{K/L} + m_3 \psi(\hat{v}) + m_5 \widehat{F/L}$ (1.4)

where $\psi(\hat{v}) = \text{sign}(\hat{v})|\hat{v}|^j$, $m_1 \geq 0$, $1 \geq m_2 \geq 0$, $m_3 \geq 0$, $m_5 \geq 0$, $0 < j$;

$\widehat{K/L} = n_1 + n_2 u + n_3(v - v_c) + n_5 Z/P$ (1.5)

where $n_2 \geq 0$, $n_3 \geq 0$, $n_5 \geq 0$, $1 > v_c > 0$;

$v = L/N$; (1.6)

$N = N_0 e^{nt}$, $n = const \geq 0$, $N_0 > 0$; (1.7)

$\hat{w} = -g_1 + rv + b_1 \widehat{K/L} + b_2 \widehat{F/L}$, $g_1 \geq 0$, $r > 0$; (1.8)

$P = wL + M + R = C + \dot{K} + Y$ (1.9)

where $C = wL + (1 - k)M + R - Y$;

$\dot{F} = Y - Z$; (1.10)

$Z = eP$, $0 < e < 1$; (1.11)

$\dot{y} = [o_1(x - f) + o_2 \hat{f}]y$, $y = Y/P \geq 0$; (1.12)

$\hat{X} = i$; (1.13)

$q = P/F,\ f = 1/q,\ q > 0;$ (1.14)

$x = X/P > 0;$ (1.15)

$\hat{e} = \hat{P}(e_1 / e - 1),\ e > e_1 > 0;$ (1.16)

$\dot{K} = kM = k[(1 - w/a)P - R] = k[(1 - u)P - R]$ where $0 < k \leq 1;$ (1.17)

$\hat{F} = (Y - Z)/F = (y - e)q.$ (1.18)

Equation (1.1) postulates a technical relation between the produced capital stock K and net output P. The output-fixed capital ratio is denoted by m, its inverse by s. Equation (1.2) relates output per worker a, net output P and labour input or employment L. Equation (1.3) describes the labour compensation share in net output or relative labour compensation u.

Equation (1.4) is an extended technical progress function. The rate of change of produced capital intensity K/L, the direct scale effect $m_3\psi(\hat{v})$, and the rate of change of natural capital intensity F/L are factors determining the rate of change of output per worker a. In special cases of HL-1, $m_3 \leq 0$.

Equation (1.6) defines the employment ratio v as a result of the buying and selling of labour-power.

Labour force grows exponentially in (1.7). In equation (1.8), the rate of change of the labour compensation rate w depends on the employment ratio v, as in the usual Phillips relation, and on the rates of change of produced capital intensity K/L and of natural capital intensity F/L, additionally. The produced capital intensity K/L is a proxy for qualification of the labourers.

Equation (1.9) presents equality between net output (national income) produced and net output (national income) finally used: on the left, wage wL, profit M and rent R comprise net output (national income) produced; on the right, C is private and public consumption, net formation of produced fixed capital is $\dot{K} = kM$ where K is produced fixed assets, Y is gross accumulation of proved mineral reserves that equals the resource rent wholly or partially. It is assumed, that workers spend the total wage on consumer goods, profit is not invested in additions to proved reserves, rent is not invested in produced capital formation, both profit and rent can be spent on private and public consumption.

The attribute *gross* at unit or whole resource rent reflects its close connection with unit or respectively total gross additions to proved reserves. It is postulated in HL-1 for simplicity that the resource rent is invested as a whole in gross additions to proved reserves $Y = R$. Then the gross resource rent is Y and gross unit resource rent is $y = Y/P$. These simplifications are removed in Section 3 where $Y \leq R$ and $y \leq \beta$.

There is a potential threat of complete depletion of remaining mineral resource (a sum of proven reserves and undiscovered resource, in Sterman's paper terminology).[6] With attentiveness to this caveat, this paper assumes that in next hundred years the remaining aggregated mineral resource in the USA (and in the world) will not be depleted wholly and that it will not become uneconomical. Equation

[6] According to that paper (Sterman, 2002: 514): "As exploration adds to the stock of proven reserves, the stock of undiscovered resource falls. *Ceteris paribus*, the smaller the stock of resources remaining to be discovered, the lower the productivity of exploration activity must be (on average), and the smaller the rate of addition to proven reserves will be for any investment rate. In the limit, if the stock of undiscovered resource fell to zero, the rate of additions to proven reserves would necessarily fall to zero…How large the resource base is, what the costs of backstop technologies are, and whether a backstop technology can be developed before depletion constrains extraction and reduces economic welfare are empirical questions, not matters of faith. The very possibility that depletion might matter cannot be assumed away, to be made untestable with models in which resources are assumed infinite, the price system always functions perfectly, delays are short and technology provides backstops at low cost."

(1.12) requires, in other words, non-zero remaining mineral resources that are of economic interest or potentially economic.

Profit is defined as a residual $M = (1 - u - y)P$. Equations (1.9) and (1.17) show that profit and incremental produced capital $\dot{K}$ are not equal in monetary terms if the investment share $k < 1$.

In equation (1.10), $\dot{F}$ is a net accumulation (loss) of the proved mineral reserves F. In this equation, Z is the depletion of proved mineral reserves – the decline in the stock F associated with extraction in the current period. This outflow is mitigated by inflow Y that represents additions to the proved mineral reserves due to improvements in recovery techniques, as well as owing to investment in resource exploration and development in the current period.

The rate of change of the capital intensity K/L in equation (1.5) is a function of the following factors: relative labour compensation u, the difference between real employment ratio and some base magnitude $v - v_c$, depletion of proved reserves in relation to net output Z/P. The rate of growth of produced capital intensity depends positively on the unit depletion of proved mineral reserves (an application of the principle 'pollution prevention pays'), in particular. A high labour compensation share and high employment ratio foster mechanisation (automation) as well. In special cases of HL-1, $n_3 \le 0$.

The inverse of output per worker $1/a$ represents a total labour input embodied in a unit of net output, so it approximates a magnitude of labour value of this unit.[7] The value of a unit labour power is $u = w/a$, unit surplus value is $1 - u$; total surplus value is the labour value of surplus product, measured by surplus labour, $S = (1 - u)L$. The sum of resource rent and total profit (abstracting from other forms of rent) is the money form of surplus product $M + R = Sa$. In hypothetical laws, net output unit price (1) is omitted for simplicity.

Calculations of u and s are done with the nominators and denominators measured in current prices. F, Y, Z are estimated in 1996 dollars. The nominator and denominator of f are measured in 1996 dollars.

The employment ratio v is for the civil labour force (without accounting the latent and stagnant unemployment). The net fixed capital K is a sum of private and governmental produced non-residential fixed assets. Data on net output P relates to net national product (NNP). The unit depletion of proved mineral reserves e is the quantity of depleted proved mineral reserves per unit of net output. The main variables with their units of measurement follow: a [millions of 1996 dollars per worker per year], e, k, u, v [dimensionless], s and f [years].

Three profit rates are defined for this economy. The first is the *average* rate of return on produced capital $(1 - u - y)/s$. The second is the *general* one; it measures a ratio of the economic surplus to the sum of produced capital and natural capital in the form of proved mineral reserves $(1 - u)/(s + f)$. The third is a *biased* profit rate $(1 - u)/s$ that is more easily calculated based on commonly available statistics.

According to equation (1.18), the growth rate of proved mineral reserves and rate of net mineral rent is $(y - e)/f$. The general rate of profit is a weighted average of the rate of return on produced capital and the rate of gross mineral rent: $(1 - u)/(s + f) = [s/(s + f)](1 - u - y)/s + [f/(s + f)]y/f$.

For the reader's convenience, Tables 1.1a and 1.1b list the variables of different kind.

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[7] Let Q is the total product, A is the direct material input per unit of total output, $l = L/Q$ is the direct labour input per unit of total output; $P = (1 - A)Q$ is the net output, while $Q = (1 - A)^{-1}P$. Then the total labour input is $L = lQ = l[(1 - A)^{-1}P] = P/a$, so $1/a = l(1 - A)^{-1}$. The labour value of an output unit is approximated by the total labour embodied in this unit: $\omega = \omega A + l = l(1 - A)^{-1} = 1/a$. The knotty problem of reduction of skilled labour to average social labour not explicit in this brief exposition requires a special consideration (not in this paper).

Table 1.1a. The main stocks and flows in HL-1

Variable	Notation
Employment	L
Labour force	N
Fixed capital (net)	K
Proved mineral reserves (net)	F
Desired proved mineral reserves	X
Real net output	P
Nominal net output	$P*1 = P$
Surplus product	$(1 - u)P = Sa$
Profit and resource rent	$M + R = (P - wL)*1 = P - wL$
Surplus value	$S = (1 - u)L$
Net accumulation of fixed capital (for $Y = R$)	$\dot{K} = kM = k(1 - u - y)P$
The gross accumulation of proved mineral reserves (the gross mineral resource rent for $Y = R$)	Y
Depletion of proved mineral reserves	Z
Net accumulation of proved mineral reserves (the net mineral resource rent for $Y = R$)	$\dot{F} = Y - Z$

Table 1.1b. The main relative variables of HL-1

Output per worker	$a = P/L$
Employment ratio	$v = L/N$
Worker's real labour compensation	w
Unit value of labour power (relative labour compensation)	u
Capital-output ratio	$s = K/P = 1/m$
Proved mineral reserves-output ratio	$f = F/P = 1/q$
Rate of capital accumulation (for profit)	k
Rate of capital accumulation (for net output and $Y = R$)	$c = k(1 - u - y)$
Fixed capital intensity	K/L
Proved mineral reserves intensity	F/L
Average profit rate (for $Y = R$)	$M/K = (1 - u - y)/s$
Rate of surplus value	$S/(L-S) = (1 - u)/u$
Unit depletion of proved mineral reserves	$e = Z/P$
Gross rent per unit of the resource extracted (for $Y = R$)	Y/Z
Minimal prospective unit depletion of proved mineral reserves	e_1
Desired proved mineral reserves-output ratio	$x = X/P$
Gross unit mineral resource rent (for $Y = R$) – potential source for royalty	$y = Y/P$
Rate of gross rent (for $Y = R$)	y/f
Net unit mineral resource rent (for $Y = R$)	$y - e$
The growth rate of proved mineral reserves (rate of net rent for $Y = R$)	$(y - e)/f$

The desired proved mineral reserves X may remain constant, decrease or increase exponentially in equation (1.13). Equation (1.12) defines an investment policy that is aimed to develop the proved mineral reserves in accordance with the desired proved mineral reserves (Figure 1.1). This investment policy substantially differs from Hartwick's rule that defines the amount of investment in produced capital apparently needed to exactly offset declining stocks of non-renewable resources (Hartwick 1977). Hartwick's rule is based in turn on the Hotelling rule that is considered in subsection 2.2.

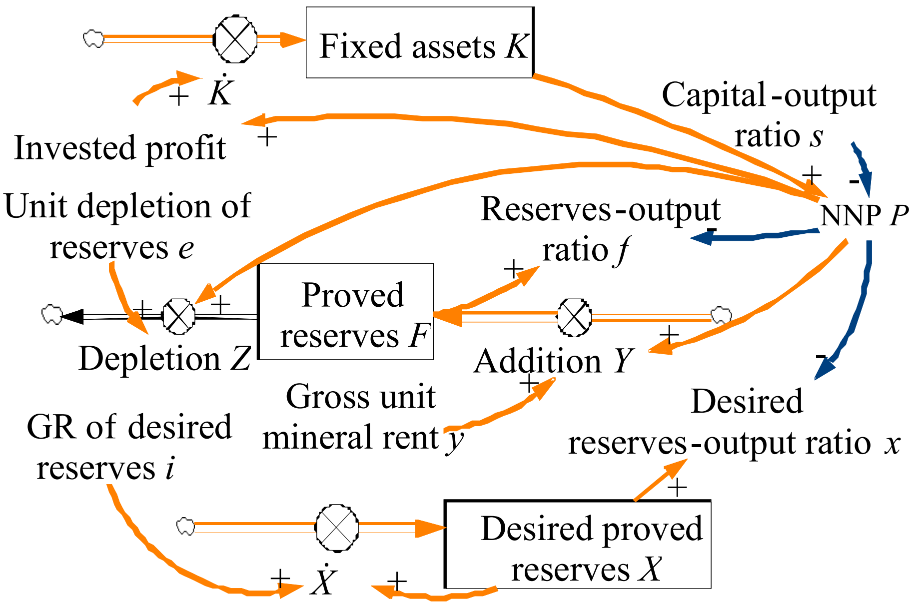

Figure 1.1 – Economy basic stocks and flows (except labour, GR stands for growth rate)

The proved mineral reserves-output ratios – real f and desired x – belong to the state variables of the HL-1 intensive form. A combination of proportional and derivative closed-loop control over this form of natural capital is attainable hereby if the first parameter is positive ($o_1 > 0$), whereas the second parameter in equation (1.12) is negative ($o_2 < 0$). It is likely that this a priory expected combination was absent in the reality on the macro scale possibly due to bounded rationality of economic subjects and (or) for another reason (Ryzhenkov 2007).

Assume that depletion Z of proved mineral reserves is directly linked to net output P by equation (1.11) whereas an increment of depletion is directly connected to that of net output

$$\dot{Z} = e_1 \dot{P} \quad \text{where } e_0 \geq e_1 > 0. \tag{1.11a}$$

Then equation (1.16) for the growth rate of unit depletion of proved mineral reserves follows. This is a better approximation in the long run than $e = const > 0$. Still equation (1.11a) neglects middle-term and short-term deviations from the direct dependence of $\dot{Z}$ on $\dot{P}$ that are reflected in the probabilistic form of HL-1 together with other similar discrepancies.

It is surmised that preferably the unit depletion of proved mineral reserves e asymptotically declines to the minimal level ($e_1 = const > 0$) due to reduction, reuse, recycling, substitution and structural change in result of technological progress, managerial practices and governmental policies according to equation (1.16) where $\hat{e} < 0$ for $\hat{P} > 0$ and vice versa. Notice that a special case of equation (1.16) for $\hat{P} = \text{const} > 0$ is a simplified logistic equation.

Equation (1.16) implies also that the absolute value of the rate of change of unit depletion of proved mineral reserves decreases to zero, and this diminution is the faster, the higher the rate of economic growth and the lower the ratio e_1/e. In practice, for particular natural resources (for example, crude oil) the relation $|\hat{e}| > \hat{P} > 0$ is possible, still it is assumed that for the aggregated proved mineral reserves the relation $0 \le |\hat{e}| < \hat{P}$ is valid.[8]

The next two equivalent equations follow from equations (1.11) and (1.16) governing the rate of depletion of proved mineral reserves for $e_1 - e \neq 0$:

$$\hat{Z} = \hat{e}\, \frac{e_1}{e_1 - e}, \tag{1.16a}$$

$$\hat{Z} = \hat{P}\, \frac{e_1}{e}. \tag{1.16b}$$

The improving resource efficiency *e* determined enables slower growth of resource use *Z* in relation to net output *P* (partial decoupling) that corresponds to global tendencies so far. A curve of $\hat{Z}$ depending on $\hat{e}$ has a negative slope whereas the slope of a curve of $\hat{Z}$ depending on $\hat{P}$ is positive.

We will see in the next Section that a widely disseminated textbook operates on a premise in its basic "neoclassical" model (NM-1) that *e* declines whatever happens with $\hat{P}$ assuming still that the lower the latter the smaller is $|\hat{e}|$. This means that a straight line of $\hat{e}$ on $\hat{P}$ has a negative slope in NM-1.

An extended Kalman filtering

The Kalman filter is a particular powerful tool for estimating unobservable part of a model (parameters and meta-parameters like variances) in one operation (Sorenson 1970, Peterson 1980, Watson 1983). Although the Kalman filter itself does not estimate the unknown parameters of the model, it provides a one-step-ahead prediction error with its covariance matrix. The prediction error decomposition of the likelihood function utilises this information. The data from the official U.S. sources fed the probabilistic form of HL-1 for the period 1958–1991. The *Vensim* professional soft-ware has served for performing such an extended Kalman filtering (EKF) in (Ryzhenkov 2007) that reports about quasi-optimal estimates obtained.

EKF realised in the Vensim software has enabled to estimate the unobservable components of HL-1. The identified parameters' magnitudes follow: $b_1 = 0.621$, $d = 0.0384$, $e_1 = 0.0054$, $g_1 = 0.0532$, $i = 0.0374$, $j = 0.211$, $k = 0.267$, $m_1 = 0.0145$, $m_2 = 0.100$, $m_3 = 0.011$, $m_5 = 0.089$, $n = 0.020$, $n_1 = -0.242$, $n_2 = 0.353$, $n_3 = 0.5$, $n_5 = 0.011$, $o_1 = -0.030$, $o_2 = -9.934$, $b_2 = -0.008$, $r = 0.061$, $v_C = 0.925$.

The inertia scenario projected the internal tendencies of capital accumulations into the XXI century. It was exploratory rather than normative. These parameters' magnitudes are used in simulation experiments based on subsequent models analysed in the next sections with few exceptions. The new exogenous growth rate of labour force is taken from (CBO 2014, OASDI 2014): $n \approx 0.005$, the stationary growth rate

[8] Fast (exponential or hyperbolic) decline of unit depletion of a particular natural resource or of a certain group of natural resources may be ecologically necessary or desirable under specific conditions. The review (Krautkraemer 1998) accentuated, in particular, the environmental impacts of non-renewable resource use that as the subject of scientific inquiries goes beyond the scope of my present work.

of net output (NNP) $d \approx 0.023$, respectively.[9] Based on BEA data, the new exogenous rate of accumulation $c \approx 0.050$ is estimated as the average ratio of fixed capital formation to net national product in current dollars over 1958–2013. Other parameters modifications are introduced in subsection 2.1.

2. The basic "neoclassical" non-equilibrium model (NM-1)

Marxian theory of commodity, surplus value and resource rent is cornerstone of HL-1. On the opposite side, the principle of marginal productivity maintains distributional equations in NM-1.[10] Similarly, technical progress function (1.4) and mechanization function (1.5) in HL-1 are in opposition to the production function (2.1) of Cobb – Douglas type in NM-1. Rent is superfluously explained mainly by scarcity of natural resources.

The basic NM-1 is a very close simplified analogue of the model proposed in (Stiglitz 1974). The critique of NM-1 in this paper is substantially relevant for the Stiglitz and Hartwick models as well as for other with similar crucial properties.

NM-1 without explicit natural resources takes the form of the Solow model (Solow 1956). The production function in the latter seemingly has a constant return to scale (according to the standard "neoclassical" definition with respect to the arguments K and L for the given values of A).

Still the standard "neoclassical" definition of economy of scale ignores feedback loops between growth rate of output per worker and other variables. Taking these feedback loops into account results in the deep definition of economy of scale (Ryzhenkov 2009: 356–357).

Without going into details, *direct economy of scale* (*direct increasing return*) manifests itself in a positive partial derivative of growth rate of output per worker $\hat{a}$ with respect to employment ratio v or growth rate of employment ratio $\hat{v}$: $\frac{\partial \hat{a}}{\partial v} > 0$ (type I) or $\frac{\partial \hat{a}}{\partial \hat{v}} > 0$ (type II). *Roundabout economy of scale* (*roundabout increasing return*) manifests itself in a positive partial derivative of growth rate of output per worker with respect to employment ratio v or growth rate of employment ratio $\hat{v}$ intermediated by other variable or variables (x_i): $\frac{\partial \hat{a}}{\partial x_1}\frac{\partial x_1}{\partial v} > 0$ or $\frac{\partial \hat{a}}{\partial x_1}\ldots\frac{\partial x_i}{\partial v} > 0$, $\frac{\partial \hat{a}}{\partial x_1}\frac{\partial x_1}{\partial \hat{v}} > 0$ or $\frac{\partial \hat{a}}{\partial x_1}\ldots\frac{\partial x_i}{\partial \hat{v}} > 0$, $i \in \{2,\ldots,I\}$.

Economy of scale (increasing return) is *reinforcing* if a *positive* feedback loop connects the growth rate of output per worker with employment ratio and (or) its growth rate. Economy of scale (increasing return) is *weakening* if a *negative* feedback loop connects the growth rate of labour productivity with employment ratio and (or) its growth rate.

The Solow model with a relaxed assumption of full employment, in particular, for n = const in equation (1.7), is a specific form of HL-1. It was shown (Ryzhenkov 2005) that the profoundly defined econ-

[9] The differences between growth rates of NNP and GDP are not to be forgotten. The U.S. Administration has projected the long-run potential GDP growth rate as 2.3 percent a year (Economic Report of the President 2015, p. 89).

URL: http://www.whitehouse.gov/sites/default/files/docs/cea_2015_erp_complete.pdf

[10] It is well-known and for this reason not presented in this paper, except equation (2.5) below.

omy of scale in a refined and generalised Solow model without full employment is linear and negative with $u \equiv u_a = 1 - \alpha = - m_3 > 0, j = 1$ in equation (1.4) and $n_3 = 0$ in equation (1.5).[11]

The above restriction of natural resources to proved mineral reserves F is applied in this Section too whereas the referred textbook offers more vague interpretations of natural resources of different types.

2.1. The deterministic form of extensive NM-1

The main structural element is the production function of Cobb – Douglas type that defines the net output

$$P = \eta K^{\alpha} F^{\beta} (AL)^{1-\alpha-\beta} \tag{2.1}$$

where $\alpha > 0$, $\beta > 0$, $\alpha + \beta < 1$ and $\eta = 1$ is a factor added here for harmonizing the variables' units of measurement, A is the index of efficiency of labour with an exogenously defined growth rate

$$\hat{A} = g > 0. \tag{2.2}$$

The rate of exogenous technical progress determines the long-term growth rate of output per worker

$$\hat{a} = m_1 / (1 - m_2 - m_5) = g \tag{2.3}$$

that equals the growth rate of wage

$$\hat{w} = \hat{a}. \tag{2.4}$$

This means that the labour share in net output u is constant and determined solely by technology – the view that the present paper exposes without any adherence. The next equation specifies the labour share consequently as

$$u = 1 - (m_2 + m_5) = 1 - \alpha - \beta. \tag{2.5}$$

Here parameters of the production function (2.1) are connected for expository purposes with parameters of the technical progress function (1.4) as $\alpha = m_2$, $\beta = m_5$, $m_1 = (1 - \alpha - \beta)g = [1-(m_2 + m_5)]g$, $g = m_1/(1 - m_2 - m_5)$. Similarly, a new magnitude of the rate of capital accumulation for profit is defined as $k = c/\alpha$.

These identities are purely instrumental and are not understood as theoretically and / or empirically valid. They are helpful for revealing substantial differences between HL-1 and NM-1 when all other conditions are the same.

Fixed capital is calculated taking into account wear and tear again. The society invests a proportion of net output c that increases net fixed capital $\dot{K} = cP$ without delay for simplicity.

After assuming exponential growth of the labour force and of output per worker, on the one hand, the exponential reduction of proved mineral reserves is assumed, on the other hand:

$$\hat{F} = -b = const < 0.^{12} \tag{2.6}$$

[11] A more complicated "neoclassical" model also relaxes the assumption of full employment and apparently includes increasing returns although preserving constant income shares. A critique of this model is developed in (Ryzhenkov 2009, 2013). These publications demonstrate: first, a logistic equation approximates the employment ratio dynamics; second, there is in fact reinforcing roundabout diseconomy of scale. Typically, direct and roundabout increasing returns characterise the simplified version of HL-1 considered that generalises this "neoclassical" model as its special non-realistic case. This generalization has been prompted by the reply (Boggio 2010) that has rejected my definition of economy of scale, yet without offering a "neoclassical" encompassing of HL-1 in turn.

This model assumes that the depletion of proved mineral reserves and their net change with the opposite sign are equal $Z = -\dot{F} = bF$. Therefore the proved mineral reserves-depletion ratio is

$$\frac{F}{Z} = \frac{F}{-\dot{F}} = \frac{1}{eq} = \frac{1}{b} = const > 0\,. \qquad (2.7a)$$

It follows that the rate of change of unit depletion of proved mineral reserves has the opposite sign and the same absolute value as the rate of change of the ratio of net output to proved mineral reserves

$$\hat{e} = -\hat{q}\,, \quad e > 0. \qquad (2.7b)$$

Rent per unit of the resource extracted is $R/Z = \beta/e$. Its growth rate equals the growth rate of rent rate determined by the previous equation with signs changed to opposite

$$\widehat{R/Z} = \widehat{R/F} = -\,\hat{e} = \hat{q}\,. \qquad (2.7c)$$

Table 2.1 traces common features and peculiarities of NM-1 and HL-1.

Table 2.1. Correspondence of equations in NM-1 to those of HL-1

Universal for NM-1 and HL-1	Specific in HL-1	Specific in NM-1	Explicit in HL-1 and implicit in NM-1
(1.1)–(1.3), (1.7), (1.14)	(1.4)–(1.6), (1.8), (1.11), (1.11a), (1.12), (1.13), (1.15), (1.16)	(2.1)–(2.7a), (2.7b)	(1.9), (1.10), (1.17), (1.18)

2.2. The intensive form of deterministic NM-1

Figure 2.1 and Table 2.2 are mappings of the NM-1 causal loop structure. It includes three first order feedback loops: B1 of unit depletion of proved mineral reserves *e*, B2 of output-produced capital ratio *m* and R1 of output-proved mineral reserves ratio *q* as well as two separate direct links that do not constitute a feedback loop.

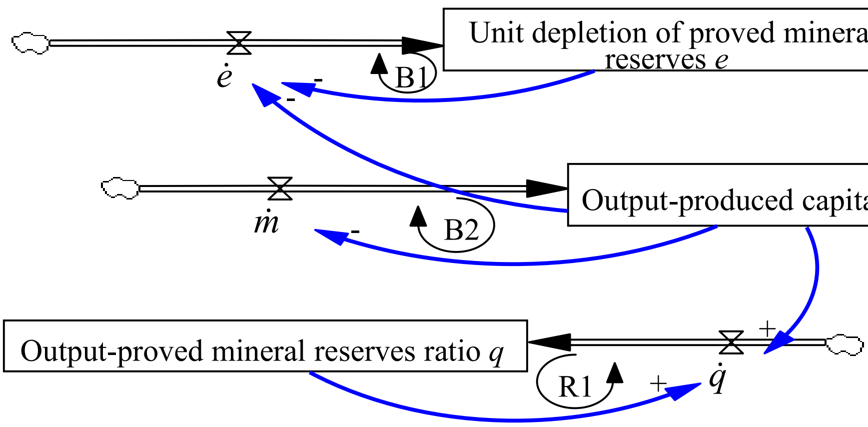

Figure 2.1 – A condensed causal loop structure with explicit stocks and flows of NM-1

[12] According to well-known data (BP 2015, OECD 2010), the world consumption of the fossil fuels has substantially increased in the long term (1965–2014), not diminished, similarly worldwide use of metal ores has been increasing rather steadily (1980–2008). Thus NM-1 mostly contradicts these factual trends contrasting with HL-1 and NM-2.

There is no feedback loop of the second or third order as these phase variables are treated in the textbook according to the single equation approach that we have already mentioned disapprovingly as methodologically unreliable in Introduction. So there is stabilizing "intra-specific" competition in e and m and destabilizing "intra-specific" co-operation in q. Still the latter does not imply hyperbolic growth in q as no hyperbolic term is a part of partial derivative $\partial\dot{q}/\partial q$. Therefore uncovered instability does not imply aggravation mode.

Table 2.2. A condensed causal loop structure of NM-1

Separate direct links	1st order negative B1	1st order negative B2	1st order positive R1
$m \xrightarrow{-} \dot{e}$; $m \rightarrow \dot{q}$	$e \xrightarrow{-} \dot{e} \rightarrow e$	$m \xrightarrow{-} \dot{m} \rightarrow m$	$q \rightarrow \dot{q} \rightarrow q$

Note. An arrow $\rightarrow$ implies a positive partial derivative, its sign + is skipped.

Thanks to equation (2.7b) that remains operative, an intensive form of deterministic NM-1 can be represented initially as a system of two (instead of three) nonlinear ODEs for output-capital ratio $m = 1/s$ and output-proved mineral reserves ratio $q = 1/f$:

$$\dot{m} = [-(1-\alpha)cm - \beta b + (1-\alpha-\beta)d]m, \tag{2.8}$$

$$\dot{q} = [\alpha cm - \beta b + (1-\alpha-\beta)d + b]q, \tag{2.9}$$

whereby growth rate of proved reserves $\hat{F} = -b$ as in equation (2.6). The parameter b is the implicit control (bifurcation) parameter in the textbook that becomes explicit in this paper.

Proposition 2.1. The system of ODEs (2.8) and (2.9) has two non-trivial quasi-stationary states (with no stationary state proper) depending on a relation of a magnitude of parameter b to its threshold value $b_c = (1-\alpha-\beta)d/\beta$ where $d = g + n > 0$. Attractors are defined by the expressions in Table 2.3.

Table 2.3. Attractors in NM-1 for $t \rightarrow \infty$ depending on parameter $b = -\hat{F}$

Indicator (variable)	Expression for		
	$b \geq b_c$	$b < b_c$	$b = -n$
Output-fixed capital ratio	$m_c = 0$	$m_c < m_b = \dfrac{-\beta b + (1-\alpha-\beta)d}{c(1-\alpha)}$	$m_n = \dfrac{\beta n + (1-\alpha-\beta)d}{c(1-\alpha)} > m_b$
Growth rate of depletion of proved mineral reserves	$\hat{Z}_c = -b$	$\hat{Z}_b = -b$	$\hat{Z}_n = n$
Growth rate of net output	$\hat{P}_c = -\beta b + (1-\alpha-\beta)d \leq 0$	$\hat{P}_c < \hat{P}_b = d - \dfrac{\beta}{1-\alpha}(d+b) < d$	$d > \hat{P}_n = d - \dfrac{\beta}{1-\alpha}g > \hat{P}_b$
Growth rate of output per worker	$\hat{a}_c = -\beta b + (1-\alpha-\beta)d - n \leq 0$	$\hat{a}_c < \hat{a}_b = \hat{P}_b - n$	$g > \hat{a}_n = \hat{P}_n - n > \hat{a}_b$
Growth rate of output-proved mineral reserves ratio	$d + b > \hat{q}_c = \hat{P}_c + b = -\beta b + (1-\alpha-\beta)d + b > 0 \geq \hat{P}_c$	$\hat{q}_c < \hat{q}_b = \hat{P}_b + b > \hat{P}_b$	$0 < \hat{q}_n = \hat{a}_n < \hat{q}_b$

When the absolute rate of change of proved reserves equals or exceeds the critical threshold b_c then no quasi-stationary state with positive output-fixed capital ratio is possible, and the system is pushed toward a rather undesirable quasi-stationary state with a zero output-fixed capital ratio. Then the absolute value of non-positive quasi-stationary growth rate of output per worker equals or exceeds the growth rate of fully employed labour force n. Therefore quasi-stationary total wage, profit and rent either stagnate for $b = b_c$ or fall for $b > b_c$. If $n > 0$, quasi-stationary output per worker and unit wage decline not only for $b > b_c$ but for $b = b_c$ as well.

The detailed proof of Proposition 2.1 will not be given. In particular, for $b < b_c$, it comes down to equivalent presentation of equations (2.8) and (2.9) as

$$\dot{m} = (1-\alpha)c(m_b - m)\,m, \qquad (2.10)$$

$$\dot{q} = [\,\hat{q}_b \; - \alpha c(m_b - m)\,]q. \qquad (2.11)$$

Notice that equation (2.10) is a logistic equation similar to its analogue in the Solow model (1956) as (Ryzhenkov 2005) has demonstrated. A joint consideration of equations (2.10) and (2.11) results clearly in $m \to m_b$, $\hat{q} \to \hat{q}_b$ for $t \to \infty$.

Corollary 1. Part 1. The rate of resource rent and the output-proved mineral reserves ratio are unbounded. Their quasi-stationary growth rates are equal: $(\beta P_b)\hat{/} F_b = \hat{q}_b > 0$.

Corollary 1. Part 2. The quasi-stationary *average* rate of return on produced fixed assets is $\alpha m_b > 0$, the quasi-stationary *general* rate of return on produced fixed assets and proved reserves, being equal quasi-stationary *biased* profit rate, is $(\alpha+\beta)m_b > \alpha m_b > 0$.

Corollary 1. Part 3. The absolute rent can be defined as $R_{abs} = F(\beta q - \alpha m)$ for $\beta q > \alpha m$ or $\beta/F > \alpha/K$. The share of absolute rent in total rent is $1 - \alpha m/(\beta q)$. This share asymptotically approaches 1 for $t \to \infty$. The share of differential rent in total rent is $\alpha m/(\beta q)$. This share asymptotically approaches 0.

Corollary 1 as a whole stipulates that the rate of resource rent βq tends to grow exponentially without an upper limit in ever bigger excess over quasi-stationary average profit rate αm_b. The Marx notion of absolute rent, not mentioned in the textbook, as the effect of monopoly of private property on natural resources can justify the real tendency of the rent rate to be higher than *average* profit rate in the epoch of free competition at least. Still this positive element in NM-1 is degraded by the unbounded rate of resource rent connected with the unrestricted output-proved mineral reserves ratio.

Corollary 2. If $\alpha > c$ and $b_c > b = b_H = \dfrac{(1-\alpha-\beta)d(\alpha-c)}{c(1-\alpha-\beta)+\alpha\beta} > 0$, the corresponding quasi-stationary output-fixed capital ratio $m_H = \dfrac{d(1-\alpha-\beta)}{c(1-\alpha-\beta)+\alpha\beta}$.

Corollary 3. The same quasi-stationary growth rates of rate of resource rent and of rent per unit of the resource extracted equal the quasi-stationary profit rate for $b = b_H$.[13] These equal magnitudes are

[13] This property is commonly referred in the "neoclassical" literature as the Hotelling rule of natural resource exploitation (Hartwick 1977: 973): "In price terms, this condition is characterized by the current capital gain on mineral deposits being equal to the…rate of return on reproducible capital." See also the original (Hotelling 1931).

higher than the positive quasi-stationary growth rate of net output: $\hat{q}_H = \alpha m_H = \hat{P}_H + b_H > \hat{P}_H = cm_H > 0$. For $b \neq b_H$ the market fails to satisfy the Hotelling rule, especially for $b > b_c$.

Corollary 4. The growth rates of total resource rent βP and of total profit αP are the same as the growth rate of net output P. Particularly, in agreement with the Hotelling rule, the quasi-stationary growth rates of total resource rent βP and of total profit αP are also the same as the quasi-stationary growth rate of net output P for $b = b_H$.

Corollary 5. An increase in the rate of accumulation c promotes the quasi-stationary growth rate $\hat{P}_H$ and fosters quasi-stationary capital-output ratio s_H : respectively, $\frac{\partial \hat{P}_H}{\partial c} > 0$, $\frac{\partial m_H}{\partial c} < 0$. This increase is associated with a lower quasi-stationary growth rate of output-proved reserves ratio q_H and a lower absolute rate of proved reserves depletion b_H: correspondingly, $\frac{\partial \hat{q}_H}{\partial c} < 0$ and $\frac{\partial b_H}{\partial c} < 0$.[14]

The growth rate of net output $\hat{P}_b$ can be negative for magnitudes of b above the threshold b_c. Restriction of $\alpha + \beta < 1$ precludes $\hat{P}_b \leq \hat{F}_b$. Contrary to the textbook claim that its basic model maintains "balanced growth path", that does not exist in NM-1 as $\hat{P}_b > \hat{F}_b$. Besides that the textbook left unnoticed the Hotelling rule and the requirement $b = b_H$ for satisfying this rule at a non-trivial quasi-stationary state in NM-1.

Proposition 2.2. The rate of growth of the ratio of output to proved mineral reserves is always positive and the rate of change of unit depletion of proved mineral reserves is always negative.

The proof:

$$\hat{e} = -\hat{q} = -\alpha cm - (1-\beta)b - (1-\alpha-\beta)d < 0. \qquad (2.12)$$

The conversion of equation (2.7b) into equation (2.12) explains the presence of the link $m \xrightarrow{-} \dot{e}$ on Figure 2.1 and in Table 2.2 where one would expect the link $\hat{q} \xrightarrow{-} \hat{e}$ instead. The latter is the element of an extensive form of NM-1 that differs from its intensive form.

Corollary. Part 1. There is negative linear dependence of $\hat{e}$ on $\hat{P}$: $\hat{e} = -b - \hat{P}$, the slope of this straight line is negative as $\frac{\partial \hat{e}}{\partial \hat{P}} = -1$.

Corollary. Part 2. For $t \to +\infty$ unit depletion of proved mineral reserves $e \to 0$, whereas output-proved mineral reserves ratio $q \to +\infty$.

Corollary. Part 3. The rate of change of depletion of proved mineral reserves is constant: $\hat{Z} = \hat{Z}_b = -b$.

Corollary. Part 4a. If $b > 0$, depletion of proved mineral reserves Z is determined by a single exponential equation – a typical case of the traditional decline curve analysis where $Z \to 0$ for $t \to \infty$.

Corollary. Part 4b. If $b < 0$, depletion of proved mineral reserves $Z \to \infty$ for $t \to \infty$.

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[14] These properties of the quasi-stationary state are similar to properties of asymptotic states for which consumption is growing exponentially in (Stiglitz 1974: 125–127). The latter applied the Hotelling rule too still without an explicit reference. The paper (Solow 1974) paid a well-known tribute to the Hotelling rule that is frequently rejected by the data (Krautkraemer 1998). The paper (Gaudet 2007) tried to bridge the impressive gap between this rule and the historical behaviour of the flow price of a number of resources.

Proposition 2.3. There is no true aggravation mode in NM-1 even for $b > b_c$ when $\hat{P}_b < 0$ – the economic decline cannot be excessively strong.

A proof boils down to the following derivatives of the first and second order for the growth rate of net output for $\hat{m} < 0$:

$$\dot{\hat{P}} = \alpha c \dot{m} < 0, \tag{2.13}$$

$$\ddot{\hat{P}} = \alpha c \ddot{m} = \alpha c \dot{m}\,[\,\hat{m} - (1-\alpha) c\, m] > 0. \tag{2.14}$$

For $t \to +\infty$, $m \to m_c = 0$ and $\hat{P} \to \hat{P}_c = -\beta b + (1-\alpha-\beta)d < 0$. Thus no true aggravation mode happens mostly due to the sorcery long term property $\hat{e} < 0$ even for $\hat{P} < 0$.

Corollary. For $b = b_c$, $\hat{P}_c = \hat{P}_b = 0$ and $m_b = m_c = 0$.

Proposition 2.4. The owners of proved reserves are interested in faster depletion of these reserves (higher b); the other capitalists and capitalist class as a whole – in their slower depletion (lower b).

The proof. Consider $b < b_c$ at first. Then the partial derivatives of quasi-stationary growth rate of rent rate, average and general profitability are $\frac{\partial \hat{q}_b}{\partial b} > 0$, $\frac{\partial(\alpha m_b)}{\partial b} < 0$ and $\frac{\partial[(\alpha+\beta)m_b]}{\partial b} < 0$, respectively. Secondly, take $b \geq b_c$. Then the quasi-stationary average and general profitability are zero whereas quasi-stationary growth rate of rent rate is $\hat{q}_c = \hat{P}_c + b = -\beta b + (1-\alpha-\beta)d + b > 0 \geq \hat{P}_c$ and the partial derivatives of quasi-stationary growth rate of rent rate is $\frac{\partial \hat{q}_c}{\partial b} > 0$. Still for $b > b_c$ total rent βP, profit αP and wage $(1 - \alpha - \beta)P$ decrease together with net output P as a whole.

Corollary. The long-term quasi-stationary growth rate of rent rate is higher than average profitability for $b > b_H$ and it is higher than general profitability for $b > b_g = \frac{(1-\alpha-\beta)(\alpha+\beta-c)d}{c(1-\alpha-\beta)+(\alpha+\beta)\beta} > b_H$ where $b_g < b_c$.

Therefore the owners of proved reserve may choose the rate of depletion of proved reserves $-b_H$ obeying the Hotelling rule only accidentally or under pressure of other capitalists not modelled yet. If the monopoly of private property on proved reserves is sufficiently strong the absolute rate of depletion of proved reserves can exceed both b_H and b_g or even b_c that may be not known with certainty. So unchecked material interests of private owners of proved reserves can slow down economic growth or drive the economy to stagnation and decline – that is not in the whole capitalist class' interests. With a decline of net output owners of proved reserves endure diminishing total rent, in particular.

The textbook has described some of the features of the NM-1 for $b = -n < 0$ in equation (2.6) without exposure of the just revealed material interests. Table 2.3 shows that in this case performance of capitalist production is superior to the case with zero investment in proved mineral reserves ($y = 0$ and $b = eq > 0$). Still the textbook left without attention the hidden explanation – the very fact that gross and net investment in proved mineral reserves became positive:

$$y = e + n/q > 0. \tag{2.15}$$

If the above equality $\hat{e} = -\hat{q}$ remains in force, Proposition 2.2 together with parts 1–3 and part 4b of its Corollary remains valid too. Because deviations from the quasi-stationary growth are of minor significance in NM-1 even now, it is sufficient to consider $\hat{e}_n = -\hat{q}_n = -\hat{a}_n < 0$.

The textbook explores neither practical feasibility of this environmental policy (for $b = -n \le 0$) nor the possibility of asymptotic validity of the Hotelling rule for certain accumulation rate

$$c = c_H = \alpha + \frac{\alpha(1-\alpha)n}{(1-\alpha-\beta)g}. \qquad (2.16)$$

The inequality $c_H > \alpha$ implies a partial investing of the resource rent, besides the total profit, in fixed capital formation – the case excluded in (Stiglitz 1974).

3. A transition to fragile balanced growth in NM-2

The model abbreviated as NM-2 differs from NM-1 by rejection of the specific equations of the latter and substituting them by relevant equations of HL-1, namely: instead of equations (2.6) and (2.7b), the equations (1.18) and (1.16) are applied (see Table 2.1). These equations reflect a more efficient policy of exploitation of non-renewable resources than the Hotelling rule, in particular. Both HL-1 and simpler NM-2 have interwoven feedback loops and can generate more sophisticated behaviour than NM-1.

3.1. A general intensive form of NM-2

The congruence in income shares in NM-2 and in NM-1 is achieved if in both models the labour share is $u = 1 - \alpha - \beta = const$, the unit rent equals $\beta = const$, and the profit share is $\alpha = const$. Then gross unit addition to proved reserves $y \le \beta$ in NM-2. The material balances are not violated if accumulation rates $c \le \alpha$ and $y \le \beta$. Still for $c = \alpha$ and $y = \beta$ the private consumption of capitalists (including private owners of proved reserves) is nil. So more realistic relations are $0 < c < \alpha$ and $0 \le y < \beta$.

Three nonlinear ODEs express an intensive form of NM-2:

$$\dot{e} = [\alpha cm + \beta(y - e)q + (1-\alpha-\beta)d](e_1 - e), \qquad (3.1)$$

$$\dot{m} = [-(1-\alpha)cm + \beta(y - e)q + (1-\alpha-\beta)d]m, \qquad (3.2)$$

$$\dot{q} = [\alpha cm + \beta(y - e)q + (1-\alpha-\beta)d - (y - e)q]q. \qquad (3.3)$$

A non-trivial stationary state of this system for $y = const > e_1$ is

$$E = (e_e, m_e, q_e) \qquad (3.4)$$

where $e_e = e_1$, $m_e = d/c > m_n > m_H$ and $q_e = d/(y - e_1) > 0$.

Notice that the expression for q_e is the hyperbolic relation *in embryo* that develops when the denominator becomes a variable. The stationary shares of absolute rent and differential rent in total gross rent are $1 - \alpha m_e/(\beta q_e)$ and $\alpha m_e/(\beta q_e)$, respectively. The stationary gross rent per unit of the resource extracted is $R_e/Z_e = const = \beta/e_1$. More effective than in NM-1 growth is, in contrast to NM-1, asymptotically balanced. Indeed, now $\hat{a}_e = g > \hat{a}_n > \hat{a}_H$ and $\hat{P}_e = \hat{K}_e = \hat{F}_e = d > \hat{P}_n > \hat{P}_H$.[15]

[15] It reflects sustainable development for this level of abstraction (particularly from explicit ecological factors and from accidents in production processes that cause injuries or even deaths of labourers especially in the coal mining). The ecological factors are emphasised in (Pope Francis 2015). It reminds us of mercury pollution in gold mining or sulphur dioxide pollution in copper mining (Ibid: 37). The U.S. metal mining is the nation's #1 toxic polluter (https://www.earthworksaction.org/issues/detail/mining accessed August 13, 2015).

Similarly to NM-1, an increase in the rate of accumulation c fosters stationary capital-output ratio s_e as $\frac{\partial m_e}{\partial c} < 0$. Contrary to the former model, an increase in rate of accumulation c has no impact on stationary growth rate $\hat{P}_e$ since $\frac{\partial \hat{P}_e}{\partial c} = 0$. Stationary ratio of net output to proved reserves q_e does not depend on rate of accumulation c: $\frac{\partial q_e}{\partial c} = 0$. Consequently, the stationary growth rate of output-proved reserves ratio ($\hat{q}_e = 0$) is not affected by an increase in the rate of accumulation c, unlike $\hat{q}_H$ in NM-1.

The net unit addition to proved mineral reserves as the difference between control (bifurcation) parameter y and specific unit depletion of proved mineral reserves e_1 determines a particular regime: the positive $y - e_1$ maintains sustainable development, the non-positive $y - e_1$ opens unsustainable path. In other words, as long as the gross unit addition to mineral proved reserves y is above a suitable threshold (e_1 namely) value, then quasi-stationary asymptotic development in NM-1 is substituted by a non-oscillatory adjustment which gives rise to a stable stationary state with a higher long-term economic growth in NM-2 than in NM-1.

Unbalanced growth evolving in aggravation mode results from $e_1 > 0$ and $0 \le y = const \le e_1$ in ODEs (3.1)–(3.3). Then development is not sustainable any more. This system does not have a non-trivial stationary state. Figures 3.1 and 3.2 together with Tables 3.1 and 3.2 display these two opposite regimes of capital accumulation.

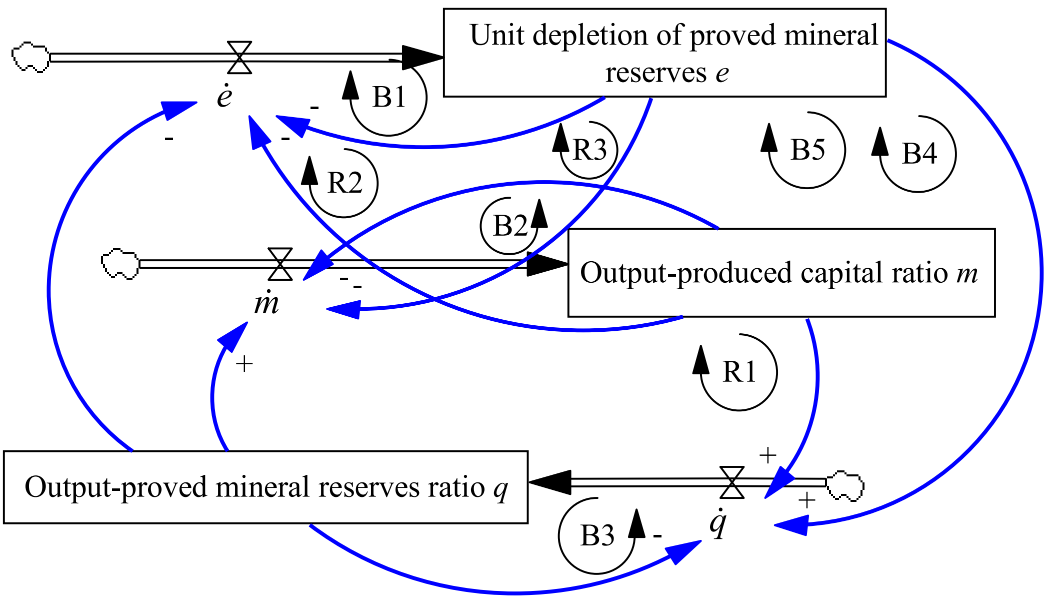

Figure 3.1 – Sustainable development in vicinity of the stationary state (3.4).
A condensed causal loop structure with explicit stocks and flows of NM-2 for $y > e_1 > 0$

Proposition 3.1. The stationary state E is locally asymptotically stable node in the linearized system of ODEs (3.1)–(3.3) for $y > e_1 > 0$.

The proof. Define Jacobi matrix J_e for E:

$$J_e = \begin{array}{|c|c|c|} \hline -d & 0 & 0 \\ \hline -\beta q_e m_e & -(1-\alpha)d & \beta d^2/(cq_e) \\ \hline (1-\beta)q_e{}^2 & \alpha c q_e & (\beta-1)d \\ \hline \end{array} \quad (3.5)$$

The parameters of the characteristic equation satisfy the Routh – Hurwitz necessary and sufficient conditions. Finding its three roots in all their beauty completes the proof:

$$\lambda_1 = -d(1-\alpha-\beta) < 0, \quad (3.6)$$

$$\lambda_3 = \lambda_2 = -d < 0. \quad (3.7)$$

We see that the higher long term economic growth rate d and value of labour power $u = 1 - \alpha - \beta$, the faster is transition to a particular stationary state E depending on d.

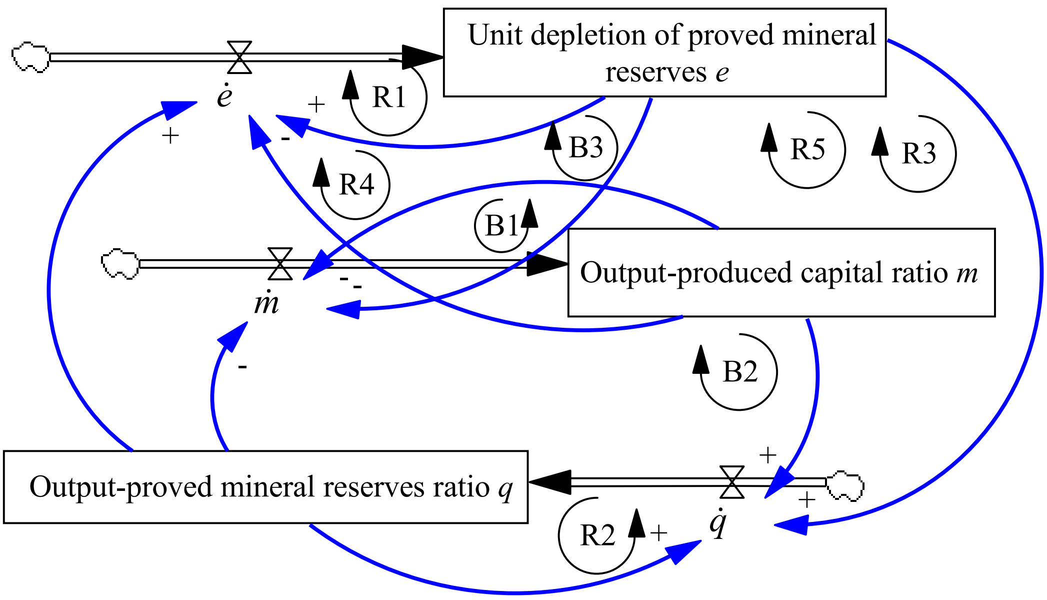

Figure 3.2 – Aggravation mode of capital accumulation.
A condensed causal loop structure with explicit stocks and flows of NM-2 for $e_1 > 0$ and $e_1 \geq y \geq 0$

Proposition 3.2. The stability property of E in the linearized system also features the original non-linear system. The rightful application of the Hartman – Grobman theorem constitutes the proof.

Establishing global stability of E goes beyond the scope of this paper. The reader can apply Olech's stability conditions for solving this task.

Corollary 1. The stationary gross and net rates of resource rent and the stationary output-proved mineral reserves ratio are constant, therefore $[\beta P_e]\hat{/}F_e = [(\beta - e_1)P_e]\hat{/}F_e = [(y - e_1)P_e]\hat{/}F_e = \hat{q}_e = 0$. So the gross and net rates of resource rent are typically limited for $t \to +\infty$ contrasting with the rate of resource rent in NM-1.

Corollary 2. Unlike NM-1, the Hotelling rule is not true as the positive stationary *average* rate of return on produced capital is higher than the stationary zero growth rate of the (gross and net) rate of resource rent: $\alpha m_e > [\beta P_e]\hat{/}F_e = [(\beta - e_1)P_e]\hat{/}F_e = [(y - e_1)P_e]\hat{/}F_e = 0$.

Table 3.1. Condensed causal loop structures of NM-2 for two accumulation regimes

Loop's order	Balanced growth with $y > e_1 > 0$		Aggravation mode with $e_1 > 0$ and $e_1 \geq y \geq 0$	
	Loop's mark		Loop's mark	
1st	B1	$e \xrightarrow{-} \dot{e} \to e$	R1	$e \to \dot{e} \to e$
	B2	$m \xrightarrow{-} \dot{m} \to m$	B1	$m \xrightarrow{-} \dot{m} \to m$
	B3	$q \xrightarrow{-} \dot{q} \to q$	R2	$q \to \dot{q} \to q$
2nd	B4	$e \to \dot{q} \to q \xrightarrow{-} \dot{e} \to e$	B2	$m \to \dot{q} \to q \xrightarrow{-} \dot{m} \to m$
	R1	$m \to \dot{q} \to q \to \dot{m} \to m$	R3	$e \to \dot{q} \to q \to \dot{e} \to e$
	R2	$e \xrightarrow{-} \dot{m} \to m \xrightarrow{-} \dot{e} \to e$	R4	$e \xrightarrow{-} \dot{m} \to m \xrightarrow{-} \dot{e} \to e$
3rd	B5	$e \to \dot{q} \to q \to \dot{m} \to m \xrightarrow{-} \dot{e} \to e$	B3	$e \xrightarrow{-} \dot{m} \to m \to \dot{q} \to q \to \dot{e} \to e$
	R3	$e \xrightarrow{-} \dot{m} \to m \to \dot{q} \to q \xrightarrow{-} \dot{e} \to e$	R5	$e \to \dot{q} \to q \xrightarrow{-} \dot{m} \to m \xrightarrow{-} \dot{e} \to e$

Table 3.2. Characteristics of feedback loops for two accumulation regimes in NM-2

Feedback loop	Number for a specific regime depending on relation of parameters	
	Sustainable development for $y > e_1 > 0$	Aggravation mode for $e_1 > 0$ and $e_1 \geq y \geq 0$
1st order	3	3
negative	3	1
positive	0	2
2nd order	3	3
negative	1	1
positive	2	2
3rd order	2	2
negative	1	1
positive	1	1
Sum total	8	8
negative	5	3
positive	3	5

Corollary 3. The stationary growth rate of total (gross and net) resource rent equals the stationary growth rate of total profit. So the Hotelling rule is not the necessary prerequisite for this property. The same stationary growth rate of each of these indicators equals also the stationary growth rate of net output P that is higher than the quasi-stationary growth rate of net output attainable in NM-1: $d > \hat{P}_b$.

Corollary 4. The stationary *gross* rent rate is $\frac{\beta d}{y - e_1}$, stationary *average* rate of return on produced fixed assets is $\frac{\alpha d}{c}$, stationary *general* rate of return on produced fixed assets and proved reserves is

$\frac{d(\alpha+\beta)}{c+y-e_1}$, stationary *biased* profit rate is $\frac{(\alpha+\beta)d}{c}$. Similarly, stationary ratio of proved reserves to depletion is $\frac{F_e}{Z_e}=\frac{P_e}{q_e}\frac{1}{e_eP_e}=\frac{1}{q_e e_1}=\frac{y-e_1}{de_1}$.

Corollary 5. The private owners of proved reserves are worse-off in terms of rent rate that is restricted in NM-2 and unbounded in NM-1.

Corollary 6. The stationary *general* rate of return on produced fixed assets and proved reserves $\alpha d/c$ equals both stationary *average* profit rate αm_e and *gross* rent rate βq_e if the stationary gross unit addition to proved reserves $y_e = e_1 + c\beta/\alpha$ where $c/\alpha \le 1 - e_1/\beta$ as $y_e \le \beta$. Then stationary ratio of proved reserves to depletion is $\frac{F_e}{Z_e}=\frac{y-e_1}{de_1}=\frac{c\beta}{\alpha de_1}=\frac{k\beta}{de_1}$ whereas the stationary absolute rent is zero.

This idealised *rule of equivalent rates of return* to fixed assets and to proved mineral reserves substitutes in NM-2 for $y > e_1$ the Hotelling rule characterized by the current capital gain on mineral deposits being equal to the rate of return on fixed assets.

Corollary 7. The stationary *average* profit rate in NM-2 is higher than quasi-stationary *average* profit rate in NM-1 under the Hotelling rule: $\alpha m_e=\frac{\alpha d}{c}>\alpha m_H=\frac{\alpha d(1-\alpha-\beta)}{c(1-\alpha-\beta)+\alpha\beta}$.

Corollary 8. The stationary *general* rate of return on produced fixed assets and proved reserves $\frac{\alpha d}{c}$ for $y = y_e$ in NM-2 is lower (higher) than the quasi-stationary *general* one in NM-1 under the Hotelling rule $(\alpha+\beta)m_H=\frac{(\alpha+\beta)d(1-\alpha-\beta)}{c(1-\alpha-\beta)+\alpha\beta}$ if accumulation rate k is higher (lower) than $\frac{\alpha}{1-\alpha-\beta}$ in equation (1.17). These featured *general* rates of return are equal if $k=\frac{\alpha}{1-\alpha-\beta}$.

Consider a partial negation of the rule of equivalent rates of return in Corollary 6.

Corollary 9. The Marx notion of absolute rent as a consequence of monopoly of private property on natural resources can justify the real tendency of *gross* rent rate βq to be higher than *average* profit rate αm over historic periods – at least under free competition capitalism. In the regime of asymptotic economic growth in NM-2, private owners of proved mineral reserves are interested in $y < y_e$ for achieving $\beta q > \alpha m$ in the long term. The capitalist class as a whole is interested in $y < y_e$ for achieving *general* rate of return on produced fixed assets and proved reserves $\frac{\alpha+\beta}{s+f} > \alpha m$ in the long term.

The economic efficiency is mostly improved in NM-2 for $y > e_1$, compared with NM-1 for $b < b_c$. Even the net total addition to proved reserves $Y - Z$ (and more so – gross total addition Y) is higher in NM-2 than the resource rent βP in NM-1 except the very beginning of the new investment policy. The output per worker a, the wage w and the profit αP become higher and higher in NM-2 than their counterparts in NM-1. Therefore the workers, the capitalists and the private owners of proved reserves are better-off in the absolute long terms owing to the new investment policy that fosters development of economic potential. A welfare-enhancement through government policies is considered in (Ryzhenkov 2000, 2005, 2007 and 2010).

Corollary 5 and Corollary 8 of Proposition 3.2 above reflect the certain advantages for private owners of proved mineral reserves and capitalists in NM-1 compared with NM-2. Still these advantages are not viable in NM-1 in the long run as the assumed zero limit of proved mineral reserves-output ratio f is not feasible. The intended negative difference $y - e$ for $y = 0$ and e falling to zero with progress of time in NM-1 would be an utterly dangerous practical recommendation for the society as a whole. We shed more light on this critical point in subsection 3.3.

3.2. A specific intensive form of NM-2 for $Y > 0$

Equation (2.6) for $b = -n$ implies that the ratio of proved mineral reserves to fully employed labour force remains constant. Substituting equation (1.18) by equation (2.15) modifies an intensive form of NM-2 as a system of three nonlinear ODEs:

$$\dot{e} = [\alpha cm + \beta n + (1 - \alpha - \beta)d]\,(e_1 - e), \tag{3.1'}$$

$$\dot{m} = [-(1-\alpha)cm + \beta n + (1 - \alpha - \beta)d]m, \tag{3.2'}$$

$$\dot{q} = \{[\alpha cm + \beta n + (1 - \alpha - \beta)d] - n\}q. \tag{3.3'}$$

Proposition 3.3. System (3.1')–(3.3') has no non-trivial stationary state. (The easy proof is skipped.)

Proposition 3.4. System (3.1')–(3.3') has formally the quasi-stationary state that is alien reality.

Confine ourselves to the explanation. For the quasi-stationary state and its local surroundings there are inequalities for $\hat{e} \to 0$: $\hat{P} > \hat{F}$, $\hat{q} > 0$, $\hat{Z} = \hat{e} + \hat{q} + \hat{F} > \hat{F}$ and $\hat{Y} > \hat{F}$. After the expiration of the limited time period, inherent real world relationships (3.8) and (3.9) are violated

$$Z \leq \zeta_1 F, \tag{3.8}$$

$$Y \leq \zeta_2 F \tag{3.9}$$

where $0 < \zeta_1 < 1$/year and $0 < \zeta_2 < 1$/year. These parameters express some maximal attainable velocities of transformation of natural resources into flows and back in the expanding economy.

3.3. Aggravation mode of environmental-economic reproduction in NM-2

The reader will see that the aggravation mode can be brought about by the very incentives for capital accumulation as its unintended consequence. Indeed, according to equation (3.4) and Corollary 4 of Proposition 3.2 in subsection 3.2, the stationary gross and net rent rates as well as the general rate of return to fixed assets and proved reserves are the higher, the lower is y for $e_1 = const$ or the higher is e_1 for $y = const$, or the narrower is the positive gap between y and e_1.

Thus NM-2 with all its shortcomings realistically reveals why the capitalist class as a whole (particularly, its group of private owners of proved reserves) is interested in at least retarding the tendency of unit depletion of proved reserves e to fall and for what material reason these large social groups prefer that both unit addition to proved reserves y and stationary proved reserves-depletion ratio F_e/Z_e are as low as possible. The material interests grasped in NM-2 (and deeper in HL-1) maintain the real global tendencies of total mineral resource use Z to increase and consequently only the partial decoupling of economic growth from the resource throughput.

Moved by their own objective interests the capitalist class as a whole (particularly, its group of private owners of proved reserves) strives to a rather small *positive* margin of y over e_1 in the long term. Due to under-shooting, this margin is ever *negative* in the basic NM-1 instead, wherein $y = 0 < e \to e_1 = 0$

for $t \to \infty$, so $y - e < 0$ – the "neoclassical" recipe for unintentional macro disaster! HL-1 and consequently NM-2 help to anticipate or perceive this problem contrasting with NM-1.[16]

How important is such a difference between positive and negative small quantities in real life everybody knows. For example, compare own feelings and real consequences of coming 30 seconds before with those of arriving 30 seconds after the termination of boarding on an airplane not destined to crash.

It is rewarding to take a breath and look back before the final ascending. The equations (1.11a), (1.16), (1.16a), (1.16b) in HL-1 and NM-2 have been derived assuming that net output P is increasing in the long run. Their application for declining net output P in aggravation mode is exploratory – it has not received a strong empirical support so far. Still the results below are not qualitatively different if the constant e_1 is substituted by e_2 such that $e_0 > e_2 > e_1$ for $\hat{P} < 0$ in these equations.

Aggravation mode could be ruled out instead if $e_2 > e_0 > e_1 > 0$ for $\hat{P} < 0$. Still then the mode of accumulation would be unfeasible (as in NM-1) since unit depletion $e \to 0$, proved reserves $F \to 0$ and output-proved reserves ratio $q \to \infty$ for $t \to \infty$ (an exposition of supporting simulations is skipped).

Figure 3.2, Tables 3.1 and 3.2 summarize information on feedback loops bringing about unsustainable regime of capital accumulation in NM-2. For brevity, consideration of unsustainable regime with $e_1 > 0$ and $0 \le y \le e_1$ in this model is restricted to its special case with $e_1 > y = 0$. There is no loss of generality in propositions and their proofs.

Proposition 3.5. Let inequality $Z > Y = 0$ is fulfilled. Then system (3.1)–(3.3) does not have a nontrivial stationary state. An elementary proof by *ruductio ad absurdum* is skipped.

Proposition 3.6. Let $Z > Y = 0$ in the system (3.1)–(3.3). Then aggravation mode sets in leading *very close* to full exhaustion of proved mineral reserves and *almost* to cessation of production within a limited period of time (up to metaphoric doomsday t_c, $t_0 < t_c << \infty$).

The proof is restricted here to the decisive moments. When $Y = 0$ the ratio of output to proved reserves q increases over time, but the plunge of net output P is accelerating when the first and second derivatives of its growth rate are negative $\dot{\hat{P}} < 0$ and $\ddot{\hat{P}} < 0$. Recall that excessively strong decline does not happen in NM-1 for $b > b_c$ and $y = 0$ since $\dot{\hat{P}} < 0$ is accompanied there by $\ddot{\hat{P}} > 0$. Equations (1.16a) and (1.16b) from subsection 1.2 illuminate the following consequences of Proposition 3.6.

Attributes "*very close*" and "*almost*" reflect the objective fuzziness of Proposition 3.6. According to the author's experience, a simulation run is always interrupted by floating point error computing.

Denote the time step in the integration algorithm as Δ. A repeated run must be stopped at $t = t_c - \Delta$ when $Z_{t_c-\Delta}\Delta > F_{t_c-\Delta} > 0$. Then the boundary of maximal velocity of transformation of proved reserves F (stock) into the depletion Z (flow) is not violated because $F_{t_c-\Delta}$ results from difference between F_0 and accumulated depletion over the segment $[0, t_c - \Delta]$ in the contracting economy. The exact equality $F_{t_c} = 0$ cannot be reproduced in simulations due to inevitable computational errors tending to build up when t moves closely to t_c.

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[16] Some authors have argued that owners, especially in the oil industry, overestimate their proved reserves on purpose. The paper limits do not allow considering this intriguing aspect that if true strengthens the real danger of under-shooting in y relative to e_1. This aspect could become a subject of modelling too.

Corollary 1. For $t \to t_c - \Delta$, $F \to F_{t_c-\Delta} << F_0$, $P \to P_{t_c-\Delta} << P_0$, $Z \to Z_{t_c-\Delta} < Z_0$, $e \to e_{t_c-\Delta} > e_0$ and $K \to K_{t_c-\Delta} > K_0$ [year].

The next two corollaries emphasise two forms of market failure. Both of these forms are manifestations of the strengthened importance of absolute rent in aggravation mode owing to the barrier for capital movement raised by monopoly of private property on natural resources that interferes with the competition of capitals.

Corollary 2. The aggravation mode amplifies disparity between the mounting rate of growth of rent rate $\hat{q}$ and decreasing profitability αm thus violating the Hotelling rule: for $t \to t_c - \Delta$, $\hat{q} \to \hat{q}_{t_c-\Delta} >> \hat{q}_0$ and $\alpha m \to \alpha m_{t_c-\Delta} < \alpha m_0$. So the market fails tremendously – much stronger than in NM-1 for the same initial conditions and magnitudes of the common parameters wherein $b > b_c$.

Corollary 3. Part 1. The aggravation mode amplifies disparity between the mounting rate of gross rent rate βq and decreasing profitability αm thus violating the *rule of equivalent rates of return* to fixed assets and to proved mineral reserves: for $t \to t_c - \Delta$, $\alpha m_{t_c-\Delta} << \beta q_{t_c-\Delta}$.

Corollary 3. Part 2. The shares of absolute rent and differential rent in total gross rent are, respectively, $1 - \alpha m/(\beta q)$ and $\alpha m/(\beta q)$. For $t \to t_c - \Delta$, the former approaches very closely 1 whereas the latter falls almost to 0.

Corollary 4. Rent per unit of the resource extracted $R/Z = \beta / e \to \beta / e_{t_c-\Delta} < \beta / e_0$ contrary to the increasing rate of gross rent and similar to decreasing profitability αm (Corollary 2).

The peculiarities of feedback loops shed more light on the roots of aggravation mode. Call to mind that there are only three first order feedback loops – two negative and one positive – in NM-1.

The first regime in NM-2 for $y > e_1 > 0$ satisfies requirements of sustainable development. The three negative first order feedback loops of all three phase variables e, m and q play the decisive role in stabilization of balanced economic growth.

The second regime in NM-2 for $e_1 > 0$ and $0 \le y \le e_1$ is the source of an apocalyptic aggravation mode. The economy dies within a restricted period of time.

For explanation, first, recall the embryonic hyperbolic element in the stationary output-proved reserves ratio q_e given by equation (3.4); second, define the Jacoby matrix for the system (3.1)–(3.3). There are, in particular, two key partial derivatives on its main diagonal that are initially positive or become positive later in time depending on the initial conditions:

$$\frac{\partial \dot{e}}{\partial e} = \beta q(e - e_1) - \hat{P}, \tag{3.10}$$

$$\frac{\partial \dot{q}}{\partial q} = \hat{q} + (1 - \beta)(e - y)q. \tag{3.11}$$

Consequently, in agreement with Figure 3.2 and Table 3.1, two first order feedback loops of phase variables e and q are positive (R1 and R2); moreover, partial derivatives $\frac{\partial \dot{e}}{\partial e}$ and $\frac{\partial \dot{q}}{\partial q}$ contain vehemently increasing hyperbolic terms $\beta q(e - e_1)$ and $(1 - \beta)(e - y)q$, respectively, implying hyperbolic growth of variables e and q in the long term.

According to equation (3.10), when $t \to t_c$ almost vertical decline of net output P promotes e ($-\hat{P} >> 0$), whereas steady increases in $\hat{q}$ benefit q additionally in agreement with equation (3.11). Besides that the second order positive feedback loops R3 (of e and q), R4 (of e and m) together with the third order positive feedback loop R5 (of e, q and m) reinforce even stronger hyperbolic growth of e and q contrasting with catastrophic decline in P and in m together with starvation of the growing labour force N.

Correspondingly, the negative first order feedback loop B1 of phase variable m essentially inherited from NM-1 turns into a fix that fails. The second order negative feedback loop B2 (of q and m), the third order negative feedback loop B3 (of e, m and q) cannot help hereby. The other feedback loops of the second (R4) and of the third order (R5) including variable m are responsible for its excessively strong decline together with similar decline in capital formation $\dot{K}$. Thus, NM-2 involves excessively strong decline in m (for $e_1 > 0$ and $0 \leq y \leq e_1$) in the absence of a hyperbolic character of partial derivative $\frac{\partial \dot{m}}{\partial m}$ contrasting with hyperbolic growth both in e and q that happens substantially due to hyperbolic character of partial derivatives $\frac{\partial \dot{e}}{\partial e}$ and $\frac{\partial \dot{q}}{\partial q}$.

It has been demonstrated already in subsection 2.2 that the traditional decline curve analysis is applicable in NM-1 for $\hat{Z} = -b < 0$. Now the traditional decline curve analysis cannot reflect adequately depletion Z for aggravation mode in NM-2.

We could conduct a partial simulation for the specific intensive form of NM-2 with $y = 0$, $q = const$ and $e = const$. In this case, the set of feedback loops would be restricted to B1. Then m would decline logistically, instead of hyperbolically – quite similar to that in NM-1.

Unfortunately, this partial simulation would violate the properties of the production function (2.1): the equalities $q = const$, $e = const$ and $y = 0$ require $\hat{P} = \hat{F} = -eq = const < 0$ that is not possible as explained in subsection 2.2. So such partial simulation would be deficient of logical purity and could not be a solid argument in support of specific hyperbolic decline of m in the unabridged NM-2 with $y = 0$. Still this thoughtful experiment yields the valuable conclusion that the equalities $y = 0$, $q = const$ and $e = const$ cannot be fulfilled together in NM-2.

NM-2 excludes the real possibility of economic decline for $y > e$ in result of over-accumulation of produced capital (Ryzhenkov 2010). Still the produced capital K becomes useless in this model for $e_1 > 0$ and $0 \leq y \leq e_1$. Only the remaining "neoclassical" assumption of full employment precludes mass unemployment of epic scale that is inevitable in reality in aggravation mode. Of course, both starvation of labour force and sharply falling *average* profitability preclude the very existence of the habitual labour market when the society is drawn into barbarism.

Notice that original HL-1 and its later upgraded modifications are free of these logical inconsistencies because full employment and constant distributional shares are generally extraneous to these models.

Conclusion

This paper reveals that the tendency in macroeconomic modelling to stick to single logistic equation or its approximation helps the "neoclassical" school to avoid uneasy questions on endemic systemic risks, tensions and antagonistic contradictions of capital accumulation. Besides that, it uncovers the shaky assumptions of the basic "neoclassical" model presented in the standard textbook for natural resources and economic growth, in particular:

1) an equal absolute rate of change for reducing the unit depletion of proved mineral reserves e and for increasing the output-proved mineral reserves ratio q;

2) a rate of change of proved mineral reserves $\hat{F} = -b$ and proved mineral reserves-depletion ratio F/Z are constant;

3) absence of aggravated environmental-economic reproduction modes, respectively, hardly a fast decline in net output P (less than 0.2% per year in absolute terms in the long term) even with the very rapid reduction of proved mineral reserves F (more than 20% in absolute terms annually).

These assumptions support the revealed properties of NM-1 where

1) the implicit Hotelling rule contradicts the interest of owners of proved reserves in a growth rate of rent rate higher than average (or even general) profitability, so this rule can be fulfilled asymptotically for $b = b_H$ only accidentally or under the pressure of the whole capitalist class not modelled yet;

2) an exaggerated power of the owners of proved reserves relative to the other capitalists manifests itself in the tendency of the rent rate to unbounded growth;

3) unchecked material interests of private owners of proved reserves can slow down economic growth or drive the economy to stagnation and decline that is not in the whole capitalist class' interests.

The strong tension between private owners of proved reserves and other capitalists represents a systemic risk of long run economic contraction in NM-1. Still this model heavily underestimates the danger of the latter and erroneously excludes the real possibility of economic decline for $b \leq b_c$ in result of over-accumulation of produced capital (Ryzhenkov 2010).

This paper critically reviews the implicit "neoclassical" premises and assumes as in (Ryzhenkov 2007):

i) direct proportionality of incremental depletion of proved mineral reserves $\dot{Z}$ to change of net output $\dot{P}$ (ignoring possible asymmetry for $\dot{P} < 0$ against $\dot{P} > 0$ mentioned at the beginning of subsection 3.3);

ii) environmental policy ensures growth of proved mineral reserves F through sufficient investment of gross mineral rent Y in their exploration and development (contrary to Hartwick's rule);

iii) velocities of flows Y and Z transformation into proved mineral reserves F and back are limited in the expanding economy;

iv) the gross and net rent rates are contained within objectively determined limits.

The original theoretical model (HL-1) is based on the Marxian theory of commodity, surplus value and resource rent. It has been tested statistically and experimentally in multiple simulations. The deep alterations of the basic NM-1 and of its close modification are realised within NM-2 that applies several equations of HL-1 (Table 2.1) reflecting development of the proved mineral reserves.

A clear description of investment required for increase of proved mineral reserves involved in social production in NM-2 eliminates the illusion hidden in the basic NM-1 that this increase appears costless as manna from heaven. The new environmental policy fosters steady growth of proved reserves contrasting with the environmental policy grounded on the Hotelling rule for exploitation of exhaustible resources. The rates of gross and net mineral rent are typically bounded in NM-2 for $t \to \infty$ contrary to the unlimited rent rate in the basic NM-1.

The asymptotic equivalence of the *average* profit rate and the gross rent rate can be satisfied in NM-2 for balanced growth. This *rule of equivalent rates of return* to fixed assets and to proved mineral reserves substitutes in NM-2 for $y > e_1$ the Hotelling rule.

Still this newly defined requirement can be fulfilled asymptotically only accidentally or under the pressure of the whole capitalist class for $y = y_e > e_1$ not modelled yet in NM-2 as it may happen with the Hotelling rule in NM-1 for $b = b_H$. The market typically fails to satisfy both because of the barrier for capital movement raised by private ownership of proved reserves for obtaining absolute rent as a transformed form of surplus value and simultaneously a part of surplus product. This part of surplus value is withdrawn from equalisation of gross rent rate and average profitability in favour of private owners of proved reserves. There is typically stronger congruence between material interests of owners of proved reserves and those of other capitalists in NM-2 than in NM-1.

The balanced growth and sustainable development in NM-2, unlike NM-1, are asymptotically achievable. Still capital accumulation is fragile at the very edge of distress because of the material interests of the capitalist class: private owners of proved reserves desire to keep a positive net unit addition to proved reserves $y - e_1$ as close as possible to zero for maximizing the gross and net rent rates, the capitalist class as a whole is interested in a minimal positive net unit addition to proved reserves $y - e_1$ for maximizing the general rate of return to fixed assets and proved reserves.

The Marx notion of absolute (ground and mining) rent, not mentioned in the textbook, as an outcome of monopoly of private property on natural resources can justify the real tendency of *gross* rent rate to be higher than *average* profit rate – at least under predominantly free competition capitalism (that is not without barriers interfering with the competition of capitals). NM-2 reflects some endemic disequilibria in capital accumulation of this type without hyperbolizing offered in NM-1. The original mathematical expressions for absolute rent, differential rent and their shares in total rent are derived in this paper for diverse regimes of capital accumulation.

The efficiency of capitalist reproduction is higher in the modified NM-1 with a constant ratio of proved mineral reserves F to fully employed labour force N than in the initial NM-1 with the steadily declining ratio of the same variables. Still such a constant ratio is hardly possible over the long term in a more realistic setting that precludes reducing the unit proved mineral reserves depletion e almost to zero.

The addition of a constant ratio of proved mineral reserves F to fully employed labour force N into NM-2 infringes important technological boundaries: in this damaged NM-2, the ratios of proved mineral reserves to depletion and to gross addition F/Z and F/Y decrease asymptotically to zero for $t \to \infty$ violating the limitations on the maximum practically achievable velocities of these flows transformation into and from such reserves in the expanding economy. Hereby the unit proved mineral reserves depletion e declines to a positive minimum e_1 unlike the modified NM-1 mentioned above.

This paper specifies conditions for balanced produced capital formation and extension of proved mineral reserves that allows higher growth in output and in output per worker in NM-2 than in the regimes of accumulation in the basic NM-1 and in its modification proposed in the textbook. However, this paper demonstrates at the chosen level of abstraction that sustainable development is not guaranteed. If we assume that proved mineral reserves F decrease in NM-2 because of uncompensated depletion Z while maintaining the above partial dynamic law (1.16) of their unit depletion e, the result is not a slow monotonous economic decline as in NM-1 for $b > b_c$ but a socio-economic collapse (*Zusammenbruch*) within a restricted period of time. The market fails to satisfy the Hotelling rule much stronger in this model for $y \leq e_1$ than in NM-1 for $b > b_c$ within $[t_0, t_c)$.

The unavoidable uncertainty in the magnitude of minimal unit proved reserves depletion e_1 complicates even prudent investment planning directed to keep regularly the difference between y and e_1 as a sensible positive quantity in expanding economy. Indeed, the magnitude e_1 is never observed and it can be estimated only with ambiguity in advance. Consequently, feed-back and feed-forward control over gross unit addition to proved reserves y based on past and expected tendencies would be recommended maintained by the equation (1.12) of HL-1 that is absent both in NM-1 and NM-2.

Yet as experience teaches, because of rational irrationality of investors disregarding essential requirements of feed-back and feed-forward control, y can happen to be lower than e_1 in not far distant future. The analysis in Section 3 demonstrates that the common view postulating there always be an adequate amount of time for corrective actions is unfounded. The exposed material interests of capitalist class, particularly those of owners of proved reserves, create possibility of aggravation mode that can unfold with breath-taking swiftness displayed. Its avoidance requires an overt well-coordinated and preemptive social control. Answering the question whether the latter can be provided by reformed capitalism or by a radically higher economic form of society goes far beyond the scope of this paper.

The presented results are products of the original system dynamics models contrasting with the traditional decline curve analysis with its separate rate-time equations (as in Arps 1945 and Höök 2010). This application of the system dynamics method, based on the substantial set of feedback loops and properties of partial derivatives, simultaneously transcends and extends the traditional decline curve analysis in interpretation of excessively strong decline.

The "neoclassical" models presented in the textbook grant no serious hitches, yet they mask the ditches. For shortening the distance between NM-1 and reality its substantial upgrading is required. The latter should at least, overcome both the infinite growth of the output to proved mineral reserves ratio q and infinite reduction of their unit depletion e to nearly zero especially for declining output.

Extraction of minerals brings about directly or indirectly environmental damage of different kinds that is still not modelled explicitly. The advanced specifications of aggravation mode in this paper will, possibly, assist to elaborate laws of motion for disaggregated proved mineral reserves in upgraded modifications of HL-1 with proper attention to the ecological aspects.

The neglect of the Marx theory in the standard courses on "Advanced macroeconomics" is detrimental for scientific education. The system dynamics is the powerful instrument for overcoming this break.

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