Mathematical modeling and computational semiotics: methodological approach to formalization of semiotic concepts

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Abstract

This paper aims to conduct a research on the state of the art of artificial intelligence techniques to investigate the relationships between cognitive actions addressed in steps of mathematical modeling and computational semiotics activities. It also briefly reviews the main techniques of artificial intelligence, with particular emphasis on intelligent systems techniques. Such analysis use semiotic concepts in order to identify the use of new techniques for modeling intelligent systems through the integrated use of mathematical and computational tools. At last, once understood that semiotics can bring contributions to the study of intelligent systems, a methodology for modeling computational semiotics based on the semiotic concepts formalization extracted from the semiotic theory of Charles Sanders Peirce is proposed.

Keywords: intelligent systems, mathematical modeling, computational semiotics.

1 Introduction

The consulted literature points out that in the last two decades there has been a rapid growth in interest in research and applications of the techniques of 'computational intelligence'. It is worth noting that the term 'artificial intelligence' was motivated to distinguish these researches from those surveys called 'classical artificial intelligence', which emerged in the mid 1950s as a study proposal for the development of machines capable of using language and perform tasks as human beings.

At the end of the 1960s the first theorem provers who developed expert systems in 1970's came up. Technology became commercial in 1980 with the so-called "shells" of expert systems.

In the area of computational intelligence, techniques such as fuzzy logic and systems, artificial neural networks and evolutionary computation (genetic algorithms, among others) have been studied, giving significant contributions to the understanding of human intelligence nature.

It is noteworthy that in parallel, the humanities have also sought a model for intelligence and intelligent behavior, given that in general, human beings have some cognitive disabilities that hinder the understanding of intelligent systems functioning and many of these difficulties stem from linear and mechanistic thinking characteristic of Western education.

More recently, partial aspects of intelligence, such as reasoning using vague or incomplete knowledge, learning and prediction have been studied in the area of computational intelligence.

According to the consulted literature intelligence is an inherently human capacity. It is said that is the characteristic that distinguishes us from other animals. Often, the word "intelligence" or the adjective "intelligent" are used to value; any product that contains some degree of automation.

However, for the purposes of this research, an intelligent system can be seen and studied as a semiotic system, where the processing of signs can be seen as the source of intelligence displayed by the system. The intelligence of this system, therefore, will depend on the amount and types of signs that it is able to process. Currently, the mathematical modeling of these systems is a major focus of interest of many researchers studying the interaction between semiotics and intelligent systems.

In this respect, it is worth highlighting the study of Peirce (1976) to design a semiotic philosophy based on universal categories of perception and thought. For Peirce (1976) thinking is an operation that takes place exclusively through signs.

In this sense, Peirce (1976) agreed in saying that "something is a sign only because it is interpreted as a sign of something by an interpreter." Therefore, it can be said, and are routinely references to these words, one does not have a thought, but if you're in thought - the thought is not an object but a semiotic process, which means that is semiosis.

For Peirce (1976) semiosis (sign action) is an irreducibly triadic phenomenon (indecomposable three-term relation) that relates a Sign (S) to its object (O) for an Interpretant (I), or effect on an interpreter . The sign is determined by the object relatively to the interpretant, and determines the interpretant in reference to the object, so as to produce the interpretant to be determined by the object through the mediation of the sign.

According to Peirce (1976) sign is something that produces in the mind of the interpreter the same idea (interpretant) that would be produced by something else (object), if it were presented to the interpreter. The sign is formed by "object" (anything, feeling, event that can generate an idea in the mind of the interpreter), "interpretant" (an idea in the mind of the interpreter) and "meaning" (that which is passed to the interpreter by the sign when causes the generation of the interpretant in the mind of the interpreter).

The "interpreter" is the key element to the understanding of a sign processing and therefore the operation of an intelligent system. While designating an object, the interpreter provides an assessment of the object significance and the smart system predisposes to an action that corresponds to a reaction to the cognition of the sign, interpreted within the context. These three tasks of the interpreter are highly interconnected.

To Silveira (apud Oliveira, 2012), what Peirce does, through its semiotic conception, is to promote the essential integration of logic in the context of the experience, giving it as an object, not merely ideal forms, as are the objects of mathematics. The signs, however, are manifest phenomenologically as thought.

Another aspect to be highlighted is that the Semiotics studies the levels of meaning, that is to say, studying the other direction, the implicit meaning, or the meaning behind the words, or beyond words, different fields of semantic structure, since it is static, while the semiotic significance levels are dynamic and variable.

It is known that the meaning as semiotic function, that is, dependency relationship between a content plan and the expression plan, is an intrasemiotics relationship, which therefore cannot be transcoded, whereas the information as a set of operated cultural clippings about the semantic amorphous continuum can be treated by any code filter. (Hjelmslev, 1975).

Therefore, the interlingual transcoding of a message does not reflect a meaning, but a designatum, a sense. What can be translated is the substance of the content, not the form of it, which is why it is up to us to ask under what conditions two words, two statements, and even two texts belonging to two different languages, are semantically equivalent to the point we can say that both are mutually translate.

Thus, the content of designative dimension is a coded representation of the interpreted object. The exact determination of this object, that is, the interpretation of the designative dimension of the sign will depend on the relationship between the sign and the object.

From this perspective, it is possible to realize that the translation of a text into another language is a process of equivalence in a conceptual level:

$$m_1 \rightarrow LN_1 \rightarrow Co \rightarrow LN_2 \rightarrow m_2 \text{ where}$$

m1 = mensagem em língua natural 1
m2 = mensagem em língua natural 2
LN1 = língua natural 1
LN2 = língua natural 2
Co = conceptualização

Source: Pottier (1974).

Therefore, the speaking subject initially decodes the m1 message in terms of the LN1 natural language, thereby arriving at a conceptual schema, which is then encoded in the LN2 natural language, resulting in a new m2 message, m1 translation.

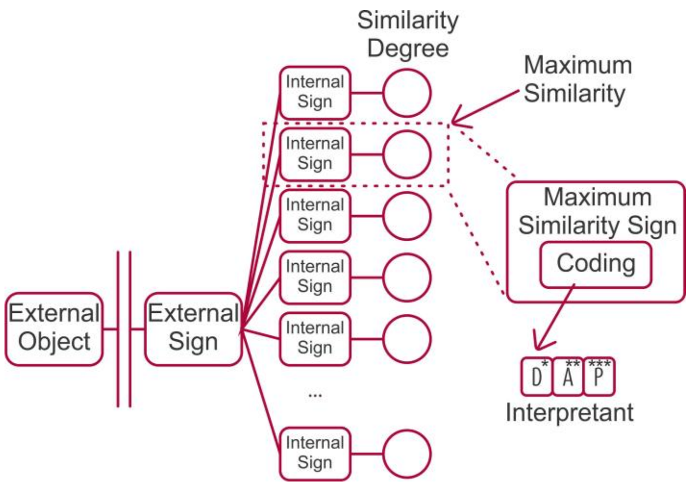
$$m_1 \rightarrow LN_1 \rightarrow Co_1 \rightarrow Co_1 \rightarrow LN_2 \rightarrow m_2$$

Source: Pottier (1974).

The entry of a sign in the system corresponds to an encoding of the information from the external world by means of sensors. This encoding is then compared to the coding prototypes of objects that match the intelligent system vocabulary, and the internal sign that is most similar to the input sign is called the sign interpretant.

To Gudwin and Gomide ([1996?]) the entry of a sign in the system corresponds to an encoding of the information from the external world by means of sensors. This encoding is then compared with the coding prototypes of objects that match the vocabulary of the intelligent system, and the internal sign that is most similar to the input sign is called the sign interpretant. Note that this interpretation is given here only in level of designative dimension. This mechanism can be seen in the following figure:

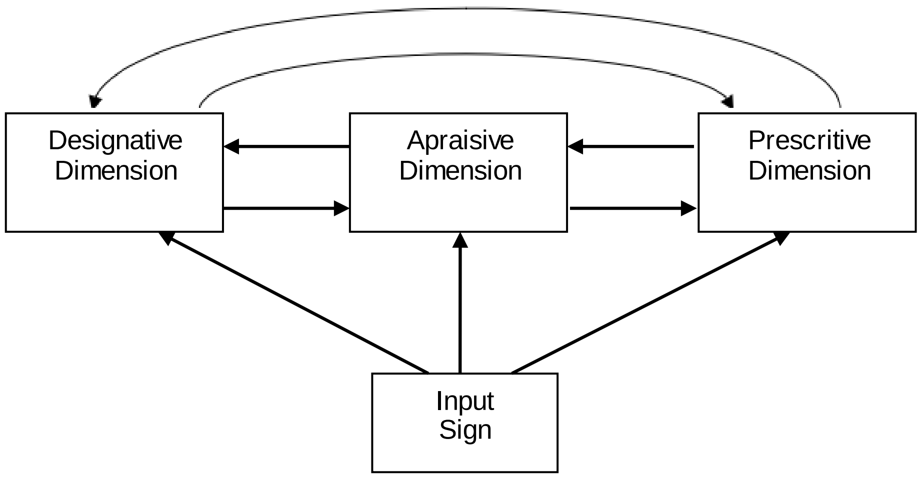
Figure 1: Input of a sign in the system



Source: (Gudwin; Gomide, [1996?])

Another interesting observation according to Gudwin and Gomide ([1996?]) is that the whole interpretation process is dynamic. That is to say, the box which is called external sign can be considered as such, if the interpretation occurs effectively. The intelligent system is continuously flooded with system data, and only certain data combinations are significant.

Figure 2: Interrelationships between the dimensions of a Interpretant



Source: (Gudwin; Gomide, [1996?])

Given these considerations, it is important to note that today, the semiotic earned dimensions that exceed the human field and can be seen as a research area that extends the semiotics of architecture, biosemiotics or cartosemiotics to zoosemiotics, and may thus be defined in a very general way as the science of signs and signification processes (semiosis) in nature and in culture.

In short, it is in this context that is presented the work developed in this paper to propose a methodology for mathematical modeling based on the formalization of semiotic concepts from the semiotic theory of Peirce (1976) drawing upon the bibliographical review to support the theoretical model. This methodological approach allows the use of new techniques for modeling intelligent systems through the integrated use of mathematical and computational tools.

2 System dynamics: methodology for investigating cognitive actions addressed in steps of modeling activities

To achieve the objective of this research, we adopt the methodology of "system dynamics" created by Jay Forrester in the 1950s. This methodology is based on the General Systems Theory (Bertalanffy, 1977) and uses computer simulation to relate the structure of a system with its behavior over time (Forrester, 1961). Currently it is used in various research centers, and various software structures that comprise the system dynamics have been developed.

Through such software, it is possible to analyze the behavior of complex systems, including all relevant relationships of cause and effect, delays and feedback loops. Originally the methodology was used in the industrial environment, but afterwards applications have been identified in other areas of knowledge, such as physics, biology, social sciences and ecology (Forrester, 1971).

To Sousa (2006) the dynamic systems (DS) are a rigorous modeling method that uses computer simulations to define more effective organizations and policies. Such tools in the design of Sterman (2000) allow the creation of management simulators - Virtual worlds where space and time can be compressed and decelerated so as to allow experimentation with long term side effects, learning, and structures design and high performance strategies.

In the context of cognitive science, van Gelder (apud Oliveira, 2012) provides the dynamics hypothesis of cognition as opposed to the computational hypothesis, stating that cognitive agents can be: 1) considered as dynamical systems, ontologically, 2) described as dynamic systems. Thus, the dynamic perspective of cognition inaugurates a third paradigm in cognitive science, artificial intelligence beyond.

According to Loula (2004), dynamic systems are systems whose state (or instantaneous description) changes in time. The state of a system at a given instant of time is the instantaneous description of him, needed to determine the values of its internal variables. The state in the following instant depends only on the state on its current instant, without needing the previous states.

The space formed by all possible system states is defined as the state space of the dynamic system. But beyond the system state, its inputs are also needed to determine their future status and from these two facts, the rule of system evolution will determine the future state.

More formally (Beer, 2000), states that a dynamical system can be defined as a triple <T; S; _t> consisting of an ordered set of time instants T, a state space S and an evolution operator _t: S! S, which transforms the state xt1 _ S _ T in time t1 in the state xt2 _ S _ t2 to time T. The space S can have finite or infinite dimension, can be numeric or symbolic, if numeric can be continuous or discrete.

The time T can be discrete or continuous. The _t rule can be defined explicitly or implicitly, can be addressed by time or events, may be linear or nonlinear, deterministic or stochastic, autonomous (not time dependent) or not autonomous. Models examples of dynamical systems are differential equations, cellular automata and finite state machines.

Among dynamical systems, a class of systems has gained increasing attention: the complex dynamic systems. But a precise and consensual definition of them has not been obtained yet, containing several authors, elaborated proposals to define this class of systems. It is still possible to define them with less formality by some common characteristics.

First, complex systems are systems composed of a large number of different interacting elements (Weisbuch, 1990). Interactions between components are the effects that one component causes on the other and in the system, that is, changes in the state or structure of components or system.

The relationships established by these interactions are responsible for the system characterization, they cannot be ignored or overlooked, preventing that the system could be decomposed, without being mischaracterized.

This impossibility of reducing the system to its components is a consequence of the nonlinearity of the interactions, the effects (in the system) are not the simple sum of the causes (on the components).

The interactions are necessarily circular processes (Bresciani; D'ottaviano, 2000), where the interaction effects are its own causes, retroactive effects on causes.

As examples of complex systems, we can mention the human brain, consisting of billions of neurons, interacting electrochemically by synapses; computational systems, constructed with a large number of electronic components such as transistors and logic gates; social and economic systems, obviously composed of several components; and language.

When the complex system has the ability to modify its structure and dynamics, either by his action and behavior or evolutionary change, it is called a complex adaptive system (Complex Adaptive System - CAS).

When the system adapts autonomously, it gets the name of self-organizing system.

Kelso (apud Oliveira, 2012), postulates that every dynamical system displays order (or collective variables) and control parameters. Very briefly in, it is possible to place an order parameter as a behavior that emerges in a dynamic system and, once established, shall direct the behavior of the system itself, in a kind of circular causality.

An order parameter can be described as a macro property or an emergent property of the interaction of the elements of a system. A control parameter is a condition in which, or the state in which a collective variable emerges, and it is not dependent on another state or condition and can be quite specific and not caused by external events.

Therefore, in the case of dynamical systems, their behavior and properties can change due to particular events (control parameters) which result in settings or emergent behaviors (order parameters) that start to drive or restrict the system itself, in its behavior.

3 Dynamical and self-organizing systems

Since its initial formulation, the concept of self-organization is continuously used in several theoretical proposals in many areas of knowledge, ranging from computational modeling to philosophy. It is worth mentioning that self-organizing systems are complex systems in which global standards are produced through local interactions, without central or external control. Global information can be used to enforce global constraints to the system, although they do not act directing the system to 'how' it must reach a state of order.

The concept of order is the opposite of entropy, for being a process by which a system tends to exhaustion, disorganization and disintegration, and finally to death. In this sense, an orderly system has invariance, redundancies, the freedom degrees of the system are restricted (the responsible parameter is called the order parameter). Self-organizing Systems – SOS cannot be seen like isolated systems, not dependent on the environment, which they constantly adapt their dynamics. Self-Organizing Systems – SOS examples are found in various social, economic, physical, biological and chemical areas.

Self-organizing systems - SOS have characteristics that distinguish them from conventional systems, such as global order, local interactions, positive and negative feedbacks, order from noise, nonlinearity, distributed control, robustness, lock, emergency, unpredictability. The components of 'lower level' interact, subject to local restrictions, spontaneously creating a global configuration ordered.

The dynamics of Self Organizing Systems - SOS is strongly based on mechanisms of positive feedback and negative feedbacks, and in a circular relationship in which each component affects the others, and is affected by others, non-linearly. Positive feedback amplifies fluctuations exploring new settings, while negative feedback stabilizes the system to reduce deviations in the system state. This keeps the system to the 'edge of chaos', between balance and chaotic activity.

The dynamics of self-organization of the system depends on fluctuations or noise, so that the system can be moved from its current state and eventually leading up to a new state of order. The noise source can be internal, generated by the system itself, or external, from the environment. The feedback loops make them robust and resilient SOS, since the deviations can be suppressed, bringing the system back to an original ordered state.

The robustness of Self organizing systems - SOS, which is characterized by its fault tolerance comes from the distributed control between the system components, through which he self corrects its behavior when its unspoilt parts recompose the activity of non-functional parts. Although not subjected to central controllers, SOS components should be observed as belonging to a coherent whole, and self-sufficient, cannot being analyzed in isolation.

Another feature of SOS is its unpredictability. It is a consequence of the intrinsic non-linearity of the system and probabilistic trajectories that can drive the system from an initial state to any of the various stable states.

Briefly, SOS are formed, in most cases, from various parts that interact by distributed manner, not predictable, non-linear, probabilistic, making it extremely difficult to analyze its parts. These properties suggest that a synthetic approach, rather than an analytical, can be an interesting strategy to study the SOS, and computer simulations play an important role when we want to design, model and experiment SOS.

Because of the difficulty of predicting the behavior of self-organizing systems, computer simulations are a useful tool for conducting 'thought experiments' and to better understand how these systems work (Camazine, 2002).

From this perspective, another fundamental question is 'where does the order come from'? According to the general laws of thermodynamics it seems that the dynamic processes tend to follow the paths of least energy consumption until the system is able to find a balance where it will remain until he has suffered disruption. There are many examples in nature of systems and organisms that present high energy and internal organization in apparent defiance of the laws of physics. Some of them are:

1. Iron filings particles that align along the lines of force of the magnetic field to which they are subjected;

2. Water particles that when suspended in air form clouds;

3. Ants or bees that grow from a zygote to form a complex system of cells which then in turn participates in a highly structured and hierarchical society.

Thus, the organization arises spontaneously from disorder and it does not seem to be driven by known physical laws. Somehow the order arises from multiple interactions between the component units and the laws that may govern this behavior are not well known.

The behavioral perspective of a self-organizing system could reveal how spatial and temporal patterns - such as roads, boundaries, cycles and succession - could arise in complex heterogeneous communities. Understanding the mechanisms of self-organization may lead to the construction of more informative and accurate models.

According to Nicolis and Prigogine (1989), it is necessary for a system to satisfy various pre-conditions and make use of various mechanisms to promote self-organization. Such mechanisms are in some way redundant and poorly defined. However, they allow intuitively assessment for the potential of systems' self-organization. They are:

1. Thermodynamics aperture: Firstly the system (a recognizable unit as an organization, an organism or a population) should exchange energy and/or mass with its environment. In other words, there must be a non-zero energy flow through the system;

2. Dynamic behavior: If a system is not in thermodynamic equilibrium, the only option left for its behavior is to assume some type of dynamic, meaning that the system is continuously changing;

3. Local interaction: Since all natural systems inherently have local interactions, this condition seems to be an important mechanism for self-organization and as such should be incorporated into models that represent it;

4. Nonlinear dynamics: A system with bands of positive and negative feedback is modeled with nonlinear equations. Self-organization can occur when there are feedback loops between the system components parts and between these components and the structures that emerge at higher hierarchical levels;

5. Large number of independent components: Since the origin of self-organization lies in the connections, interactions and feedback loops between the systems parts, it becomes clear that self-organizing systems must have a large number of components;

6. General behavior independent of the components internal structure: This means that no matter what or how are made the system components, since they do the same things. In other words, this means that the same property will arise emerging in completely different systems;

7. Emergence: Emergence is probably the least known among the notions that relate to self-organization. The Emergence Theory says that the whole is greater than the sum of the parts and the whole displays patterns and structures that emerge spontaneously from the conduct of the parties;

8. General conduct organized and well defined: Disregarding the internal structure of a complex system and seeing it only as an emergent phenomenon, it is observed that its behavior is quite accurate and regular;

9. Effects at multiple scales: The emergence also points to interactions and effects between multiple scales on self-organizing systems. The small-scale interactions produce the large-scale structures which in turn modify the activity on a small scale.

Based on these assumptions, for purely didactic purposes, we describe the example of Pereira (2000) three possible systems based on the logic of meanings production: 1) the first, considered low complexity, performs its meaning productions without the participation of a creative memory, 2) the second, which admits a median complexity and, just like the first, would be carrying a memory, but in this case already with creative dimensions, although of medium complexity, and 3) the third, with high rates complexity, concierge highly creative memory and can thereby generate meanings rich in diversity, even ambiguous.

From this angle, Pereira (2000) states that differentiated systems as the degrees of low, medium and high complexity possessed some complexity proportional to their degree of complexity memories, varying as to their production of meaning, the mere possibility of recognizing elements which they can interact (recall stored information that enables recognition) to the possibility of furthering their operations with mnemonic content, and this may create new meanings in the face of information with which they interact.

It is also important to observe that the latter system models, by their richness of its variability of semiotic productions can sometimes within the set of self-organization, promote disorders within the system as an effect of over-complexity. Such excess may form themselves into new content for a growing organization in terms of complexity, or in some cases, promote a radical disruption of the system itself.

Thus, the higher the rates of creative memory and complexity, the greater the possibility of meanings produced are rich and varied, organizing, expanding and complexifying the system further. And on the other hand, the excess of significant possibilities of the system itself produces a greater willingness to instability and chaos, which may or may not be the resumption of the eternal game of self-organization.

In this sense, Peirce (1976) advances on the dynamic view. Therefore, closely related to the notion of representation, the role of interpretation is explicit in his semiotics theory. That is, through interpretation a meaning to the sign is joined. In addition, representations are subject to additional representations of operations on the representations, the interpretation that become successively new representations and so on.

Therefore, in designing Nadin (2011) once known objects levels and interpretants, the sign is no longer a synchronic entity, which means, the sign comes to life, in the sense that the process of interpreting injects dynamic to their reality. To Nadin (2011), we never deal with signs, but with representations, aggregates of signs whose dynamic meaning is a function of the context, not the alphabet.

Thus, under this assumption, we can infer that beyond a representation to be able to cause the generation of different meanings, each sign in turn can generate meanings from themselves. And this chain of ideas continues until the receiver is satisfied or until a new chain idea begins. Peirce's semiotics in this chain of attribution of meaning is known as unlimited semiosis.

From this perspective, it is necessary to emphasize that the representation of dynamic and non-linear systems, a properly systemic language should be used, given that our linear language and Cartesian is insufficient. And as language shapes the perception, a new language would bring new ways of thinking that would facilitate the understanding of complex dynamic systems.

Thus, there would be a break with linear thinking, which mirrors our written and spoken language and presupposes cause and effect relations that prevent the perception of situations involving dynamic complexity.

Therefore, if the thesis that dynamic systems and semiotic systems are correlated is relevant, the property of self-organization as a property of dynamical systems should also manifest in semiotic processes.

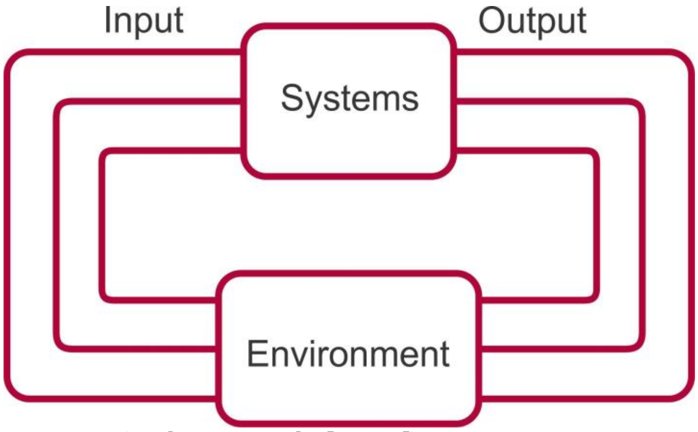
4 Semiotic systems and the systems dynamics

In this paper we postulate that a semiotic system is a dynamic system. The arguments set forth herein is proposed on the assumption that there is a relationship between the concepts of self-organization and the semiotics of Peirce logic from the premises of Gonzalez and Haselager (2003) which established a relationship between the concepts of self-organization and Peirce’s semiotics logic, Peirce's in the study of creativity in natural and artificial systems.

A system receives information from the environment through inputs, and can act in this environment through outputs:

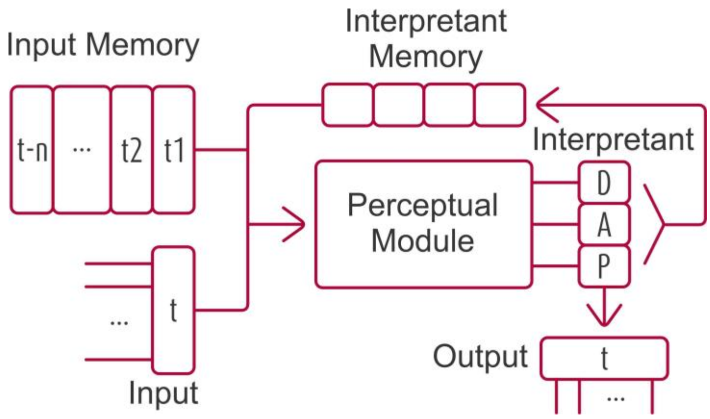
A semiotic system is a system that generates, transmits and interprets signs of different types. This system is subdivided into "iconic, indexical and symbolic semiotic systems." In terms, summarily iconic, indexical and symbolic semiotic systems use signs as means for communication of similarity patterns, correlations, timelines and legal relations. These systems exhibit self-correcting behavior, or some kind of activity driven by a purpose.

Figure 3: System and Environment



Source: (Gudwin; Gomide, [1996?])

Figure 4: Process signic scheme in an Intelligent System



Source: (Gudwin; Gomide, [1996?])

The effectiveness of a sign process corresponds to the execution of an interpretation, that is, a function that takes a particular sign facing a given context, to an interpretant. In some cases, this function can be a simple function. The interpretant is only a function of some coordinates of the input vector, normally finding very complex functions, where the interpretants could fuel the input vector, creating interpretation chains, where a sign refers to an interpretant, which in turn leads to another interpretant and so on.

5 The categories of logical reasoning and adaptive computational methods

The pragmatism of Peirce points to the conceptualization of three categories of logical reasoning: 1) deduction, 2) induction and 3) abduction. Abduction is the process of constructing a hypothesis for the generation of an initial model as an attempt to understand and explain a perceived phenomenon. Induction tests this model as opposed to other factual information and performs the necessary adjustments.

Deduction applies the established model of the observed phenomenon. This model will be used by deductive reasoning until new information will put the credibility of this model at risk, or that the reality represented by this model is changed. At this time, a new process of abduction, induction and deduction starts, where a new model of thinking is established.

Deduction corresponds to deterministic methods, which can present predictable solutions to a problem. The induction relates to the statistical methods, since they have not only a single, but also a range of possible solutions to the same problem as well as abduction relates to adaptive methods, which can be automatically reset and recreated based on new understandings of the problem they are modeling, or on its dynamic change.

Usually, theories of cognitive science rely on inductive-deductive inferential models. However, the originality of Peirce's conception of thought comes primarily by abduction, sometimes called retroduction or hypothetical inference.

6 Semiotic representation systems and mathematical modeling

Although computational representations are similar to the semiotic representations, they are not of the same nature. Semiotic representations are conscious representations, closely linked to the idea that we have anything as a result of the action of the object, while the computational representations are internal and independent representations of the vision that one has regarding the object.

A computational internal representation can be conscious or not, whereas a conscious semiotic representation can be or not externalized.

Every semiotic register of representation is supported on the most important semiotic system. That is, the "natural language". It could be added that the role of this natural language is decisive and structuring on the other records that are part of formal languages.

Normally there are four types of sign systems in which language operates: 1) Semantic systems that constitute the semantic structure of a sentence; 2) semiotic systems that produce the senses by the levels of semiotic significance; 3) logic systems whose senses are processed by meaning levels and the logical relationship, for example, the mathematical operation, and; 4) Symbolic systems.

In this regard, in relation to mathematical language, Peirce (1976) denies that mathematics is a branch of logic by indicating that this knowledge involves a semiotic problem that exceeds the notional subject of writing. In this sense, the consulted literature indicates that no formal language is sufficient enough to entirely sustain from the nature of mathematical doing, since its character is "diagrammatic", which clearly articulates its own internal logic. Therefore, the diagrams are the main, if not the only, way to acquire new information about spatial relationships.

Thus, diagrams and graphs, which represent their objects through the relationships between the constituent parts of them, are semiotic artifacts designed to reveal information about relationships. Although a description of spatial relations can be based on linguistic structures, for example, we know that the best technology is based on the manipulation of lines, arcs, and vertices.

Overall, in mathematics, the aspects related to representation are of great importance, for this reason several authors claim that there is no mathematical knowledge that could be mobilized by a person without the aid of a representation.

However, the existence of multiple semiotic registers of representation for the same object, and the inability to access the objects perceived by the material (requiring a representation) uniquely define their own cognitive activity proper of mathematical procedures, determining their learning.

In his manuscript entitled "The essence of mathematics" Peirce (apud Campos, 2007) presents two definitions of mathematics. Initially, like his father, Benjamin Peirce, he defines mathematics as "the science that draws necessary conclusions". The following defines it as the study of what is true of the hypothetical state of things. These two definitions contain the essence of the concept of Peirce (apud Campos, 2007) in mathematics.

To Hookway (1985) the thought of Peirce (1976) about math is systemic, considering that he sees mathematics as the core discipline in the classification of the sciences, and this fundamental position of mathematics is a result of his method of reasoning. The mathematical thinking proposed by Peirce (1976), which forms a diagram or model of the problem to study and experiment on him, may be employed in any science that is at a lower hierarchical level, and in fact such reasoning is required for lower sciences while pure mathematical reasoning must remain free of the individual methods of the sciences below it.

According to Hookway (1985), Peirce (1976) considers the virtually foolproof mathematical method, producing certain conclusions and pre-logical, in other words, not subject to logical criticism. The mathematical reasoning is a priori in the sense that its objects of study are entia rationis - we create the objects, namely mathematical forms. Their findings are right, even when experimentation and observation are inductive, that is, are not reactive, thus the diagrammatic instances are the objects of study. Namely, the math is not subject to the error than the study of perceived truths introduce in our scientific reasoning, since by understanding the meaning of diagrammatic instance we immediately understand the form of general mathematical relationship being studied. For these reasons, the pure mathematical reasoning is fundamental to Peirce (1976).

From this perspective, numeration systems, geometric figures, and formal algebraic writings, graphical representations and natural language itself, are examples of semiotic representations. In this sense, Almeida, Tortola and Merli (2012) based on Duval (2011) admit the 'semiotic representation' as part of a semiotic system, a system composed of signs.

To Almeida, Tortola and Merli (2012) these different semiotic representations constitute from the use of different languages, and thus are associated with different language games whose meanings are mediated by signs or instruments that represent them.

According to Duval (2008), the use of different semiotic representations, that is to say, the plurality of semiotic systems enables diversification of representations of the same object. This fact contributes to a reorganization of the person's thinking and influences his cognitive activity. Semiotic representations are essential to the understanding of mathematical concepts.

According to Duval (2003), the registers of semiotic representation are characterized by three cognitive tasks: the first is the formation of an identifiable representation, that is, when it is possible to recognize this representation of what it represents, within a system of signs socially established; the second is the treatment, which is a transformation that takes place within one system record, for example, solving a system of equations; the third is the conversion, which is the transformation of representation of a mathematical object into another representation of the same object.

Conversions are transformations of representations that consist in changing registry keeping the same objects denoted: for example, to move the algebraic writing a function to its graphical representation.

According to what Rosa (2009) claims**,** the conversion is generally considered a simple operation, whose finality lies in finding a record in which the treatment is more economical. However, in general it does not happen.

To perform conversions it is necessary to make joints between the cognitive variables that may be specific to the operation of each of the records systems. These are variables which allow determine which units of relevant meaning should be taken into account in each one of them.

There are many factors that influence the success of a conversion, as the phenomenon of congruence, the order of conversion, the record's nature and the knowledge that the student has of the records. Thus, the conversions can be more or less complex, depending on these factors. To deepen the analysis of cognitive activity required for mathematics, the author deepens the analysis in relation to the different registers of semiotic representation, since they are of different natures. Such nature is important in the conversion process of the records.

In this context, Duval (2003) ranks the representation registers in multifunctional, monofunctional and its forms in discursive and non discursive. These records are characterized by their form of treatment: the monofunctional have treatments that can be my made by using algorithms, that is, they are derived and specialized in some kind of therapy and have formal characteristics while the multifunctionals are used in different fields of cultural and social means. They cannot be made by algorithms.

For a representation record to be in the discursive form it needs to allow arguments, deductions, symbolic writing (natural language, numerical system, algebraic) while the non-discursive form is the geometries and Cartesian graphs. For example, writings in natural language are a multifunctional record of discursive representation and a fraction is a monofunctional recording of discursive representation.

According to Duval (2003), in solving a problem one record can appear more privileged than the other, but the important thing is the existence of a possibility of mobilizing at least two records of representation at the same time, or the possibility of constantly interchanging the register and to see, in different registers, the same mathematical object represented.

That is, having the coordination between records. To the author it is the articulation and the coordination of at least two records which constitute a condition of access to understanding in mathematics.

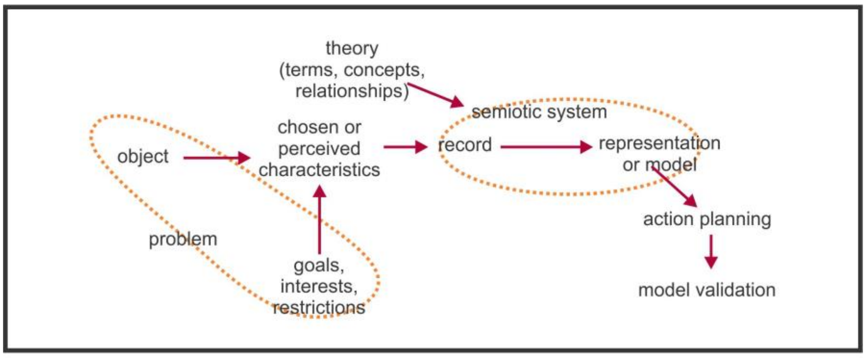
In general, mathematical modeling activities involve several steps. The first begins when the individual is faced with a problem situation that he wants to investigate. Then, the action follows to the identification of the characteristics and variables that directly influence the problem.

The second step is the simplification of variables. Afterwards, the stage where they are introduced to the formal mathematical concepts and notations; this step is the abstraction, which involves the selection of mathematical objects needed to represent the situation under study.

The next step involves the manipulation with the representations of mathematical objects in order to obtain a model. Finally, the last step is the model validation and interpretation of the response found, taking into account the initial problem situation.

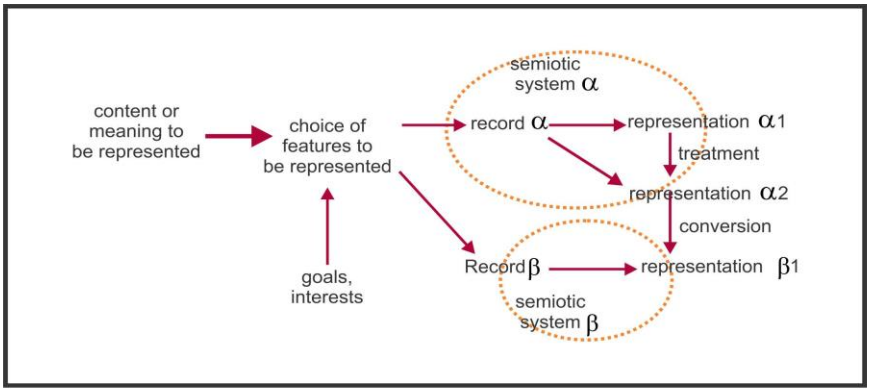
To Bassanezi (2002), more important than the answer to the problem and the mathematical model are the discussions promoted during the development of the activity, both on mathematical objects as on the actual situation itself. The modeling process can be described by the diagram shown in the figure 5 below:

Figure 5: Modeling a problem



Source: Silveira (2005)

Figure 6: Semiotic representations of content



Source: Silveira (2005)

In this sense, for example, we are going to describe here one of mathematical modeling activities that was developed by a group of students from the 1st year of high school, during math classes at a public school in the state of Paraná, Brasil. Which, for example, we extracted from Rose's (2009) text.

Firstly, the group of students sought a topic of their interest. They found an article on the benefits and evils of the use of photocopies, which featured the title "Xerox: a necessary evil." The story relates the millionaire dispute between publishers and copiers. From there, the group decided to research prices of copies in various copiers from the city. Table 7 shows some proposals found by them, although in order to preserve traders, the names of their copiers were changed.

Table 1: Data collected by students on 07/13/2008 in Sarandi, PR.

COPIER	PRICES
Copier A	R$ 0,20 each and R$ 0,15 each, up to 100 copies
Copier B	R$ 0,15 each up to 300 copies; R$ 0,05 for each copy beyond 300 copies
Copier C	R$ 0,15 each and R$ 0,10 each, up to 10 copies
Copier D	R$ 0,20 each and R$ 0,15 each, up to 10 copies
Copier E	R$ 0,10 for any number of copies
Copier F	R$ 0,15 each and R$ 0,10 each, up to 50 copies

Source: Rosa (2009)

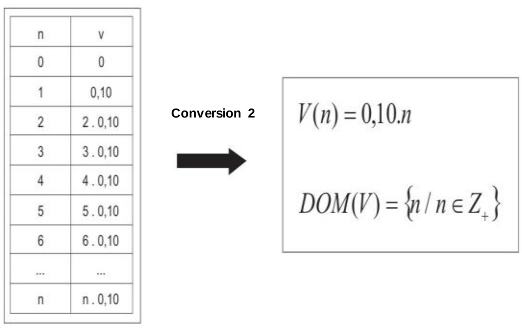
According to what Rosa (2009) argues in analyzing the data, it became clear the curiosity of the group in relation to the copier "B", thus came the problem to be investigated "from many copies, is it financially feasible to use the copier" B "? Since the number of copiers, the group decided to choose only two to be analyzed: the copier "B", for its characteristics, and the copier "E" for having the lowest price for a small number of copies.

This was the step that the group presented more difficulties, supporting Duval's (2003) argument that the congruence phenomenon and the fact that the difference in the nature of records of a conversion determines the degree of difficulty that students face to perform the conversion.

The next step of the activity of mathematical modeling is the formulation of hypotheses. They considered that the growth rate against the number of copies for the two copiers was linear. And so, they considered doing two functions, one to represent each copier, and then they would discover the number of copies that would equalize the price charged by the two copiers.

Rosa (2009) mentions that when observing the formulation of hypotheses by the group, she noted that when they referred to the " representation of a function to each copier", they were referring to the same algebraic record, thus identifying at that time, the mathematical object "function" by their algebraic representation. To obtain the model, they began by the E copier, and set off a record tabular.

Table 2: Conversion between record tabular and algebraic entry – E Copier



Source: Rosa (2009).

From the data in Table 2, the group held a congruent conversion of the registration tab to the algebraic registry because the output (tabular) registry reveals the arrival record (algebraic). The nature of the two records in this case is monofunctional and the two representations are discursive. The difficulty encountered by students at this stage was in relation to the characterization of the function domain, given that they were unaware of the numerical sets. The complete characterization of the algebraic function to copier "E" prices passed to the copier "B", using the same procedures.

Table 3: Conversion between algebraic and tabular log record (for a number less or equal to 300 copies) - "B" Copier.

n	v
0	0
1	0,15
2	2 . 0,15
3	3 . 0,15
4	4 . 0,15
...	...
300	300.0,15

Conversion 3 →

$V(n) = 0{,}15n$

$n \leq 300$

.

Source: Rosa (2009)

In Rosa's (2009) opinion, this first part happened analogously to the previous one, namely a conversion of the registration tab to the algebraic register, congruent, an activity of simple coding records of the same nature. The second part started the same way, first the tab record, then the algebraic tabular record.

Table 4: Conversion between algebraic and tabular log record (for a larger number than 300 copies) – B copier..

n	V
301	300.0,15+0,05
...	...
350	300.0,15+0,05.50
...	...
400	300.0,15+100.0,05
...	...
600	300.0,15+300.0,05
...	...
700	300.0,15+400.0,05
...	...
n	300.0,15+0,05(n-300)

Conversion 4 →

$V(n) = 0{,}05.n + 30$

$n > 300$

Source: Rosa (2009)

For Rosa (2009) that conversion did not happen the same way the previous ones, because the author found that the degree of difficulty was higher, influenced, in her view, by the level of knowledge of students. Therefore, we have:

Copier E: $V_E(n) = 0{,}10\,.\,n \ ; \ \forall\, n \in Z_+$ (1)

Copier B: $n) = \begin{cases} 0{,}15\,.\,n \ ; \ n \leq 300 \\ 0{,}05\,.\,n + 30 \ ; \ n > 300 \end{cases} \forall\, n \in Z_+ \ ;$ (2)

Source: Rosa (2009)

Thus, Rosa (2009) states that: once defined the functions that represent the "E" Copier situation and the Copier "B" situation, the group went to the next step, the validation of models. Numerical records were used, obtained with the development of models and presenting the results in the form of tabular records.

Table 5: Validation of Models

NUMBER OF COPIES	REAL DATA COPIER E IN R$	FUNCTION: V(n):=0.10.n	REAL DATA COPIER B IN R$	FUNCTION:
0	0	0,10.0=0	0	0
1	0,10	0,10.1=0,10	0,15	0,15.1=0,15
2	0,20	0,10.2=0,20	0,30	0,15.2=0,30
...		...	...	...
50	5,00	0,10.50=5,00	7,50	0,15.50=7,50
100	10,00	0,10.100=10,00	15,00	0,15..100=15,00
200	20,00	0,10.200=20,00	30,00	0,15.200=30,00
300	30,00	0,10.300=30,00	45,00	0,15.300=45,00
400	40,00	0,10.400=40,00	50,00	0,05.400+30=50,00
500	50,00	0,10.500=50,00	55,00	0,05.500+30=55,00
540	54,00	0,10.540=54,00	57,00	0,05.540+30=57,00
560	56,00	0,10.56=56,00	58,00	0,05.560+30=56

Source: Rosa (2009)

By comparing the observed values and the values estimated by functions, the group concluded that the two models referring to tables 1 and 2 represent both “E” and “B” copiers’ situation respectively.

To answer the original question, "from how many copies is it feasible to use the "B" copier? the group drew up the table 3.

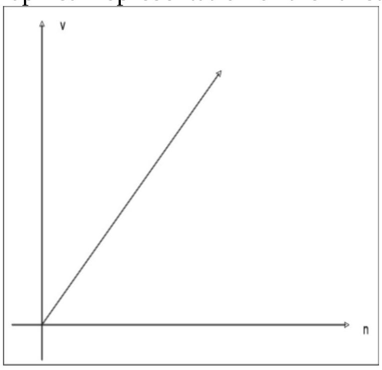
Table 6: Calculations to answer the initial question

N	Related Function Copier E (R$)	Related Function Copier B (R$)	Worth
0	0	0	
2	0,10.2=0,20	0,15.2=0,30	E
200	0,10.200=20,00	0,15.200=30,00	E
300	0,10.300=30,00	0,15.300=45,00	E
400	0,10.400=40,00	0,05.400+30=50,00	E
500	0,10.500=50,00	0,05.500+30=55,00	E
600	0,10.600=60,00	0,05.600+30=60,00	IGUAIS
700	0,10.700=70,00	0,05.700+30=65,00	B
800	0,10.800=80,00	0,05.800+30=70,00	B
900	0,10.900=90,00	0,05.900+30=75,00	B
1000	0,10.1000=100,00	0,05.1000+30=80,00	B

Source: Rosa (2009)

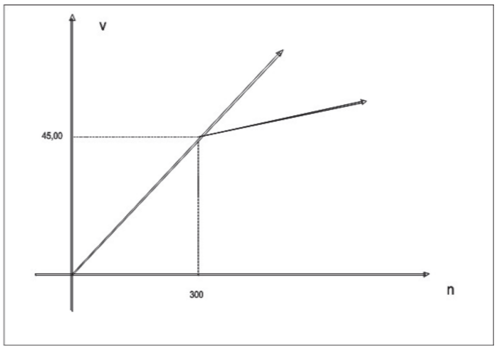
According to Rosa (2009), based on the calculations shown in Table 6, the group concluded that from 600 copies it was worth using the services of the "B" copier. Here another conversion took place (from the registration tab to the record in natural language). As none of the group members 'thought' in using graphic recording to find the problem solution, or even to visualize the function behavior, Rosa (2009) suggested them to do a graph sketch of each function found and to show the solution graphically:

Graphic 1: Graphical representation of the function E Copier



Source: Rosa (2009)

Graphic 2: Initial graphical representation for "B" copier’s function



Source: Rosa (2009)

In this respect, Rosa (2009) noted that, in figure 1, "E" copier, the starting point entered as a point belonging to the function, and, in graph 2, "B" copier, beyond the initial point belonging to the function they failed to make the function graph defined by sentences. For the construction of these charts they used the records of tables 2, 3 and 4, namely there was a conversion of the registration tab to the graphic record. Graph 2 shows that they failed to graph the algebraic function found for the "B" copier, since graph 2 does not have the characteristics of a "set of sentences."

For Rosa (2009), this fact shows that the students failed to understand that the registration of the algebraic function (2) and the chart record shown in figure 2 represented the same function. Therefore, they need to preserve some features. So we can say that there was no full apprehension of mathematical object under study, because according to Duval (2003), to be apprehension of the mathematical object by the student, he needs to make coordination between records.

In summary Rosa (2009) states that she found seven conversions in the activity of mathematical modeling ‘Xerox, a necessary evil’.

Table 7: Conversions performed by the group.

Conversion	Output Record	Arrival Record
1	Record in Natural Language	Algebraic Record
2	Tabular Record	Algebraic Record
3	Tabular Record	Algebraic Record
4	Tabular Record	Algebraic Record
5	Tabular Record	Record in Natural Language
6	Tabular Record	Graphic Record
7	Tabular Record	Graphic Record

Source: Rosa (2009)

Regarding the conversions performed by the students, described in the table above, Rosa (2009) claims to have verified much difficulty by the students, fact which can be evidenced mainly in the error occurred in the conversion 7, when converting one tabular record for the chart record for "B" copier.

Given these considerations, Rosa (2009) states that in the undergoing preparation and problem-solving process, it was found that each member of the group of students felt somewhat responsible, referring to the activity as "my problem"; states the author, students seemed to be solving something particular and of self-interest. This demonstrated interest influenced in the development of the activity, since they 'wanted' to solve the problem.

In this sense, says Rosa (2009), that from the diversity of records that emerged during the development of the activity, the focus of their study was the analysis of the conversions performed during the stages development of mathematical modeling activity that approached the mathematical object 'function '.

Taking into account all these aspects, Rosa (2009) noted that in general, students do not make coordination between records as well as being clearly the choice of algebraic record, while the graphic record is not even remembered (unless when they are suggested).

Thus, the success of the conversion activity can be compromised and consequently a conversion is not always carried naturally by the students. In this regard, we note that it is in the conversions where are present the major cognitive difficulties students.

The work of Fischbein (1994) presents the explanation of the difficulty in conversions when he introduces the idea of figural concept with two components: conceptual and figural. The conceptual component, varying its degree of formalism from top to bottom is presented in natural and/or symbolic language; the figural component is visual in nature (shape, position, size), and is expressed through drawing. It is the proper fusion of these components that ensures the construction of geometrical ideas.

The idea of Fischbein (1994) brings advances in understanding the complexity of the mathematics learning process, because his theory shows that it the work with many conversions between different records that will subsidize the knowledge construction.

Therefore, Rosa (2009) concludes that student involvement in activities which nurture a diversity of records and the coordination between them is essential for learning the concepts involved.

Given these considerations, we agree with Ernest (2006) that, semiotics being the signs study which participates in different contexts of human activities, it is natural to consider the process of learning mathematics also from this perspective.

Ernest (2006) advances in defining what would be a semiotic system in the specific context of mathematics, highlighting three components: a set of symbols that are expressed through speech or text, and design; a set of production rules of signs, including, here, those that deal with the organization of discourse that makes use of the signs composition; a set of relationships between signs and their meanings.

This definition seeks to embrace the texts, symbols and designs characteristics that integrate the logical discourse that produces and crystallizes mathematical knowledge.

It is important to note that the development of mathematical knowledge depends on representation systems that crystallize and generate new concepts and ideas, but it is these same representation systems that must be learned by the student, so that he can have access to mathematical knowledge.

That is, on one side there is the mathematics professor in the teaching process of representations that convey ideas and mathematical procedures; and on the other hand, there is the student in the position of apprentice concepts and procedures that depend on understanding the representation systems.

In short, these reflections made in the light of Peirce's semiotic theory, we believe that the development of a mathematical modeling activity relates to a 'quality' (a phenomenon), a 'reaction' (the identification of a problem and setting goals resolution) and a 'representation' (associated with the solution for the identified problem).

In this sense we can associate this development to the phenomenological categories established by Peirce (1976) (Firstness, Secondness and Thirdness) and, consequently, the levels of identified relationships for signs (meaning, objectification and interpretation).

7 Interaction model text subject x computing environment

In this topic, we present a model made by Behar (1999) to analyze both logical and infralogical operations, as well as the subject and the computational tools. One study goal is to show the model construction process of the subject-object interaction (tool), arising from semiotics elements.

Note that the terms "logical and infralogical operations" have the following meaning conceived by Piaget (1971):

"Logic operation is the one which deals with individual objects considered as invariant and merely assemble them or relate them regardless of their neighborhoods and spatiotemporal distances that separate them."

Infralogical Operation consists in 'making the object through its own elements, not achieving classes nor independent spatial relations, but, overall objects of different types. It deals with, for example, gathering the parts of an object in an all or putting them in a certain succession order.

Infralogical Operation - In general terms, the infralogical operation consists of 'composing the object through its own elements, not achieving classes nor independent spatial relations, but, overall objects of different types. "It deals with, for example, gathering the parts of an object in an all or putting them in an order of certain succession (Piaget, 1971).

For the author, by using a graphical editor, we can highlight the logical operations that are found in a text development, in which regards to the parts relationship of a text with their final product, the sequence should be followed in a text to arrange a coherent whole, the correspondence that should exist between the same parts, among others.

According to her interpretation, when a subject, for example, develops an activity in which the graphical editor is working on the representational space, forming an overall figure through their parts. This means that he will be operating, mainly in an infralogical level, composing, from the partial objects, entire objects.

This does not mean that the person is not making logical operations. For sure he will be. From the formal standpoint, there is no superiority of logic level in relation to the infralogical. These two does not only mutually assume themselves, but also deal with one same operational system, namely, the group applied to different operational modes with the objects.

Therefore, regarding the operative analysis of computational tools, the proposed study by Behar (1999) shows one of the paths found as to how they can be seen in relation to the logical and infralogical operations.

Given these considerations, it is worth highlighting that in this topic, in order to better illustrate the interactional model construction subject x computing environment, we reproduced below the extracted sample of Behar (1999):

In this sense, Behar (1999) states that, in order to view the simplest interactional subject (S) model with a computational environment (C) which, in this case is the object (O), shall be a first draft of the same, in figure 7.

Figure 7: External view of the Interaction Subject - Computational Environment

Source: Behar (1999)

The object in question (C) consists of the computational tool (FC), that is, the system itself (hardware environment) and the representation that the subject performs on the computer (RC - software environment). This figure is detailed as it follows:

Figure 8: Internal viewing of Interaction Subject - Computational Environment

Source: Behar (1999)

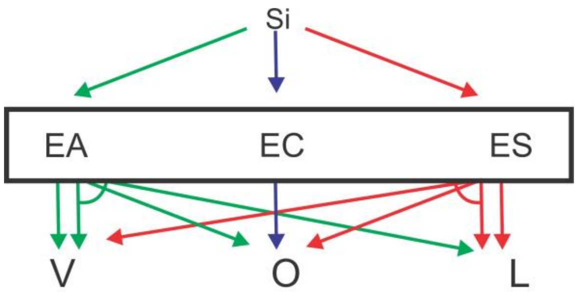
To define the elements that are part of the subject, Behar (1999) took into account the mental factor of the same which is formed according to Piaget (1973) by three inseparable aspects: structural (cognitive), energetic (affective) and symbol (symbolic) systems, serving to these significant operational structures or to these individual values. Thus, it can be said that a subject is composed of affective, cognitive and symbolic structures.

Affective structures relate to the values of the subject. The cognitive ones refer to the object itself, that is, they are responsible for the operations carried out in relation to objects, such as ratings, measurements, seriations, sum, subtraction, etc.

Finally, the symbolic structures are the ones who give representative meaning to the objects, using, for this, the signals, like the speech. Inherent in these structures are also the mental image of the subject, which will not be presented schematically. This model can be seen in figure 9.

For Behar (1999) these objects, plus those arising from semiotics, gave rise to the necessary elements for the construction of the interactional model that is used to perform the operative analysis of the computational tools for individual and collective use. Therefore, the result of the composition of figures 8 and 9 can be seen in the model of figure 10.

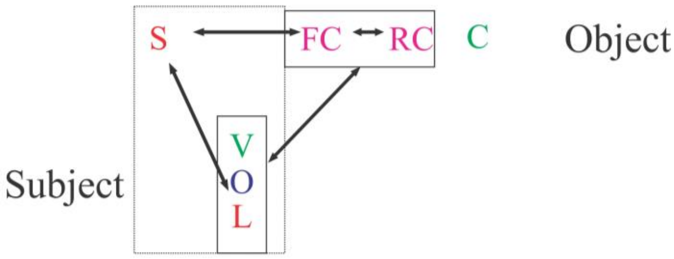
Figure 9: Structures that make up an individual subject



Source: Behar (1998)

Where: Si: individual subject; EA: affective structure; EC: cognitive structure; ES: symbolic structure; V: values; O: objects; L: language.

Figure 10: Individual interaction assisted by computer



Source: Behar (1998)

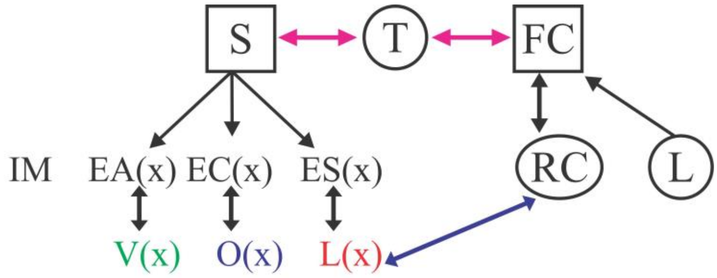
When the subject has to use some kind of computational tool to represent anything, he is led to think about his thinking in order to then be able to transcribe or express his ideas. At the moment in which this one has to express in writing or figurative manner of his thought, he can reflect on it and often restructure it, by building or rebuilding his mental image.

This process may lead the subject to construct new knowledge, but everything will depend on how the subject relates to the environment. That is why we used a bidirectional arrow connecting the values, objects and language to the computer. As the mental image of the subject is inherent in the structures, through interacting with the tool and/or the computational representation, this image can be changed constantly.

In the model defined in figure 10 were added other elements that are part of the interactional model, using the notation of "Petri nets" to detail the elements that are active (ð) and/or passive (O) in the environment in question. This result is shown in figure 11.

Importantly, Petri nets or simply PNs were created from the doctoral thesis of Carl Adam Petri, entitled 'Kommunication mit Automaten' (Communication with Automata), presented at the University of Bonn, Germany, in 1962. From the beginning, PN aimed modeling systems with competing components (Marranghello, 2005).

Figure 11: General interactive model subject x computing environment



Source: Behar (1998)

Where: S: subject (Subject User and/or programmer); 'FC': computational tool '; L: language used for the computational representation; 'RC': computational representation (value, object and/or language represented in the form of textual/figural graphical image); 'RIM': representing the mental image; 'IM': mental image; T: Communication channel: screen, keyboard. image; 'IM': mental image; T: Communication channel: screen, keyboard.

Therefore, according to what Behar (1999) claims, this model can be understood as it follows:

Relying on the mental picture (IM) that the subject (S) has in relation to "something" which wants be to represented, it can be values, objects and/or a particular language, it uses a language (L) for representing your IM. This language, which may be its natural language (drawing or writing) or computer language, allows the representation of this "something" on your computer (RC). To accomplish this representation, the subject uses the computational tool (FC) manipulated via the keyboard, screen, etc.

In summary, this is the general model that was built by Behar (1999) as one of the ways found to explain the interactional process of a subject and its structures with any computational tool, on the individual level.

8 Results and Future Work

This study had two main objectives: first, to conduct an analysis of the state of the art regarding the techniques of intelligent computational systems. Also, present the theoretical foundations in order to identify the use of new techniques for modeling intelligent systems through the integrated use of mathematical and computational modeling.

During the process of conducting this research we looked through the consulted literature that the basic triad of semiotics, that is, the triple (sign, object, interpretant) can be mapped in models of artificial intelligence. In the semiotics triple, the sign is used to represent the object, whose understanding by an intelligent mind corresponds to the interpretant.

That is to say, the interpreter is the intellectualization of the object. In triple of artificial intelligence, a phenomenon of the environment (which corresponds to the triple object of semiotics), interpreted as knowledge about the environment is represented by a model of knowledge representation, which corresponds to the sign.

We also saw that "semiotic representation" consists of signs belonging to a given semiotic system, which gives it a particular meaning within a given context. How different records will be accepted, that is, different semiotic systems referred to the same content, you must consider that the various systems are part of a compound semiotic system, referred to the same content (or slight variations of a same content), endowed with conversion rules between different records.

In this sense, the same content can be represented in different records, each recognizing different representations. This relationship between representations of the same object in different records is what Duval (2003) calls "conversion" between records. Thus, in a given record, it can be constructed different representations related to each other. Duval (2003) calls them "treatment".

The didactic use of Duval’s (2003) theory assumes that the student may not have fully appropriated himself of the represented content, and it can treat a representation as a game without clearly understand the terms meanings. Everything happens as if the understanding that the vast students majority had of content was limited to the form of the representation used.

For this reason, the use of an appropriate representation avoids continuous symbols remissions to their meaning, saving the thinking job through the purely symbolic treatment - and this is one of the main pragmatic representational functions, under the domain hypothesis of the associated content, transforming the reasoning in a "calculation". But at the same time, a single representation reduces the concept to a symbolic calculus.

Based on these arguments, we believe that the construction of intelligent systems semiotically inspired or not, constitutes a great challenge to computer science and other sciences, because scientific and technological evidence points to the increased interaction between computing and virtually all areas of human knowledge.

However, for the hypothetical promise of technological impact is performed, it is necessary that researchers and practitioners are prepared to change and adapt their work practices. This supports the need for further research practices, requiring multidisciplinary and strong fundamental scientific knowledge teams. In our opinion, this knowledge is essential to allow the construction of artificial intelligent computer systems.

9 Thanks

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