Desired Supply Line Value Calculation for Multi-Supplier Systems[1]

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Abstract
Desired Supply Line is the product of expected acquisition lag and desired acquisition rate. This calculation ensures that supply line would produce the desired acquisition rate given that it is at this desired level. A wrongly calculated Desired Supply Line value leads to a steady-state error preventing stock approach its goal. Therefore, correct calculation of Desired Supply Line values is crucial. Desired acquisition rate is equal to the expected loss flow in a single-supplier system. However, it is not easy to decide on the desired acquisition rates for a multi-supplier system. We give a general formula for the calculation of Desired Supply Line values based on the supplier utilization priorities and supplier production/shipment capacities.

Keywords: acquisition lag; constant loss; multi-supplier system; steady-state error; stochastic loss; stock management; supply line.

1. Introduction

The sourcing success of a manufacturer does not only depend on the ordering strategies, but it also depends on supplier selection. A firm should use multiple-sourcing strategy instead of a single-sourcing strategy in order to reduce procurement risk (Arda and Hennet, 2006; Chiang, 2001; Chiang and Benton, 1994; Jokar and Sajadieh, 2008; Minner, 2003; Ramasesh, 1991; Sculli and Shum, 1990; Sculli and Wu, 1981; Thomas and Tyworth, 2006). In the presence of stochastic lead times, multiple-sourcing strategy reduces the effective lead time (Minner, 2003). Multiple sourcing also reduces the dependency on a single supplier, thus the power of supplier over the buyer (Burke,

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Carrillo and Vakharia, 2007; Newman, 1989). Some firms give scores to its suppliers, and decide on their priority levels. They give orders according to the priority levels of its suppliers (Burke, Carrillo and Vakharia, 2007).

Determining the target value for a supply line, which is called *Desired Supply Line*, is important when anchor-and-adjust heuristic is used. *Desired Supply Line* is calculated by multiplying expected acquisition lag and desired acquisition rate (Sterman, 2000). In a single-supplier stock management problem, desired acquisition rate for a stock is equal to the expected loss flow from that stock. Therefore, *Desired Supply Line* is equal to expected acquisition lag times expected loss flow. However, determining desired acquisition rate, thus, calculating *Desired Supply Line*, is not that straightforward in the presence of multiple suppliers.

A stock having multiple supply lines can be seen in inventory management, human resource management, capacity management, and personnel training. For example, firms have different human resources management processes. Some firms use an internal human resources department. Some outsource their human resources needs to private agents. Additionally, some of them use both an internal department and private agents. Each entity who deals with human resources management has a different operational mechanism which has its own working capacity and hiring lead time. The hiring/firing process of each entity corresponds to a different supply line for the human workforce of a firm. Firms using multi-sourcing strategies need to decide on the utilization level of each supplier. They should also determine *Desired Supply Line* values for each of those supply lines, which is the main issue examined in this work.

In a stock management task, the goal is maintaining a stock at a desired level. This is achieved by adjusting for the supply line and the corresponding stock at the same time. To adjust for the supply line, its desired level should be chosen appropriately. We developed formulations for *Desired Supply Line* value calculation for multi-supplier systems in the presence of constant *Loss Flow* and stochastic *Loss Flow* with a known stationary mean.

2. Generic Formulations of Desired Supply Line

The general formula for *Desired Supply Line* is seen in Equation 1:

$$Desired\ Supply\,Line = Acquisition\,Delay\,Time \times Loss\ Flow \qquad (1)$$

Equation 1 is valid for a stock with a single supply line. It needs to be adjusted when a stock has multiple suppliers and, thus, multiple supply lines. When there are *n* supply lines attached to a stock, each supply line needs to have its own *Desired Supply Line* value. As an example, a stock management system having 2 supply lines attached to a stock is presented in Figure 1.

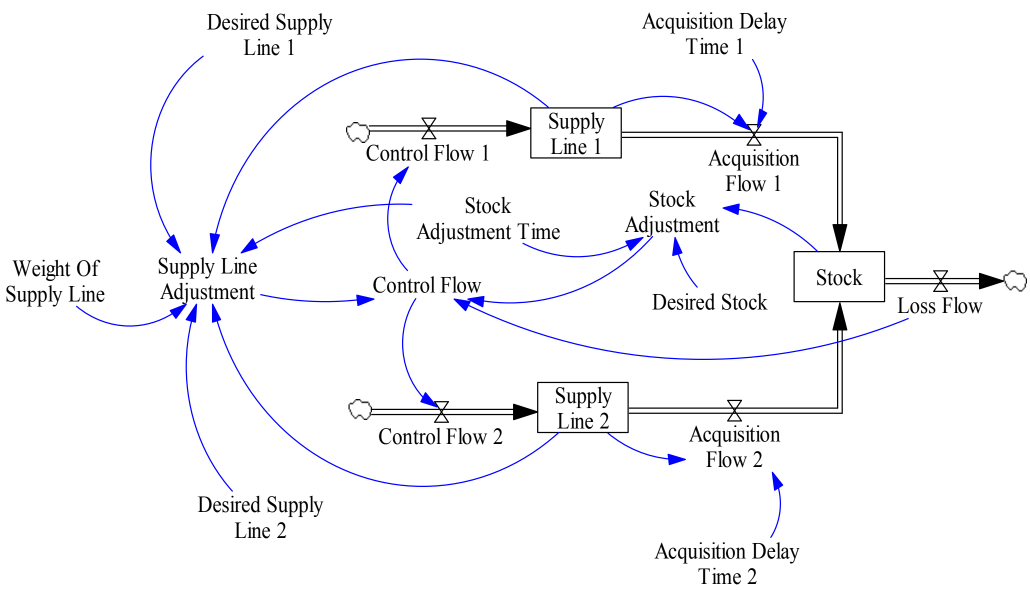

Figure 1. Stock-Flow Diagram of the Stock Management Task with 2 Supply Lines.

If there is an error in determining the desired supply line values, there will be a steady-state error. Therefore, desired supply line values should correctly be selected. If the selected values are proper, the average value of *Stock* subtracted from its desired level will be equal to zero in the long run.

In order to balance the inflows to and outflow from the stock, the total average acquisition flow should be equal to the average loss flow:

$$\sum_{i=1}^{n} E[\mathit{Acquisition\ Flow}_i] = E[\mathit{Loss\ Flow}] \tag{2}$$

Average value of an inflow (i.e. control flow) attached to a supply line should balance the outflow (i.e. acquisition flow) from that supply line. This is needed to maintain each supply line around its desired value.

$$E[Control\ Flow_i] = E[Acquisition\ Flow_i] \qquad for \quad i = 1, 2, \cdots, n \tag{3}$$

Equations 2 and 3 yield Equation 4:

$$\sum_{i=1}^{n} E[Control\ Flow_i] = E[Loss\ Flow] \tag{4}$$

According to Equation 4, the total average control flow should also be equal to the average loss flow in the long run. *Desired Supply Line* values should be selected so as to satisfy Equations 2 and 4.

It is known that the expected outflow from a supply line (i.e. expected acquisition flow) is equal to the average value of the supply line (i.e. desired supply line) divided by the delay time of that supply line (i.e. acquisition delay time).

$$E[Acquisition\ Flow_i] = \frac{Desired\ Supply\ Line_i}{Aquisition\ Delay\ Time_i} \tag{5}$$

Equations 2 and 5 yield Equation 6:

$$E[Control\ Flow_i] = \frac{Desired\ Supply\ Line_i}{Aquisition\ Delay\ Time_i} \tag{6}$$

From Equation 6, Equation 7 can be obtained:

$$Desired\ Supply\ Line_i = Aquisition\ Delay\ Time_i \times E[Control\ Flow_i] \tag{7}$$

The expected value of a control flow can be obtained using the priority assigned to the related supplier, and the probability distribution function of the loss flow. Once the expected control flow values are obtained, *Desired Supply Line* values can be obtained using Equation 7. Although Equation 7 is always valid, its application may not be that straightforward due to the difficulties in obtaining expected *Control Flow* values.

3. Multiple Supplier Examples

In this part, we give the applications of proposed *Desired Supply Line* calculation method for constant *Loss Flow* case and stochastic *Loss Flow* with a known stationary mean case. The distribution of *Loss Flow* (i.e. demand), supplier capacity limitations, and supplier priorities are the factors to be considered in control decisions. Handling stochastic demand is more problematic than handling constant demand in supply chain management (Nahmias, 2009; Jokar and Sajadieh, 2008, Schmitt, 2007). One other important concern in supply chain management is the capacity of suppliers. Production and shipment capacity constraints of suppliers lead to more oscillatory stock behaviors (Goncalves and Arango, 2010; Minner, 2003; Schmitt, 2007; Springer and Kim, 2010). The following control flow equation is used in both of the examples:

$$Total\ Control\ Flow = \begin{pmatrix} Expected\ Loss\ Flow + Stock\ Adjustment \\ + Supply\ Line\ Adjustment \end{pmatrix} \tag{8}$$

Note that, our examples assume three-supplier stock management system. The following individual orders to the three suppliers are calculated as given below. The priority of a supplier is represented by the index assigned to that supplier (low index represents high priority level).

$$\begin{pmatrix} Control \\ Flow_1 \end{pmatrix} = \begin{cases} Total\ Control\ Flow, & Total\ Control\ Flow \le \begin{pmatrix} Capacity\ of \\ Supplier_1 \end{pmatrix} \\ Capacity\ of\ Supplier_1, & Total\ Control\ Flow \ge \begin{pmatrix} Capacity\ of \\ Supplier_1 \end{pmatrix} \end{cases} \tag{9}$$

$$\begin{pmatrix} Control \\ Flow_2 \end{pmatrix} = \begin{cases} \max\left(\begin{pmatrix} Total\ Control \\ Flow \\ - \\ Capacity\ of \\ Supplier_1 \end{pmatrix}, 0\right), & \begin{pmatrix} Total \\ Control \\ Flow \end{pmatrix} \le \begin{pmatrix} Capacity\ of \\ Supplier_1 \\ + \\ Capacity\ of \\ Suppplier_2 \end{pmatrix} \\ Capacity\ of\ Supplier_2, & \begin{pmatrix} Total \\ Control \\ Flow \end{pmatrix} \ge \begin{pmatrix} Capacity\ of \\ Supplier_1 \\ + \\ Capacity\ of \\ Suppplier_2 \end{pmatrix} \end{cases} \tag{10}$$

$$\begin{pmatrix} Control \\ Flow_3 \end{pmatrix} = \max\left(Total\ Control\ Flow - \begin{pmatrix} Capacity\ of\ Supplier_1 \\ + \\ Capacity\ of\ Supplier_2 \end{pmatrix}, 0 \right) \quad (11)$$

3.1. Three Supplier System with a Constant Loss Flow

In a single-supplier system, *Desired Supply Line* is calculated by using Equation 1 when *Loss Flow* is constant. Desired acquisition rate of the supply line is equal to *Loss Flow* given that supply line reaches its desired level.

In a multi-supplier system, desired acquisition rate of each supplier is selected by the decision maker depending on their priority levels and production/shipment capacities. The sum of the desired acquisition rates must be equal to *Loss Flow* in order to prevent a steady-state error. To calculate *Desired Supply Line* of a supplier, desired acquisition rate of that supplier must be multiplied by *Acquisition Delay Time* of the same supplier.

Let's assume there is a three-supplier system. The stock to be managed has a constant *Loss Flow* equal to 60. Acquisition delay times of the suppliers are 8, 12, and 16 in order. Target level of *Stock* is 0. The decision maker wants to receive 28 units from the first supplier, 22 units from the second supplier and 10 units from the third supplier. The desired value of each supply line is found by Equation 7. So, desired values of supply lines become 224, 264 and 160 in order. Notice that desired acquisition rate of a supplier must also be equal to expected control flow of that supplier for supply line stability.

As it can be seen from Figure 2, *Stock* stays on its desired level when *Stock* and its supply lines start at their desired levels. It is also observed from Figure 3 that even though *Stock* does not start at its desired level (starts at 250), both *Stock* and its supply lines seek their desired levels.

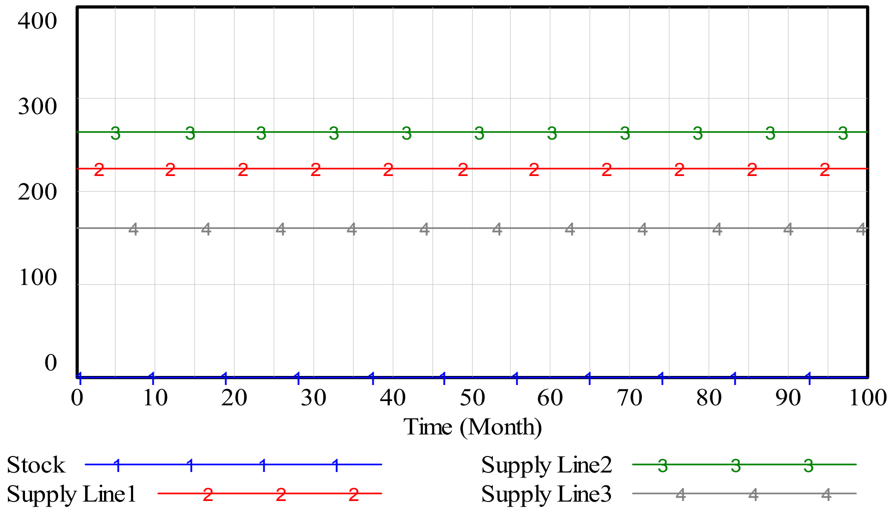

Figure 2. Stock and Supply Line Behaviors in a Three-Supplier System when *Loss Flow* is Constant.

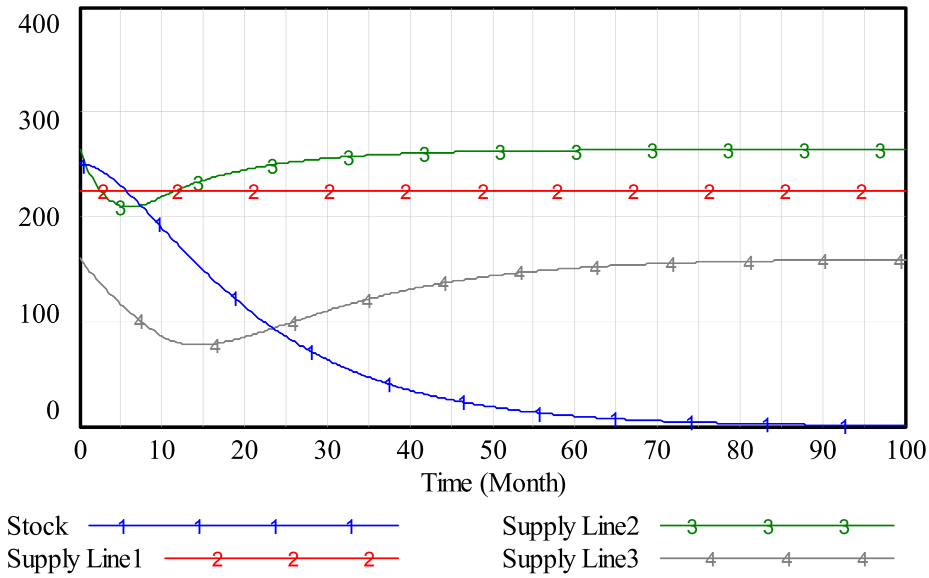

Figure 3. Stock and Supply Line Behaviors in a Three-Supplier System when *Loss Flow* is Constant and *Stock* does not start at its Desired Level.

3.2. Three Supplier System with a Stochastic Loss Flow

We have a three-supplier stock management model which has a stochastic *Loss Flow*. *Loss Flow* has a normal probability distribution with mean 60 and standard deviation 12. Simulation runs are obtained in discrete time; unity is used as the simulation time step. In this example, *Stock Adjustment Time* and *Weight of Supply Line* are taken as one. Under these assumptions, the distribution of *Control Flow* is equal to the distribution of *Loss Flow* (see Appendix). In our model, the first supplier has priority over the second supplier and the second supplier has priority over the third. First and second suppliers have limited shipment, their capacity limits are 40 and 25 in order. The decision maker gives the orders up to 40 from the first supplier, orders between 40 and 65 from the second supplier, and orders above 65 from the second supplier. If order exceeds 40, first supplier provides 40 units and, if order exceeds 65, second supplier provides 25 units while first supplier still provides 40 units. *Desired Supply Line* depends on desired acquisition rate and acquisition lead time. Desired acquisition rates do not depend on acquisition lead time or the order of the supply line. However, they are affected by the capacity limitations of the suppliers. The upper limit of *Control Flow* of a supplier is its production/shipment capacity.

$$f(x) = \frac{1}{\sigma \times \sqrt{2 \times \pi}} \times e^{-\frac{1}{2} \times \left(\frac{x-}{\sigma} \right)} \tag{12}$$

$$E[Control\,Flow_1] = \int_{-\infty}^{40} x \times f(x) \times dx + \int_{40}^{+\infty} 40 \times f(x) \times dx \tag{13}$$

$$E[Control\,Flow_2] = \int_{40}^{65} (x-40) \times f(x) \times dx + \int_{65}^{+\infty} 25 \times f(x) \times dx \tag{14}$$

$$E[Control\,Flow_3] = \int_{65}^{+\infty} (x-65) \times f(x) \times dx \tag{15}$$

Equation 12 shows the probability distribution function of normal distribution. Equations 13, 14, and 15 are valid because the distribution of *Control Flow* is equal to the distribution of *Loss Flow* (see Appendix). According to Equations 13, 14, and 15, desired acquisition rates are consecutively equal to 39.76207, 17.54096, and 2.696963. *Desired Supply Line* values become 318.0966, 210.4915, and 43.15141 consecutively for the first, second, and third suppliers (see Equation 7). If *.Desired Supply Line* values were calculated assuming constant *Loss Flow* (i.e. equal to mean of *Loss Flow* which is 60), they would be 320, 240, and 0 instead. This would lead to steady state error causing higher penalties.

In Figure 4, "DSL" on the x-axis corresponds to our base run which uses the calculated *Desired Supply Line* values of 318.0966, 210.4915, and 43.15141. Penalty values are generated by using Equations 16 and 17 and the average of five different seeds is taken. The length of the simulations is 250. There are three lines in Figure 4 for the three suppliers. As one moves to the right on a line, *Desired Supply Line* value corresponding to that line increases while the other two *Desired Supply Line* values remain at their base levels. As one moves to the left on a line, *Desired Supply Line* value corresponding to that line decreases while the other two *Desired Supply Line* values remain at their base levels. An increase or a decrease in the proposed *Desired Supply Line* values results in an increase in the total penalties according to Figure 4. These results approve the appropriateness of our desired supply line value calculation method:

$$Total\ Penalty_0 = 0 \quad [item \cdot time] \tag{16}$$

$$Total\ Penalty_{t+DT} = Total\ Penalty_t + \left| \begin{matrix} Desired\ Stock\ Level \\ -\ Stock \end{matrix} \right| \times DT \tag{17}$$

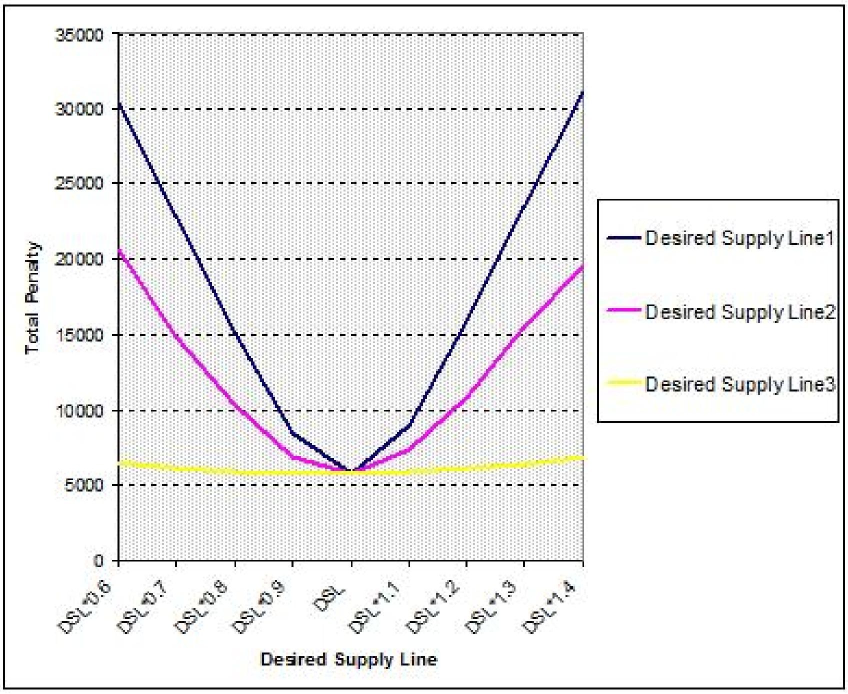

Figure 4. *Total Penalty* vs. *Desired Supply Line* Values.

4. Conclusion

A wrongly calculated *Desired Supply Line* value leads to a steady-state error preventing stock approach its goal. Therefore, desired supply line values should correctly be selected. In this work, the calculation of *Desired Supply Line* values in multi-supplier systems is examined. Using multiple suppliers instead of a single supplier reduces the procurement risks in stock management. However, determining *Desired Supply Line* values in a multi-supplier system is not that straightforward compared to a single-supplier system.

In steady state, inflow (*Control Flow*) to a supply line has to be equal to the outflow (*Acquisition Flow*) from that supply line. Also, the total inflow (sum of all acquisition flows) to a stock has to be equal to the outflow (*Loss Flow*) from that stock. Note that, outflow from a supply line is, at the same time, an inflow to the corresponding stock. Eventually, this brings the deduction that the sum of all inflows to the supply lines in a stock management system (i.e. control flows) has to be equal to the outflow (i.e. *Loss Flow*) from the main stock of that system. Therefore, the selection of *Desired Supply Line* values must ensure that different supply lines in total produce the total desired acquisition rate. We give a general approach in obtaining proper *Desired Supply Line* values in a multi-supplier stock management system. The desired values obtained by using this approach make the average value of *Stock* subtracted from its desired level equal to zero in the long run.

According to the general approach in determining the *Desired Supply Line* values in a multi-supplier stock management system, once the expected control flow values are obtained, *Desired Supply Line* values can be obtained using Equation 7. Although Equation 7 is always valid, its application may not be that straightforward due to the difficulties in obtaining expected *Control Flow* values. In this study, this approach is applied to two cases: one under constant *Loss Flow* assumption and the other one under stochastic *Loss Flow* (normally distributed with known mean and variance) assumption. As a continuation of this study, we are planning first to extend the application of this approach to a case under stochastic *Loss Flow* (normally distributed with unknown mean and variance) assumption with exponential smoothing heuristic used in expectation formation. Secondly, the generality of the results obtained from the first extension of the study will be discussed for other *Loss Flow* distributions.

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Appendix

Stock Adjustment Time and *Weight of Supply Line* are both chosen as 1. With these settings, two consecutive control flows are calculated by Equations 18 and 19.

$$Control\ Flow_i = Expected\ Loss\ Flow + \begin{pmatrix} Desired\ Stock \\ - \\ Stock_i \end{pmatrix} + \begin{pmatrix} Desired\ Supply\ Line \\ - \\ SupplyLine_i \end{pmatrix} \quad (18)$$

$$Control\ Flow_{i+1} = Expected\ Loss\ Flow + \begin{pmatrix} Desired\ Stock \\ - \\ Stock_{i+1} \end{pmatrix} + \begin{pmatrix} Desired\ Supply\ Line \\ - \\ SupplyLine_{i+1} \end{pmatrix} \quad (19)$$

Stock formulation is seen in Equation 20 and Supply Line formulation is seen in Equation 21 with these settings.

$$Stock_{i+1} = Stock_i + Acquisition\ Flow_i - Loss\ Flow_i \quad (20)$$

$$Supply\ Line_{i+1} = Supply\ Line_i + Control\ Flow_i - Acquisition\ Flow_i \quad (21)$$

Difference of two consecutive control flows is shown in Equation 22.

$$Control\ Flow_{i+1} - Control\ Flow_i = Stock_i - Stock_{i+1} + Supply\ Line_i - Supply\ Line_{i+1} \quad (22)$$

When Equations 20 and 21 are plugged in to Equation 21, Equation 23 is obtained.

$$Control\ Flow_{i+1} - Control\ Flow_i = Loss\ Flow_i - Control\ Flow_i \quad (23)$$

Equation 23 yields Equation 24. Equation 24 shows that *Control Flow* follows *Loss Flow* from 1 time unit behind with these settings. This means that their probability distributions are exactly the same. Therefore, our expected control flows can be calculated by using the probability distribution of *Loss Flow*.

$$Control\ Flow_{i+1} = Loss\ Flow_i \quad (24)$$