Modeling the survival of Athenian owl tetradrachms struck in the period from 561-42 BC from then to the present

Harald Ulrik Sverdrup[1]
Physical Geography and Environmental Systems, Earths Sciences
Stockholm University, SE 106 91 Stockholm, Sweden
Harald.sverdrup@chemeng.lth.se, harald.sverdrup@ka-rasmussen.no,

Peter Schlyter
Physical Geography and Environmental Systems, Earths Sciences
Stockholm University, SE 106 91 Stockholm, Sweden
Peter.schlyter@natgeo.su.se

Abstract

The TETRA-model has been built to predict the modern occurrence of ancient Athenian tetradrachm silver coins in quantitative terms, based on their original minting volumes an antiquity and the processes of their loss and destruction, as well as the process of finding them in modern times. The TETRA model was tested against independent estimates and seems to work well. The approximate number of coins surviving until today for the different types such as archaic owls, classical owls, transitional owls, heterogeneous owls and new style owls were predicted well within the estimates derived through other means and museum inventories (r^2=0.82).

Key words: *Athens, tetradrachms, Laurion, owls, ancient coins, coin production, archaic owls, classical owls, transitional owls, heterogeneous coinage, the symbols series, new style tetradrachms, integrated dynamic modeling, ancient coin survival*

Introduction

Athenian Owls were arguably the most influential of all coins, and the Classical Owl tetradrachm is the most widely recognized ancient coin among the general public today. Owls were the first widely used international currency. They remained thematically unchanged through 500 years, Athena on the obverse, her owl on the reverse, through great changes in the ancient world (Figure 1). Ancient texts report that Athens started to mint their own coins as early as 575 BC, however, no coins from that age can be ascertained. Prior to the introduction of coinage the Athenians had used iron spits or elongated nails as money, maybe these are the "coins" alluded to. Athenian tetradrachms are traded in the market, being a popular object of collection amongst specialists. However, great uncertainty exists in the market concerning their true rarity of these coins, the relative rarity between the coin types and the prices to be expected. Little is published on this, and many believe such estimates to be impossible (Buttrey 1993, 1994). From the estimates of coin circulation, hints may be gained in the size and vigor of the Athenian economy, as well as indicate the size of the money circulation.

[1] corresponding author. +46-708 855 683

Objectives and scope
The objective is to get a grip on Athenian minting volumes and through that get a glimpse of the Athenian state economy. This is done by using literary sources, estimates made from remaining coins in the world's collections and museums and through integrated systems modeling. Some claim this cannot be done (Buttrey 1993, 1994), others see possibilities (Newton 2006, Carter 1983, de Callatay 1995). We will go on to do it anyway with a tool not available to any of these people; Integrated modeling assessment.

Methods and data sources
Thuchydides, Xenofon, Herodotos, Aristotiles all left important quantitative information. The basic method is that of systems analysis and systems dynamics. We have developed a model based on constructing analyzing the system with flowcharts, causal loop diagrams and use these for developing a dynamic model for coin minting volumes, coin circulation, re-melting, losses and rediscovery over time. A model tool called STELLA® was used as the modeling developing environment. Some data on minting volumes are available (Tab. 1)

Historical background
Discovery of a rich seam of silver in the Laurion Mines in southern Attica occur 490 BC (Flament 2007, Kroll 1981, 1998, Seltman 1924, Starr 1973). Themistocles persuaded the Athenians to use some of the proceeds from the mine to build a fleet of 200 warships in preparation of the anticipated Persian wars that were to follow 485-478 BC. Later, during the period from 220 BC to about 50 BC the minting was significantly higher than Laurion silver alone would indicate. Increased trade and subsequent exports with inflow of Roman silver increased the turnover significantly during this period as compared to what the diagram shows. For the official minting, the Athenians relied on a number of sources of silver: (1) From mining: (a) Silver mined at Laurion in Attica, (b) From mines in Northern Greece, in Macedonia and Cheronnessos, (c) From the mines on Thasos. (2) From abroad: (a) Taken from the Persians in war, (b) Given by the Persians in bribery and political corruption, (c) Money earned in trade and exports as silver or by re-minting foreign coin silver (Snell 1995). Mining at Laurion commenced possibly as early as about 3,000-2,500 BC, however on a small scale. This date has been ascertained because lead found in Egypt and dated to that time has been found to have the same isotope ratios as lead from Laurion[6]. Similarly, lead from Laurion was also used in the Minoan culture in Crete in about 1,100 BC. The mines were worked variously by the Greeks, Minoans, Phoenicians, and then the Greeks again. The various classical period Greek writers, Perikles, Theofrastos and others have reported on the mines from time to time, they being described by one historian as 'the treasure house of the country'. The miners first started working the surface ores and discovered that silver was in the galena mineral by noting that when the lead oxidized, the silver precipitated as pure metal with argentite on top. Accordingly, the miners followed first the galena mineral veins and then the cerussite, knowing that where lead was found there would also be silver. Laurion lead ores contained about 0.5% silver. By the second millennium BC the mining had reached a depth of 20-30 meters using stone tools. By 483 BC, according to Aristotle, mining had penetrated down to 100 meters using metal hammers and chisels by that time. Private persons in Athens acquired silver mines in the Chalchidike and on the Strymon River in the Macedonian area quite early (700-600 BC), silver also came from mines on Thasos in the northern Aegean. All these sources were used for minting silver coins. The mines in the north were lost early during the Persian wars (512 BC) and only partly recovered afterwards.

Fig. 1. *The major different types of Athenian coins 561-42 BC separated in this model. Specimens are from the H. U Sverdrup collection, photographs were taken by the author.*

After the Peleponnesian war closed the mines in 413 BC, the slave-based workforce was dispersed and the production did not restart until 353 BC because of lack of capital to restart it on any substantial scale. It regained a large production again in the years 338-320 BC for financing the war campaigns of Alexander, in the next few decades the mines reached the ground water table 100 m below and were exhausted. Small-scale mining continued until 103 BC when it stopped altogether. Nearly 2,000 shafts have been found there. Diagnostic is that the Laurion silver was high in lead and very low in gold and copper, and can thus this source can be identified in the coins. Athens issues an edict in 425 BC ordering all foreign coins to be handed in to the Athenian mint and compelling all her allies to use the Attic standard of weights, measures and money (Mattingly 2003). However, no Athenian coin dated with certainty older than 560 BC has been found so far. Prior to the introduction of coinage the Athenians had used iron spits or elongated nails as money. The first owls were minted just after the rule of the Peisistratids 546-527 BC. In the period leading up to Kleistenes democratic reform, the regular owls became standard in 526 BC. Discovery of a rich seam of silver ore in the Laurion Mines occur 483-482 BC. Athens issues an edict in 454 BC moving the Athenian League treasure from Delos to Athens, and in 425 BC ordering all foreign coins to be handed in to the Athenian mint and compelling all her allies to use the Attic standard of weights, measures and money. Albeit it was felt a bit oppressive at the time, it did wonders for trade and prosperity.

Owls were also minted in a host of smaller denominations, including di-drachms, drachms, hemi-drachms, tetra-obols, di-obols, tri-hemi-obols, obols, hemi-obols, tri-tartemorions, tri-hemi-tartemorions, tetra-tartemorions, hemi-tartemorions, and several bronzes. The smaller fractions, used for everyday market transactions and hoarded less, typically were struck less carefully, circulated more, and are found in worse condition than the tetradrachms. The owl silver coinage ended in the middle of the 1st century BC, but some Athenian bronzes featuring an owl continued well into Roman Imperial times until the end of ancient Athenian coinage in 267 AD. Much later the Classical Owl tetradrachm was widely remembered, and honored, on coinage and elsewhere. In 483-482 BC, rich lodes of silver ore was discovered at the lead mines at Laurion in southern Attica, and this contributed greatly to the economic wealth of the Athenian state. It is not known when the Athenians started to coin money. The term obol goes back to earlier iron tokens that were used as money, they date at least from the time 650-580 BC. In imperial Athens (475-400 BC), every state employee, a hoplite, rower, or worker of any kind (free or slave) each received a daily wage of 1 Attic drachm. A workday was from 7 to 18. In the period of 400-320 BC, the daily wage rose to 1.5 drachm. Craftsmen and masons working on public projects in 447-408 BC, received daily wages from 2-2.5 drachm. Bricklayers in 395-391 BC received 3-4 tetradrachms per 1,000 bricks laid; in 329-328 BC they received 4-6 tetradrachms for the same amount of work.

Table 1. *Estimated number of owls minted according to contemporary sources*

Date	Minted number of coins	Treasury balance	Reference
477 BC	2,760,000	?	Thuchydides. I. 96
454 BC	3,000,000	48 million	Meiggs, AE, p. 253, Diodorus. XII. 28. 2
431 BC	3,600,000	60 million	Thuchydides I. 99. 3 & II. 13. 6-7, Diodorus . XII. 30. 1-2
428 BC	4,800,000	54 million	Meiggs, AE, 325, Plutarch, Aristophanes 24.3, Thuchydides II. 13. 6-7
425 BC	9,000,000	36 million	Meiggs, AE, p. 343, Thuchydides II. 13. 6-7
421 BC	7,200,000	?	Andoc. III. 9
406 BC	0	0 million	Thuchydides, Aristophanes IG II 2.2, 665, The Frogs, ll. 725-726
337 BC	2,650,000	?	Lycurgus
338 BC	1,070,000	0	Lycurgus
325 BC	2,650,000	10 million	Lycurgus
304 BC	1,770,000	?	Xenokrates

Table 2. *Estimation of relative proportion of tetradrachms, based on a survey of auction catalogs. The absolute number of surviving coins were scaled on the estimate of 1402 archaic coins in 1924 and 6,800 coins of the new style in 1961, estimated at 8,100 in 1997. The price does not only reflect the scarcity, but also the fact that the classical Athens owls are the most popular.*

Coin type	Number of survivors	Rarity %	Years minted	Period	Price in US $ in 2008-2009
Wappenmünzen	176[1]	0.5	33	561-528 BC	2,500-9,000
Archaic owl	1,400[2]	4-9	49	527-478 BC	1,000-5,000
Classical owl	8,100-17,700[3]	38-50	74	478-407 BC	400-1,000
Official fourrees	150-300	0.5	3	406-404	300-1,000
π-type owl	4,000-15,500	10-30	98	383-285 BC	300-700
Quadridigité	1,400-1,900	4-8	23	285-262 BC	200-400
Heterogeneous	93[4]	0.3	42	262-220 BC	500-2,000
First neutral new style annual series	40	0.1	23	220-197 BC	1,000-2,500
Symbol new style annual series	100	0.3	27	196-169 BC	1,000-2,500
New style annual series	8,100[5]	23-38	122	168-46 BC	400-1,000

Estimating mining volumes

In 450 BC Athens received an annual tribute of 500 talents or 750,000 tetradrachms, which was tripled to 1,500 talents or 2.25 mill tetradrachms in 425 BC. Profits from mines, justice, customs netted another 2,000 talents or 3 million tetradrachms per year. This was only part of the wealth of Athens. It can be argued that total revenues might have exceeded 9 mill tetradrachms, an impressive sum for a commercial city whose annual revenues might have been half of the revenues of King Artaxerxes I (465-425 BC). In contrast to the Persian Great King, the Athens had high expences every year. In 431 BC, the service of 200 triremes for six months cost 1.2 million tetradrachms. In 483-410 BC Athens commissioned 1,500 triremes at a cost of 22.5 million tetradrachms. In 460 BC, Athens had to pay out in wages at least 90,000 tetradrachms for the 180 days marked for jury service. In 408 BC this sum rose to 135,000 tetradrachms. The building costs recorded in 447-425 BC totaled at least 12 million tetradrachms for the construction of the main monuments on the Akropolis (Parthenon, Propylaea, Temple of Athena Nike, and cult statues), the Middle Walls linking the city to the Piraeus, and the expansion of harbor facilities at the Piraeus. Silver was valued at 28.2 kg of silver to 2 kg of gold, a ratio of 1:14 (Table 1). Vast quantities of Owls have been found overseas, particularly in hoards in South Italy, Sicily and Egypt (van Alfen 2002, Flament 2007, Kraay 1964, 1976, Mörkholm 1982, Robinson 1960). Owls, in addition to their local usefulness, were minted for international trade as a surplus commodity for export abroad. In this respect they reflect a remarkable change in Athens' fortunes and monetary policy. At the time of minting, it is estimated that something like 28,000 tetradrachms could be produced per die (Mattingly 2003).

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[2] Seltman 1924
[3] Sverdrup (2010) based on Flament (2007) and Starr (1973)
[4] Price 1983
[5] Thompson 1961

Table 3. *Overview of issue volumes and coins remaining for the wappenmünzen for the period 561-527 BC. The wappenmünzen were mostly of smaller denomination, thus amounting to almost insignificant amounts of silver metal*[6].

Date	Symbols on the coins	Seltman group	n	Dies	Estimated volume, mill	Period sum, mill
561-556 BC	Amphora	A	25	5	1.130	1.130
555-552 BC	Amphora	B-I	14	6	1.360	1.360
551 BC	Triskeles Bent leg	B-II	2	2	0.452	0.452
550-546 BC	Horse fore	B-III	5	2	0.452	1.580
	Beetle		1	1	0.226	
	Pommegrenade		3	2	0.452	
	Oracle bone		1	1	0.226	
	Cartwheel		1	1	0.226	
545 BC	Bull	B-IV	1	1	0.226	0.226
544-542 BC	Horse	B-V	1	1	0.226	0.904
	Horse fore		5	2	0.452	
	Eye		1	1	0.226	
542-541 BC	Cartwheel	B-VI	8	2	0.452	0.904
	Horse behind		4	2	0.452	
540-538 BC	Chariot wheel	D-I	8	4	0.904	0.904
537-536 BC	Owl	D-II	3	2	0.452	0.904
	Bull's head		4	2	0.452	
535-530 BC	Gorgon	D-III	45	8	1.808	1.808
533-529 BC	Owl Bull's head Chariot wheel	J-I	21	9	2.034	2.034
529 BC	Bent leg	J-II	1	1	0.226	0.678
	Triskeles		3	2	0.452	
	Bull's head		3	1	0.226	
538-527 BC	Panther	K	12	3	0.678	2.034
	Gorgon and small panther		6	2	0.452	
	Gorgon		6	3	0.678	
34 yrs			176	64	14.464	

Other estimates show that not all dies have survived, and that something like 230,000 tetradrachms were minted for every die that has survived (Tab. 5). The relative rarity of different types of coins may also be addressed from some simple surveys done in the web-shops for ancient Greek coins. We surveyed 141 coin dealers, that displayed 158 tetradrachms of the kind we are looking at we find. Ancient imitative issues are commonly sold as the real Athenian ware by many dealers, and this is nor entirely due to ignorance. Occasionally and mostly in the obvious cases are they marked as imitative issue. This is partly due to ignorance, and that is less than satisfactory.

[6] By Sverdrup 2010 after data from Seltman 1924

Table 4. *Overview of the issue volume of the archaic style tetradrachms 527-478 BC (Adopted and derived from Seltman 1924)*

Seltman group	Coins remaining 1924	Dies 1924	Estimated issue volume, mill	Modern dating	yr	Annual issue, million tetra-drachms
H	38	14	3.16	527-515 BC	12	0.263
L	21	15	3.39	519-515 BC	4	0.848
J	21	6	1.36	515-510 BC	5	0.271
F	26	15	3.39	510-500 BC	10	0.339
G-I	76	42	9.49	510-500 BC	10	0.949
M	76	43	9.72	500-490 BC	10	0.972
G-II	85	40	9.04	490-485 BC	5	1.81
C E	72	41	9.27	485-479 BC	6	1.54
P	7	6	1.36	478 BC	1	1.360
Museums	422	225	50.85		49	1.04

Table. 5. *Approximate tetradrachm minting volumes as estimated from the available dies-volume relationship based on the observation data in Tab. 5. The year group 419-407 BC is heterogeneous, after the Sicilian disaster in 413 BC, the economy rapidly deteriorated and tetradrachm minting volume decline the years after, with very small volumes 410-407 BC. In total, it is estimated that 141 mill tetradrachms were minted.*

Time	Classification	yrs	Dies	Estimated drachm issue, million	Annual drachm issue, mill/yr	Tetradrachm issue mill/yr
477-474 BC	Starr I	4	14	12.4	3.1	0.775
473-471 BC	Starr IIa	3	15	13.3	4.4	1.108
470-468 BC	Starr IIb	3	8	6.9	2.3	0.191
467-463 BC	Starr IIc	4	25	22.5	5.6	1.406
462-458 BC	Starr III	4	12	10.6	2.7	0.663
457-455 BC	Starr IV	3	13	11.5	3.8	0.958
454-453 BC	Starr Va	2	13	11.5	5.8	1.438
452-449 BC	Starr Vb	4	35	31.7	7.9	1.981
448-441 BC	Flament I	8	22	19.8	2.8	0.619
440-432 BC	Flament IIa	9	58	52.9	5.9	1.470
431-425 BC	Flament IIb	7	105	96.4	10.6	3.442
424-420 BC	Flament IIc	5	95	86.9	17.4	4.345
419-407 BC	Flament III	13	166	152.2	11.7	2.927
406-404 BC	Emergency	3	8	6.9	2.3	0.575
Sum		65	589	532.9	8.2	2.100
394 BC		1	10	3.0	3.0	0.750
393-383 BC		10	28	6.4	0.64	0.160
Sum		76	627	541.3		

Table 6. *Observed data on dies and minting volumes in drachms and tetradrachms. The data was used to estimate from the number of surviving dies and historical records of minting volumes in silver talents and drachms, the number minted per surviving die. On the average, 226,000 tetradrachms were minted per die preserved today. We van estimate from Roman coin data that approximately 25-30,000 tetradrachms of the actual size could be minted per die, this implies that we have specimens from about 1/8 out of the dies that once existed*

Year	Dies preserved	Observed minting, million drachms	Million tetradrachms per year	Tetradrachms minted per surviving die, million tetradrachms
477 BC	3	2.8	0.70	0.233
454 BC	5	6.0	1.50	0.300
431-425 BC	105	96.4	24.10	0.230
421 BC	19	14.2	3.55	0.187
Sum	132	119.4	29.85	0.226

As a comparison, the auction house CNG is their own auction that day displayed 32 tetradrachms of this type, where 9% were archaic tetradrachm type, 38% classical tetradrachms, 6% imitations, 9% π-type tetradrachms, and 38% were of the new style tetradrachm type. The auction house CNG are among those that identify the imitative issues at times, though not consistently. These are probably biased towards the rarer, as CNG features their more exclusive tetradrachms in their own independent auctions. As a comparison, in the new search engine for ancient coins (http://www.acsearch.info/), new style coinage made up 16.6% out of a sample of 1,154 coins found. Using earlier estimates, if each die could produce approximately 28,000 tetradrachms, then originally there should have been something like 5,060 dies used. This suggests that 12.4% of all dies have survived. The period estimate is 141 million tetradrachms. The heterogeneous and the proto-new style coins are very rare, and their abundance is harder to estimate. But by looking at the different archives available, an estimate would be: Heterogeneous appear to make up 0.2% of all Athenian tetradrachms, the Proto-new style appear at approximately 0.3% of all Athenian tetradrachms preserved. The imitations and fakes in the market, excluding the modern, are approximately 30% of all tetradrachms relevant. The price in the market is of course made up of more than just rarity, aestetic tastes, historical relevance, what the collectors on the average can afford, amount of information available about the coin type, etc also play a large role. In Tables 2-5 we have put together estimates of minting volumes based on estimates of coins minted per surviving die as shown in Tab. 6. Table 7 shows the time periods they were minted.

Model description

The model is a mass balance model. It was formulated in flow charts and causal loop diagrams (CLD's). We operate with several types of tetradrachms that were issued as is shown in Tab. 6. In the model, each of these tetradrachm types have their own mass balance system as illustrated in Figs. 2 and the causal loop diagram shown in Figure 3. Each tetradrachm type is minted and issued, and then several fates may befall it: Tetradrachms may be lost to the soil: Some lost tetradrachms are found and added to the found stock (The amount of coins found is 0.7% of the available soil stock after 1610. Before that it is only 0.07%); Some of the found, are re-melted for the silver; Some are placed in museums; Some are kept in collections; Some are lost again. Some tetradrachms are destroyed by corrosion

(Copper coins by 0.1% per annum, the silver coins by 0.01% per annum). Tetradrachms may be remelted because it is worn or taken for its metal (Rate of withdrawal is varying as shown in Figure 3, and of this, 8% is lost to the soil); It may be never lost or re-melted and thus remain. The tetradrachms of Athens were minted mainly from Laurion silver, but also from re-melting of old coins and external sources like revenues from trade, war booty and foreign donations. During some periods (404-355 BC, 286-265 BC) the Laurion works had no or very low output because of military conflicts. Tetradrachms are lost in several ways, by trade, loss into the ground and by selective re-melting. The lost tetradrachms are retrieved by finding them in the soil. The input value for the minting rate to the TETRA model is shown in Figure 6 together with the intrinsic tetradrachms loss rate to re-melting and loss to the soil. The bottom curve shows rates on intrinsic loss to re-melting and physical loss. The curve is determined by occurrences of such factors as warfare, social unrest and random factors. They were parameterized by a survey of the Athenian history from 561 BC to present, looking into narratives and estimates of minting volumes, silver volumes and trade volumes. The upper curve shows the annual tetradrachm minting rate of the Athenian mint. The curve was drawn up in relative terms and then level calibrated using existing data on minting volumes from ancient literature. Input data for the TETRA model simulations. The curve is determined by occurrences of such factors as warfare, social unrest and random factors. The upper curve shows the annual minting rate of the Athenian mint. The curve was drawn up in relative terms and then the level was calibrated using existing data on tetradrachm minting volumes from ancient literature.

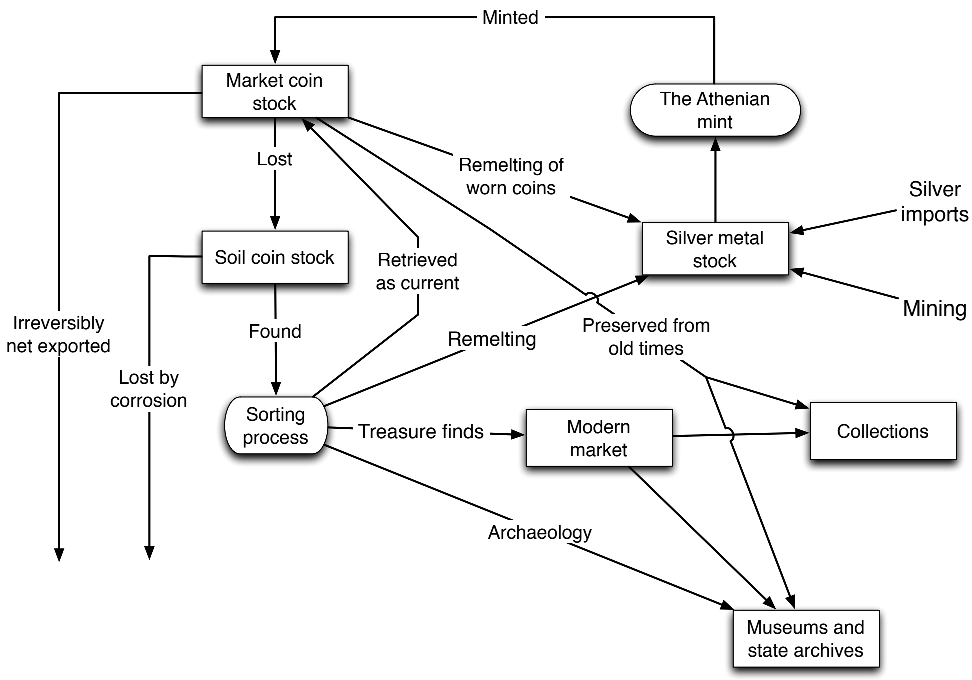

Fig. 2. *Modell overview. The flow chart for each specific tetradrachm type in the TETRA model. The tetradrachms are minted mainly from Laurion silver, but also from re-melting of old worn tetradrachms and from coin and bullion silver from external sources, like revenues from trade, war booty and foreign donations. Tetradrachms are lost in several ways, by trade, loss into the ground and by selective re-melting. The lost tetradrachms are retrieved by finding them in the soil. These tetradrachms have 3 fates; in older time they were re-melted for their metal content, archaeologists would put them into museums and private people would keep them or sell them to collectors. A fraction of the tetradrachms in the soil are also lost to corrosion.*

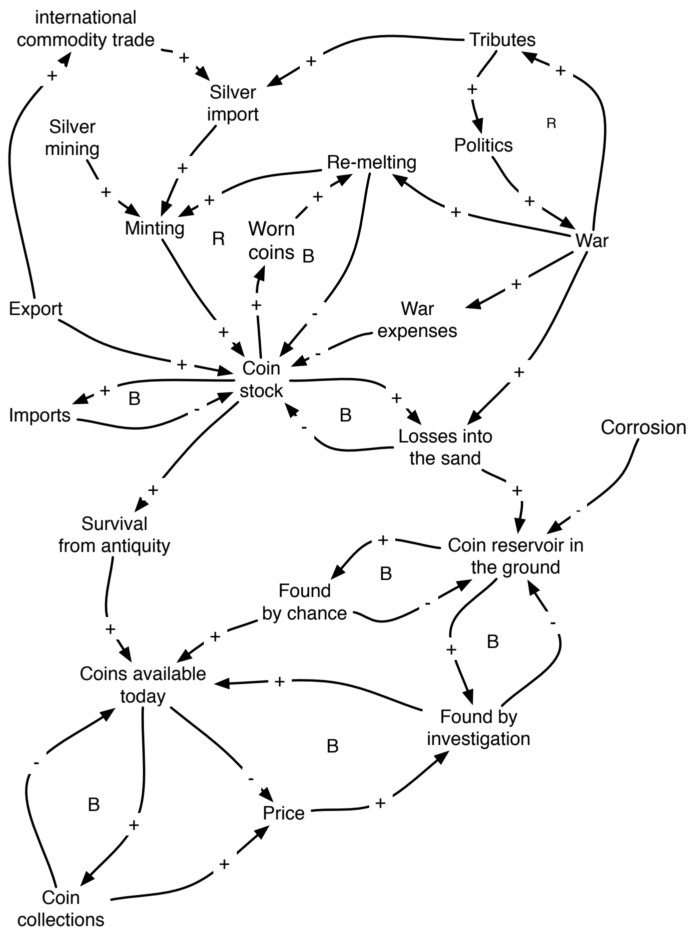

Fig. 3. *Modell overview II: Causal loop diagram for the TETRA model. The diagram shows the causal relationship employed in the modell, consistent with the flow chart in Fig. 2.*

The TETRA model was realized in the STELLA® environment and solved numerically in a 2-step Runge-Kutta algorithm, using a time step of 0.25 years (The flow chart in Figure 2, the causal loop diagram in Figure 3 and the STELLA diagrams in Figures 4-5). The outputs are taken as diagrams or tables from the STELLA® modeling environment. The TETRA model has 3 levels. It was designed with a control deck (Figure 7). With the control deck, the underlying structure is controlled, as well as it allows interactive control with some of the inputs. The diagram shown in Figure 9 is underlying the control deck. At the bottom level, all equations can be displayed. The most important output diagrams are pegged on the control deck, normally they rest on the level below.

Table 7. *Periods of minting for the different tetradrachm types.*

Type	Minting period
Archaic owls	527-477 BC
Classical owls	476-407 BC, 394-383 BC
Fourrees	406-404 BC
Transitional owls	383-286 BC
Quadridigité owls	285-262 BC
Heterogeneous owls and first series	261-169 BC
New style series	168-42 BC

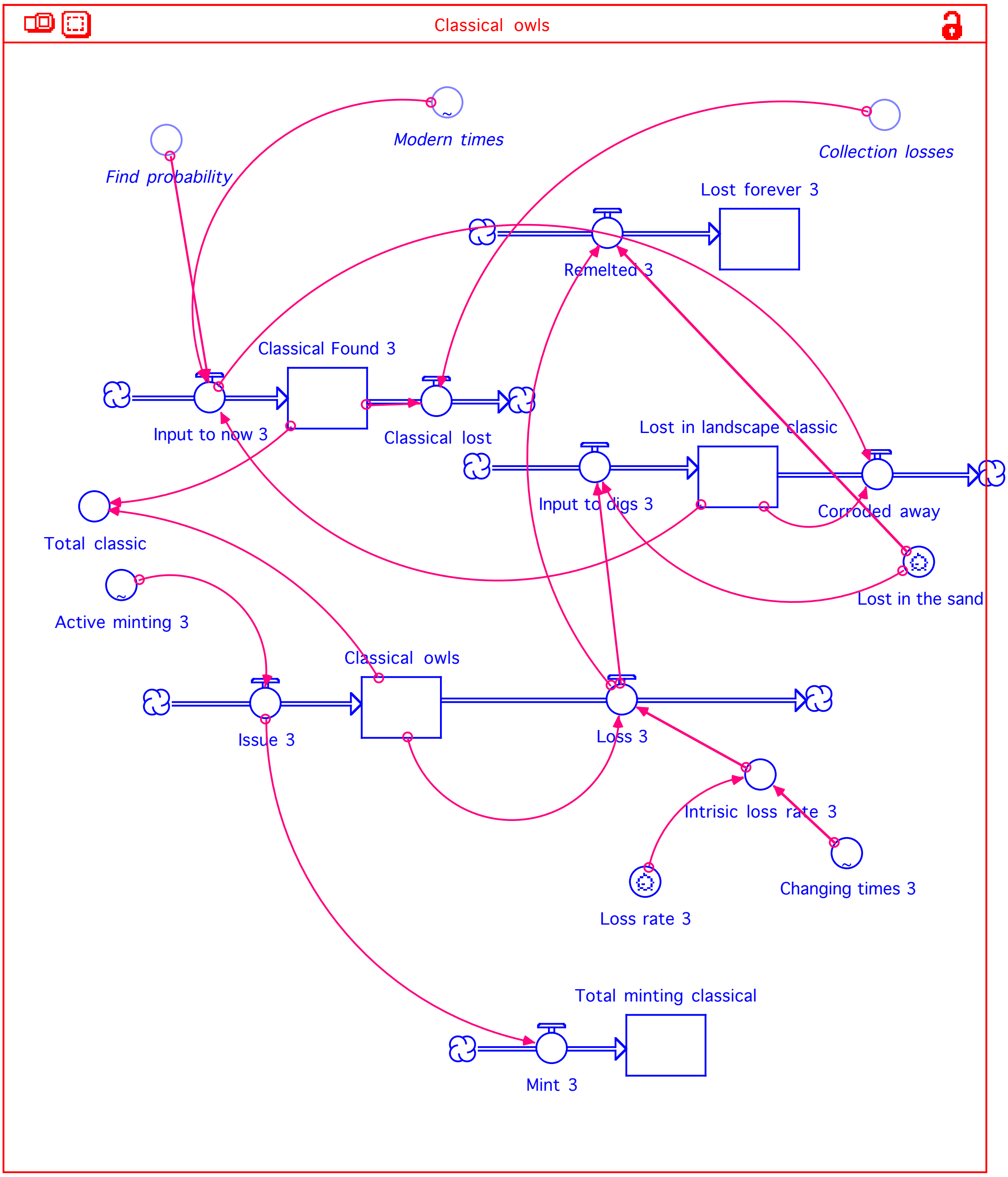

Fig. 4. *STELLA® modell diagram for each type of tetradrachms, the unit component of the model. For each tetradrachm type we have 5 stocks: market stock, soil stock, re-melted and found from soil, as well as a stock for counting up the total minted value.*

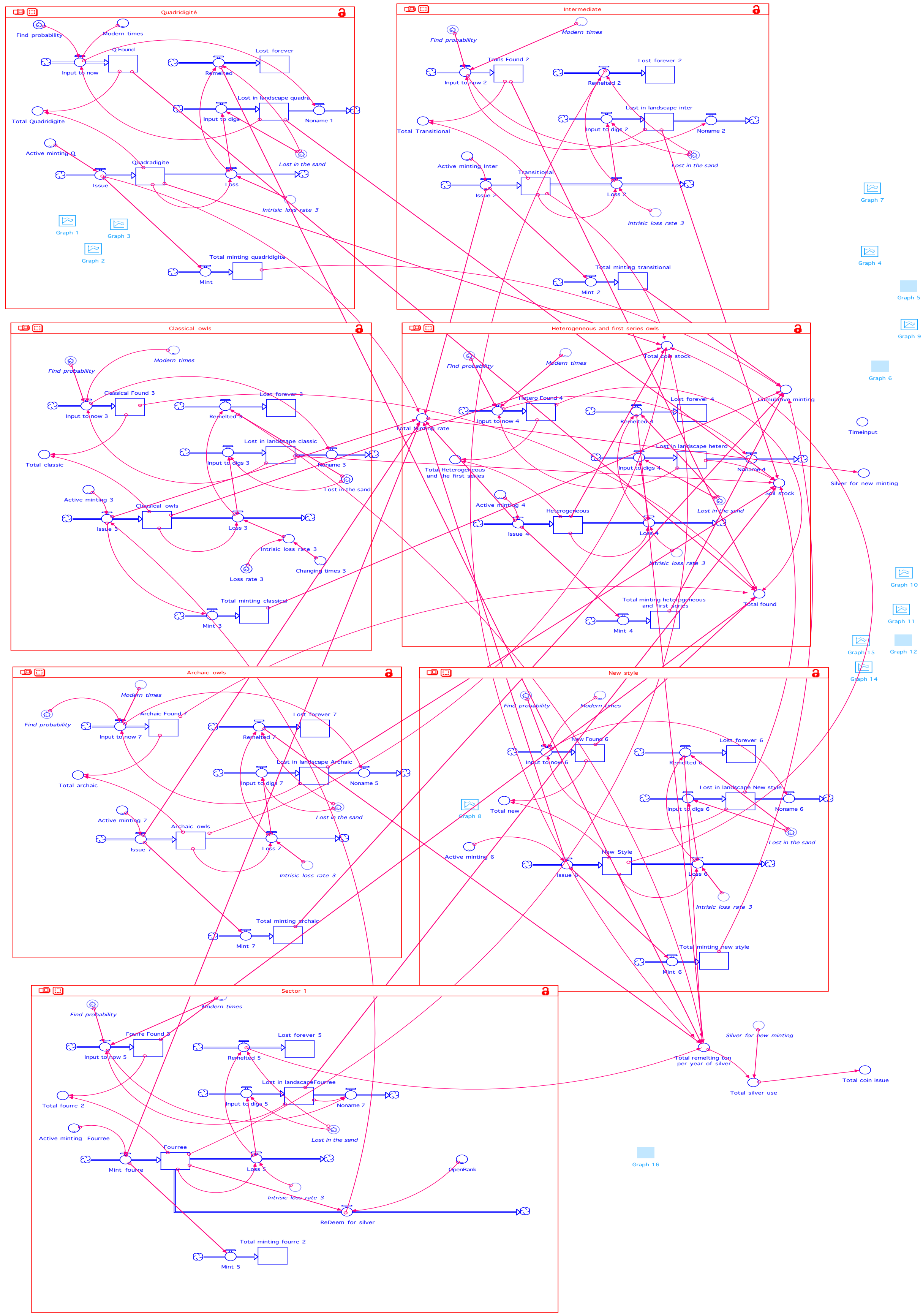

Fig. 5. *Modell overview III: STELLA® modell diagram for all types of coin: archaic, classical, fourree, transitional, heterogeneous and new style tetradrachms.*

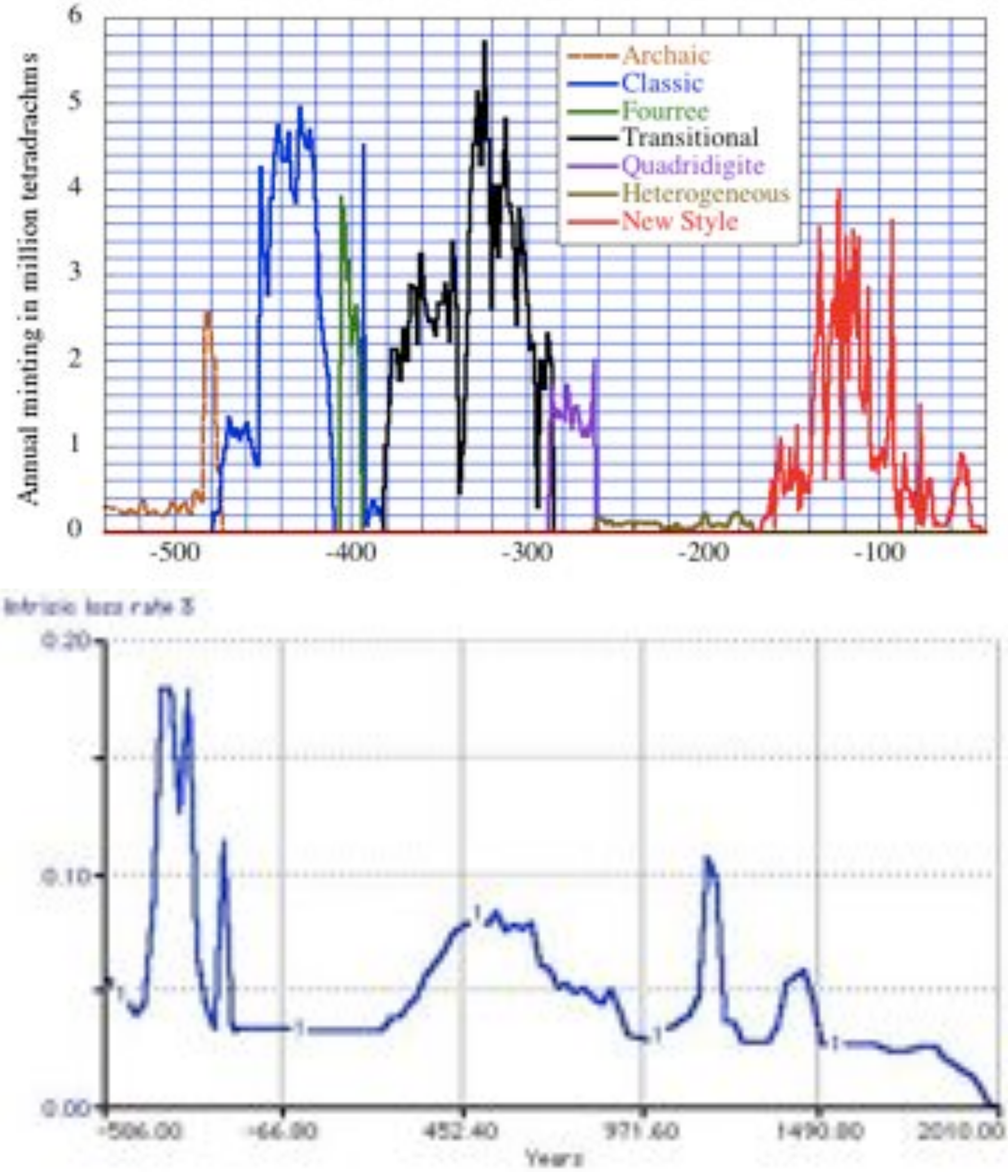

Fig. 6. *The upper curve shows the annual minting rate of the Athenian mint distributed among the diefferent coin types. The curve was drawn up in relative terms and then level calibrated using existing data on minting volumes from ancient literature. The bottom curve shows rates on intrisic loss to re-melting and physical loss. The curve is determined by occurrences of such factors as warfare, social unrest and random factors. They were parameterized by an extensive survey of the Athenian history from 561 BC to present.*

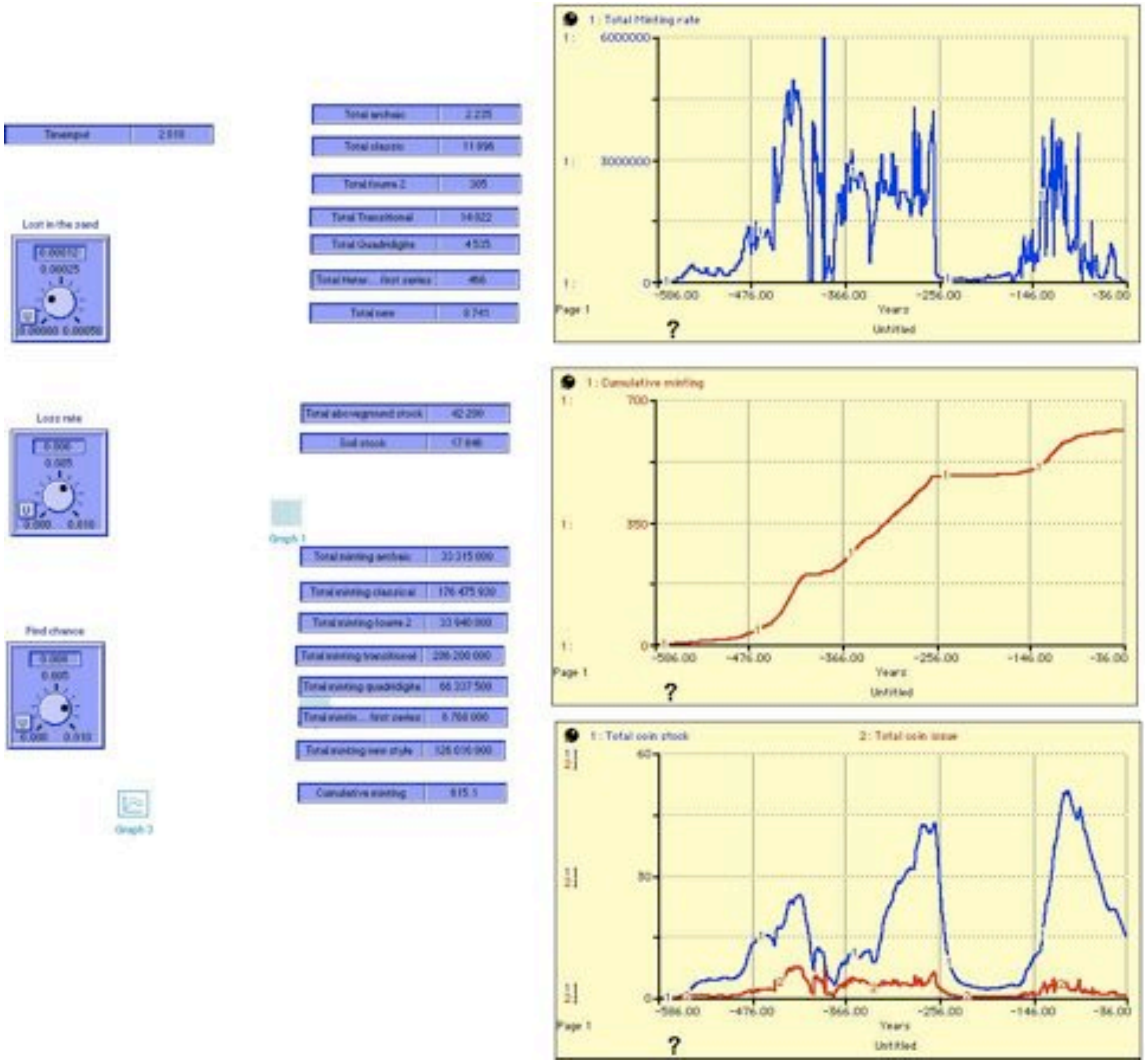

Fig. 7. *The TETRA model control deck. With the control deck, the underlying modell structure is controlled, as well as it allows interactive control with some of the inputs. The diagram shown in Fig. 8 is underlying the control deck. At the bottom level, all equations can be displayed.The most important output diagrams are pegged on the control deck, normally they rest on the level below.*

Results

The simulation was run for the years 560 BC-2010 AD. The results of the runs are presented as a number of graphs. Fig 8 shows the amount of coins of different types at any time in the Athenian market space. It is evident that Athens had 3 periods of economic flourishment; The classical period 470-410 BC; The age of Alexander 330-255 BC; The Roman rise to power 135-20 BC. Figure 8 shows the stock of tetradrachms in antiquity and how many were in circulation of each particular type at any time. Figure 9 shows the number of tetradrachms minted in total, the amount above ground in modern times, and how this has developed over time as well as the stock in the ground. Figure 10 shows the development of above ground stock of the different types 1601 to present. Finally, the model was tested against independent field estimates as is shown in Figure 11. In 1930, more coins had been found than what remains in the ground according to the estimates.

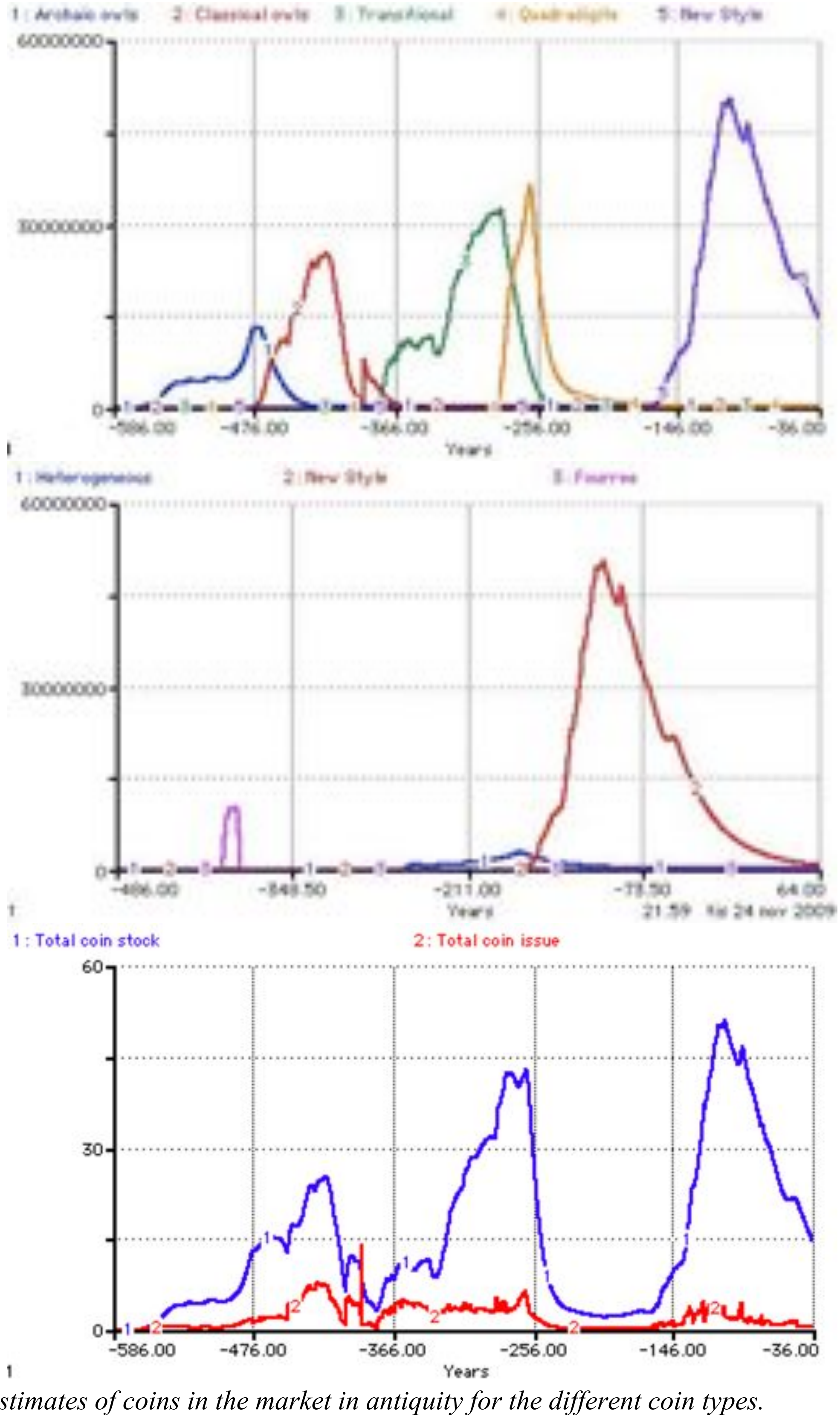

Fig. 8. *Estimates of coins in the market in antiquity for the different coin types.*

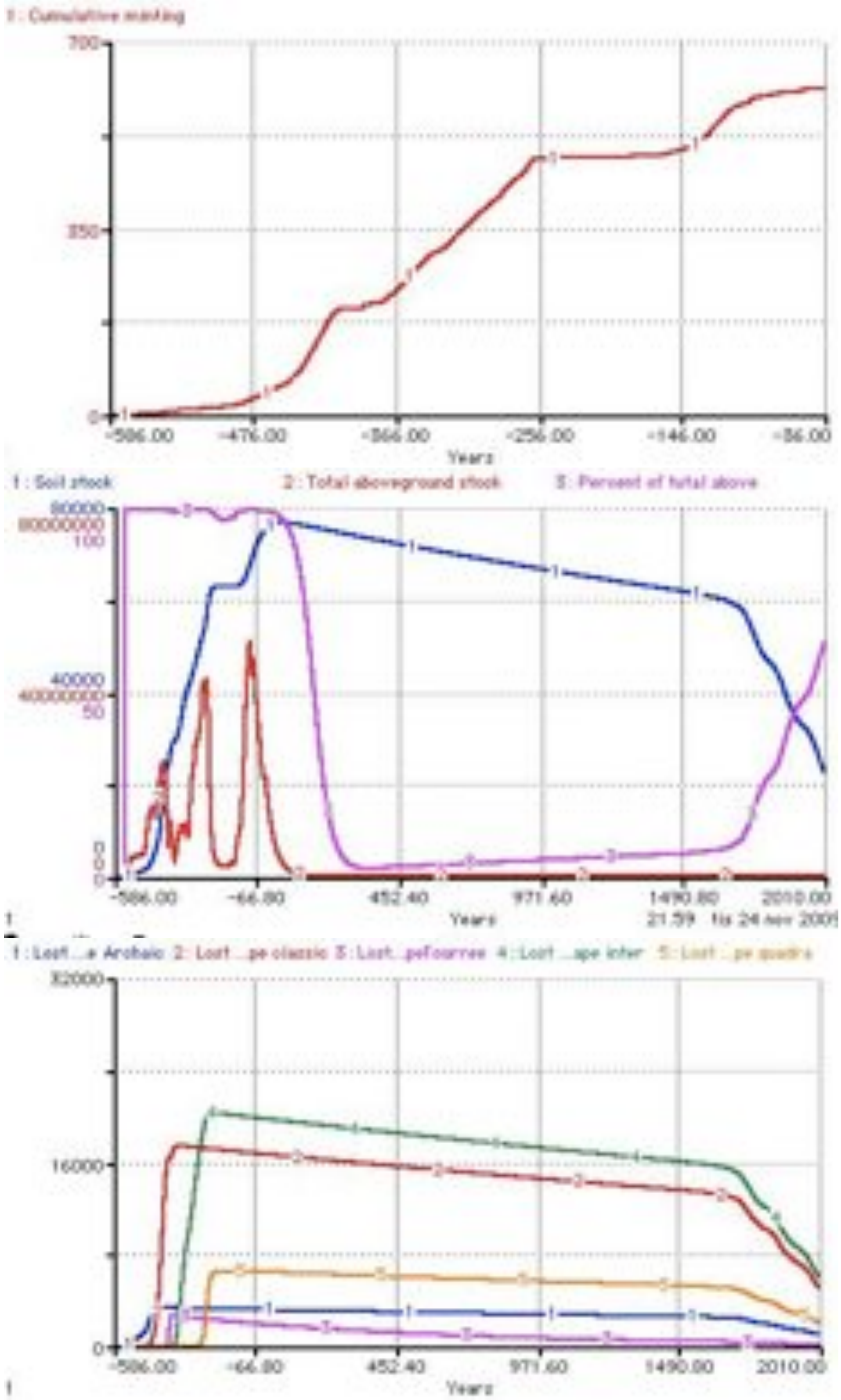

Fig. 9. *Estimated cumulative minting volume, the middle panel shows soil stock 586 BC-2010, total above-ground stock and percent of total stock as above ground. The bottom panel shows how the soil stock declines with time as coins are found and deteriorate by corrosion. The fourrees deteriorate faster as they are of copper which has a larger corrosion rate.*

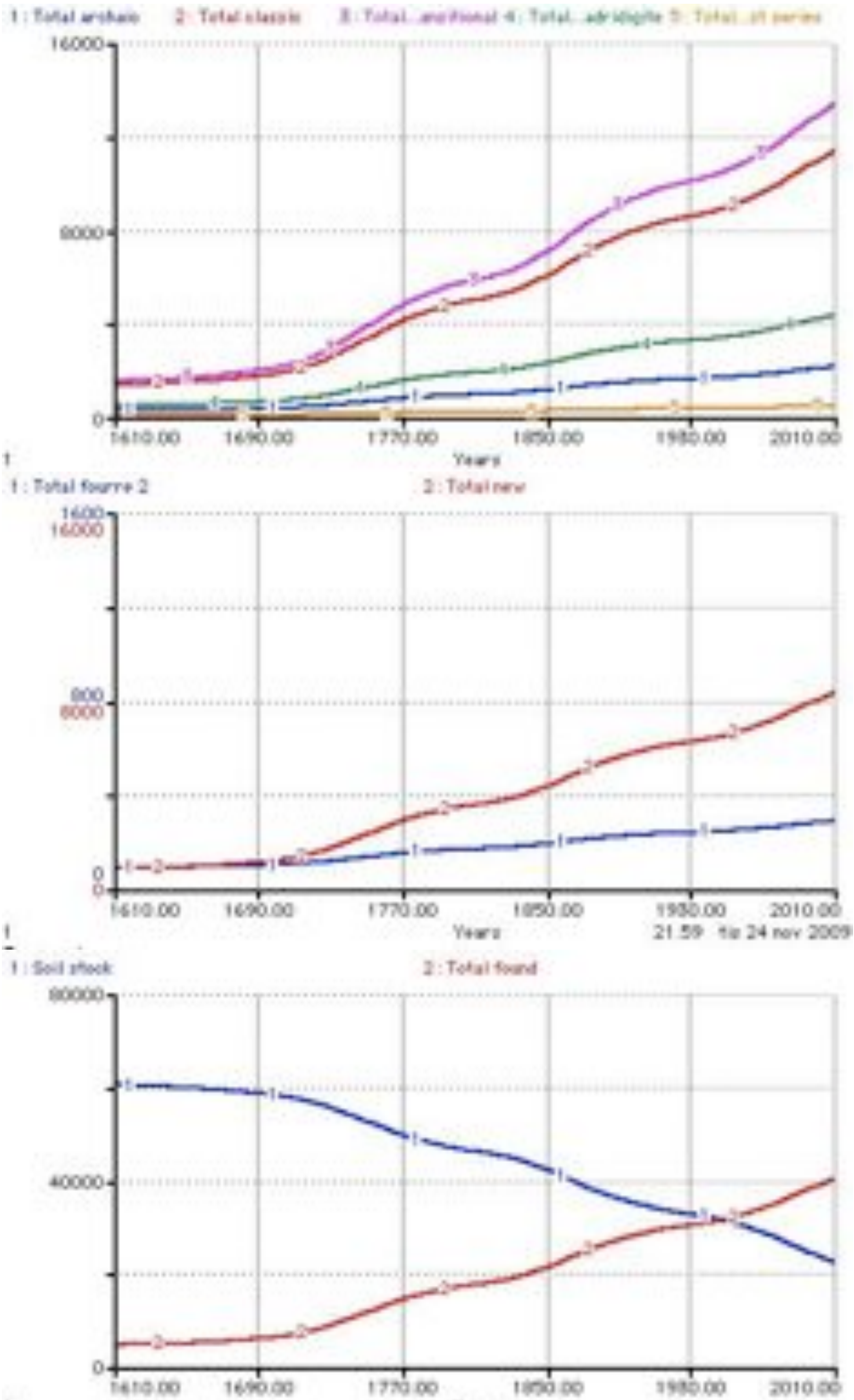

Fig. 10. *The amount of tetradrachms available above ground as a function of time between 1610 and 2010 for the different coin types, and at the bottom, the development of the soil stock and the total number found.*

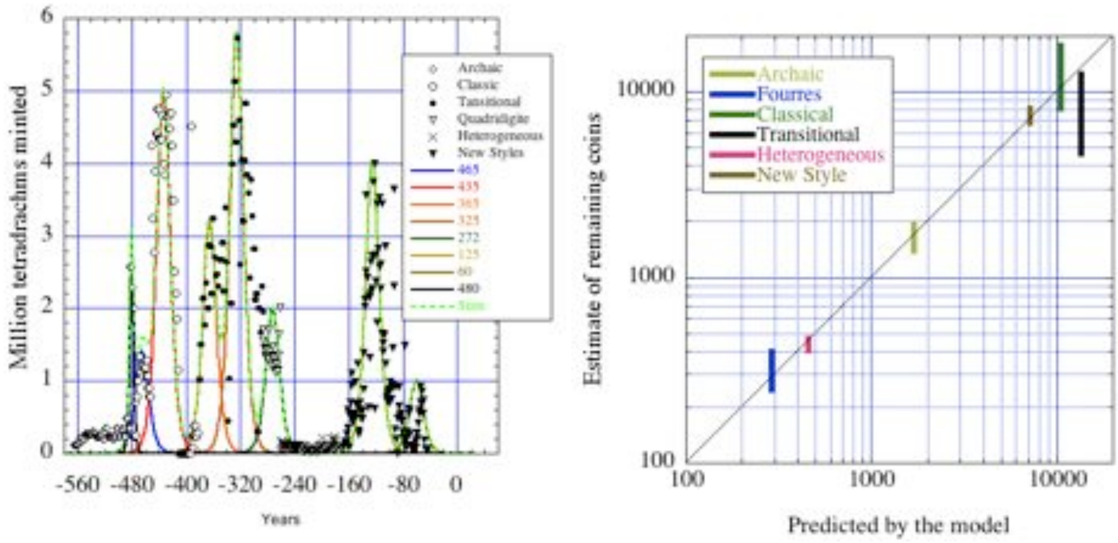

Fig. 11. *How well does the TETRA model work? This diagram shows the result of test made on independent estimates of above ground number of coins by Seltman (1924) and as can be deducted from the data of Starr (1973), Mattingly (1997), Thompson (1961, and Flament (2007). See Sverdrup (2010) for more detail and a catalogue with comments.*

We estimate that approximately 40,000 tetradrachms are above ground in 2010, while 22,000 tetradrachms are remaining in the ground. The model estimates that approximately 1,800 tetradrachms of all kinds possibly have survived above ground without ever having been in the ground. The test of the model on estimates is shown in Figure 11 and the data used for the test is Tab. 8. How well does the TETRA model work? Test made on independent estimates of above ground number of tetradrachms (Seltman 1924) and as can be deducted from the data of others (Starr 1973, Mattingly 1997, Thompson 1961, Flament 2007). The predicted values correlate to the independent estimates with a bias of 0.88 and an r^2=0.818. It implies we are making fairly accurate predictions.

Tab. 8. *Doing the field test, the testing database. The estimated surviving coin estimates come from museum inventories and surveys of auction catalogs, as well as estimates from number of dies in surviving coins.*

Type	Predicted above ground in 2010	Observed amounts above ground	Predict at reference time	Reference time
Archaic	2,336	1,400[7]-1,900	1,677	1924
Classical	11,418	8,100-17,700[8] 16,900[9]	10,500	1973 2007
Fourrees	290	250-400	287	2007
Transitional	14,022	4,600-15,500	13,450	2007
Hetero+Symbol	446	40 + 300-400	453	1990
New style	8,380	6,805[10]-8,100	7,100	1961

[7] Seltman 1924
[8] Based on a combination of information from Starr 1973 and Flament 2007
[9] Flament 2007
[10] Thompson 1961

The economy of the Laurion mining operation

The total minted volume, corresponds to 10,500 tons of silver. There was 0.5% silver in the lead extracted at Laurion, but not all silver came from there. Assuming 75% of the silver still to be Laurion silver, we can estimate that approximately 1.5 million tons of lead was extracted from those mines. This is mining on an industrial scale, and estimating for the approximately 330 years the mines were actively operated, that amounts to an average production of 4,600 tons of lead and 32 tons of silver annually. Considering that the mines employed approximately 12,000 slaves and workers, the productivity per person for the operation was approximately 330 kg lead and 2.7 kg silver per slave and year. The revenue was thus 644 drachms or 161 tetradrachms per year per worker, assuming the price of lead and silver to be 1:40. Annual revenue before costs would be for Laurion 8.65 mill drachms per annum or 2.16 mill tetradrachms per year.

Conclusions

The model works in the field tests, suggesting that we have possibly captured the important processes. Weaknesses of the model is that we have poor information of coin leakage through trade to distant lands, however, this may be seen as embedded in the loss rates. Neither can the effect of single large events be included as these cannot be quantified. The model will perform satisfactorily for a range of loss rates, and these rates cannot be estimated with great accuracy. The model may be used to estimate the relative rarity above ground for the different coin types. The model lumps coins in collections and those at museums.

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