Simulation-based Analysis of Service Levels in Stable Production-Inventory Systems

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Abstract
The performance analysis of a general production–inventory control system under uncertain demand is presented. In the model, the production order releases are determined based on the information feedback on the forecasted demand, work-in-process discrepancy and inventory discrepancy. Stability conditions are obtained in terms of the control parameters that manage the rate at which the above discrepancies are corrected. The service and cost performances of the system in terms of order fill rate, item fill rate and average system cost are analyzed for various values of the control parameters within the stability region. Additional safety stock is considered to help achieve a desired level of service (desired order fill rate). Results based on numerical simulations are presented and their implications are discussed.

Keywords: production ordering; stability; order fill rate; inventory discrepancy; safety stock.

1. Introduction

Production–inventory systems are integral to any manufacturing enterprise. An efficient and effective control scheme for such systems becomes crucial for industries to maintain their competitive edge, especially in the face of uncertain market conditions. The foundations for a control system view of production–inventory systems were laid down by Forrester (1961) in his book *Industrial Dynamics*, which is based on system dynamics methodology. The system dynamics models have since been used to capture the production-inventory order behavior at an aggregate level using information feedback structures, with the model represented by differential/difference equations (Forrester 1961, Towill 1996, Sterman 2000, Venkateswaran and Son 2007). Past works in literature have mainly focused on the stability and controllability of generic production–inventory control system models, and their implications (Ortega and Lin 2004, Disney and Towill 2002, Riddalls and Bennett 2002, Disney *et al*. 2006, Venkateswaran and Son 2007, Bijulal *et al*. 2011).

A popular production–inventory control model in literature, which is comparable to the Forrester's model, is the Automatic Pipeline and Variable Inventory and Order based Production Control System (APVIOBPCS), well studied by Disney *et al*. (2006), Dejonckheere *et al*. (2003) and others. APVIOBPCS models a production system with

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pipeline delay, and considers the discrepancies in work-in-progress (WIP) and in end-inventory to determine the production order releases. These policies have been defined in continuous domain, and the equivalent discrete-time models come under the family of order-up-to policies (Axsäter 2000).

Inventory control and production planning have also been well researched over the years from the Operations Research perspective, where the focus has been on the development of prescriptive models primarily aimed at minimizing inventory related costs (Hax and Candea 1984, Axsäter 2000). Some key performance measures identified in literature are the inventory holding costs, backlog costs, lost sales costs and service level measures of order fill rate and item fill rate. To achieve the desired level of service in face of random demand, the system typically stocks additional inventory, i.e. safety stock, which tends to increase the system cost.

A few works can be found on the cost performance of different types of production–inventory control systems (Disney and Grubbström 2004, Disney *et al.* 2006, Chen and Disney 2007, Cannella and Ciancimino 2010). All these works have analyzed the inventory and ordering cost performance of the APVIOBPCS model under different demand patterns. They have, however, focused their finding only along the Deziel and Eilon (D-E) setting (Deziel and Eilon 1967). The D-E setting corresponds to the scenario where the rate of adjustment for WIP discrepancy equals the rate of adjustment of inventory discrepancy. Bijulal *et al.* (2011) have attempted to establish the variations in order fill rates obtained throughout the stability region of a slightly different production–inventory control system. However, the applicability of their model is restricted by the definition that a period's demand is considered fulfilled only if there is sufficient inventory at the start of the period to satisfy the demand.

In this paper some results from ongoing research work on the performance analysis of a general production–inventory control system model under uncertain demand is presented. In the model, the production order releases are computed based on the forecasted demand, adjustments for WIP discrepancy and adjustments for inventory discrepancy. The rates of adjustments of these discrepancies are identified as the system control parameters. Stability conditions based on these parameters are derived. The service and cost performances of the system in terms of order fill rate, item fill rate and average system cost are analyzed for various values of the control parameters within the stability region. Additional safety stock is handled in the system to help achieve higher levels of service. Results based on numerical simulations are presented and their implications are discussed.

2. Production–Inventory Model

2.1 Notations Used

Symbols, notations and abbreviations used in this article are summarized in Table 1.

2.1 Model Description

The production–inventory control structure model discussed in this paper is similar to the classical industrial dynamics model by Forrester (1961), and those studied by Bijulal *et al.* (2011) and is also comparable to the APVIOBPCS family of models. The equations underlying the model are as follows:

$$FD_n = FD_{n-1} + \rho \cdot (CD_{n-1} - FD_{n-1}) \quad (1)$$

$$INV_n = INV_{n-1} + PCR_{n-1} - CD_{n-1} \quad (2)$$

$$WIP_n = WIP_{n-1} + PREL_{n-1} - PCR_{n-1} \quad (3)$$

$$PCR_n = PREL_{n-L} \tag{4}$$

$$PREL_n = FD_n + WIPADJ_n + INVADJ_n \tag{5}$$

$$WIPADJ_n = \alpha \cdot (DWIP_n - WIP_n) = \alpha \cdot (L \cdot FD_n - WIP_n) \tag{6}$$

$$INVADJ_n = \beta \cdot (DINV_n - INV_n) \tag{7}$$

Table 1: Notations Used

Symbol	Description	Units
α	Fractional rate of adjustment of WIP discrepancy	(*rate*) 1/*time*
β	Fractional rate of adjustment of inventory discrepancy	(*rate*) 1/*time*
ρ	Smoothing factor (constant) for forecast	
L	Production delay	*time period*
T_w	Time to adjust WIP discrepancy	*time period*
T_i	Time to adjust inventory discrepancy	*time period*
CD_n	Demand in period *n*	*units/ time period*
$DINV_n$	Desired Inventory in period *n*	*units*
$DWIP_n$	Desired work-in-process in period *n*	*units*
FD_n	Demand forecast for period *n*	*units/ time period*
INV_n	Inventory at the start of period *n*	*units*
$INVADJ_n$	Adjustments for inventory discrepancy in period *n*	*units/ time period*
PCR_n	Production completion rate in period *n*	*units/ time period*
$PREL_n$	Production order release in period *n*	*units/ time period*
WIP_n	Work-in-process at the start of period *n*	*units*
$WIPADJ_n$	Adjustments for WIP discrepancy in period *n*	*units/ time period*
b	Cost of backordering one unit per period	*Rupees/unit/time*
h	Cost of on holding unit of inventory per period	*Rupees/unit/time*
k_{DOFR}	Scaling factor for safety inventory	
s_D	Estimated standard deviation of demand	
ASC_n	Average system cost until period *n*	*Rupees.*
ABC_n	Average backorder cost until period *n*	*Rupees.*
AHC_n	Average inventory holding cost until period *n*	*Rupees.*
$DOFR$	Desired order fill rate	
IFR_n	Item Fill Rate until period *n*	
OF_n	0-1 variable. 1 indicates demand is fulfilled in period *n*	
OFR_n	Order Fill rate until period *n*	

The forecasted demand (FD_n) for period *n* is based on the first order exponential smoothing of the previous period's demand (CD_{n-1}), with smoothing constant ρ (Equation 1). The finished goods inventory INV_n at the start of any period is the previous period's starting inventory (INV_{n-1}) plus the difference of the previous period's production PCR_{n-1} and demand CD_{n-1}, as shown in Equation (2). Negative inventory represents backordered quantities. Similarly, system WIP at the start of a period is the previous period's starting WIP (WIP_{n-1}) plus the difference of the previous period's production orders $PREL_{n-1}$ and production, as shown in Equation (3). Equation (4) represents the production completion rate PCR_n in the system as a pipeline material delay process to the production release, with fixed delay L. The production order release for period *n*, $PREL_n$, is the sum of demand forecast, FD_n, the adjustment for WIP discrepancy, and the adjustment for inventory discrepancy, as shown in Equation (5). The WIP discrepancy is adjusted by a fractional rate α (Equation 6). The inventory discrepancy is adjusted by a fractional rate β (Equation 7). It is noted that feedback gains modeled here as α and β are modeled as $1/T_w$ and $1/T_i$, respectively, in the past literature.

Typically, the desired WIP and desired inventory levels are computed based on the expected system performance in steady or equilibrium state. In steady state, the system inflows balance the outflows such that the stock levels (i.e. system state) remain the same (stable). Thus, in steady state the forecasted (expected) demand (FD_n) will equal the mean end customer demand, and production order release (and production rate) will tend towards the expected demand. Little's Law states that in steady state the expected WIP in the system is the product of the expected throughput rate and the lead time (L). Hence the desired WIP is set as the product of forecasted demand and the production lead time (Equation 6).

Now, the desired inventory ($DINV_n$) can be set to 0, in a bid to operate lean and minimize inventory holding costs. However this settings ($DINV_n$=0) may not provide adequate inventory coverage in the face of random demand. In order to achieve higher inventory coverage (and hence higher service level) the desired inventory in the system has to been modified by adding a safety stock.

2.3 Calculation of Safety Stock

Suppose that the demand (CD_n) in each period n is independent and Normally distributed with mean μ_D and standard deviation σ_D. The enterprise may carry some safety stock to cover the random variations in demand. The amount of variations covered by the safety stock will depend on the desired probability of stock out or the customer service level. Desired order fill rate (*DOFR*), a service level measure, is defined as the proportion of periods in which demand is fulfilled entirely. Since there is some lead time involved to produce the orders and replenish the end inventory, the *DOFR* can also be viewed as the probability of not having a stock out situation during the replenishment lead time (stock-out occurs when demand exceeds available stock). Thus safety stock is the amount of additional inventory to be stocked to achieve the desired fill rate, and is given as $SafetyStock = k_{DOFR} \cdot s_R$. In the above equation, safety factor, $k_{DOFR} = \Phi^{-1}(DOFR)$, where $\Phi(\cdot)$ is the standard normal cumulative distribution function, and s_R represents the standard deviation of demand during the replenishment lead time. In the system dynamics model under study, there is a one period ordering delay and L periods of production delay, resulting in a total of L+1 periods of replenishment delay. Since demand in each period is independent, the safety stock can be written as shown in Equation (8), where s_D represents the estimated standard deviation of demand.

$$SafetyStock = k_{DOFR} \cdot s_D \cdot \sqrt{L+1} \tag{8}$$

2.3 Calculation of Desired Inventory

Based on how the demand is being fulfilled, different settings of desired inventory (*DINV*) becomes necessary: (i) A period's demand is fulfilled from the inventory available at the start of that period. That is, the production in a period is not used to satisfy the demand in that period. In this case, there may not be sufficient inventory to meet the demand in that period, but still there could be a positive inventory holding at the start of the next period when the latest production quantity is included to inventory. The desired inventory required would be the mean demand plus safety stock (a detailed analysis of this scenario has been presented in Bijulal *et al.* 2011). (ii) A period's demand is fulfilled (say, at the end of the week) from the inventory available at the start of that period *plus* the production in that period. In this case, when the demand in a period is not fulfilled, the excess demand is backordered (captured as negative inventory). This latter scenario is modeled and analyzed in this paper.

Since the demand is fulfilled (say, at the end of the week) from the inventory available at the start of that period *plus* the production in that period, desired inventory (*DINV*) can be set to cover only the variability in the demand, as follows:

$$DINV_n = SafetyStock = k_{DOFR} \cdot s_D \cdot \sqrt{L+1} \tag{9}$$

2.3 Measuring System Performance

Order fill rate (*OFR*), Item fill rate (*IFR*) and average system cost per period (*ASC*) are the system performance measures selected to analyze the system behavior under stable parameter settings. *OFR* is the proportion of periods that the demand was satisfied in entirety, and is modeled using Equation (10) and (11).

$$OFR_n = \frac{1}{n}\sum_{i=1}^{n} OF_i \tag{10}$$

$$OF_i = \begin{cases} 1 & PCR_i + INV_i \geq CD_i \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

IFR is the average fraction of demand units fulfilled during any period of time, and is modeled using Equation (12).

$$IFR_n = 1 - \frac{\sum_{i=1}^{n} INV_i^-}{\sum_{i=1}^{n} CD_i} \tag{12}$$

The system cost has been assumed to have two components: the holding costs and the backorder costs. Every unit stocked in inventory and carried over to the next period incurs cost *h* per period and every demand unit backordered incurs cost *b* per period. The average system cost (*ASC*) has been estimated as:

$$ASC_n = \frac{1}{n}\sum_{i=1}^{n}(\underbrace{h \cdot INV_i^+}_{AHC_n} + \underbrace{b \cdot INV_i^-}_{ABC_n}) \tag{13}$$

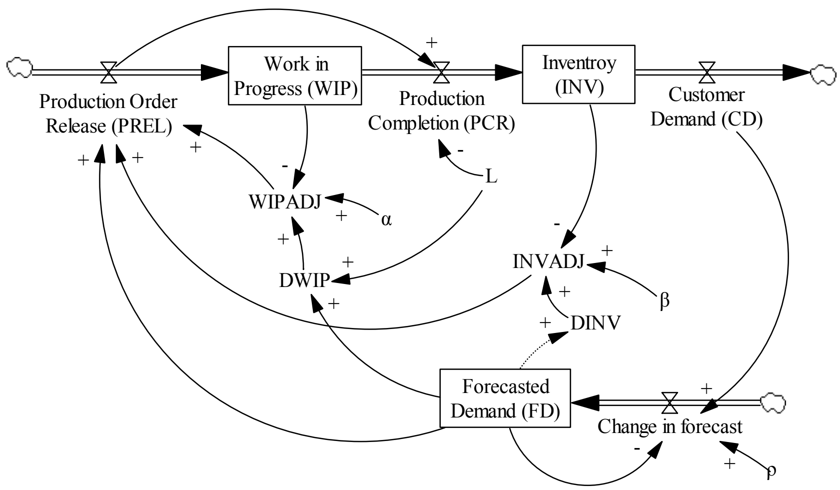
where INV_i^+ and INV_i^- represents the excess inventory and backorder quantities, respectively.

$$INV_i^+ = \begin{cases} INV_i & INV_i \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad INV_i^- = \begin{cases} -INV_i & INV_i < 0 \\ 0 & \text{otherwise} \end{cases}$$

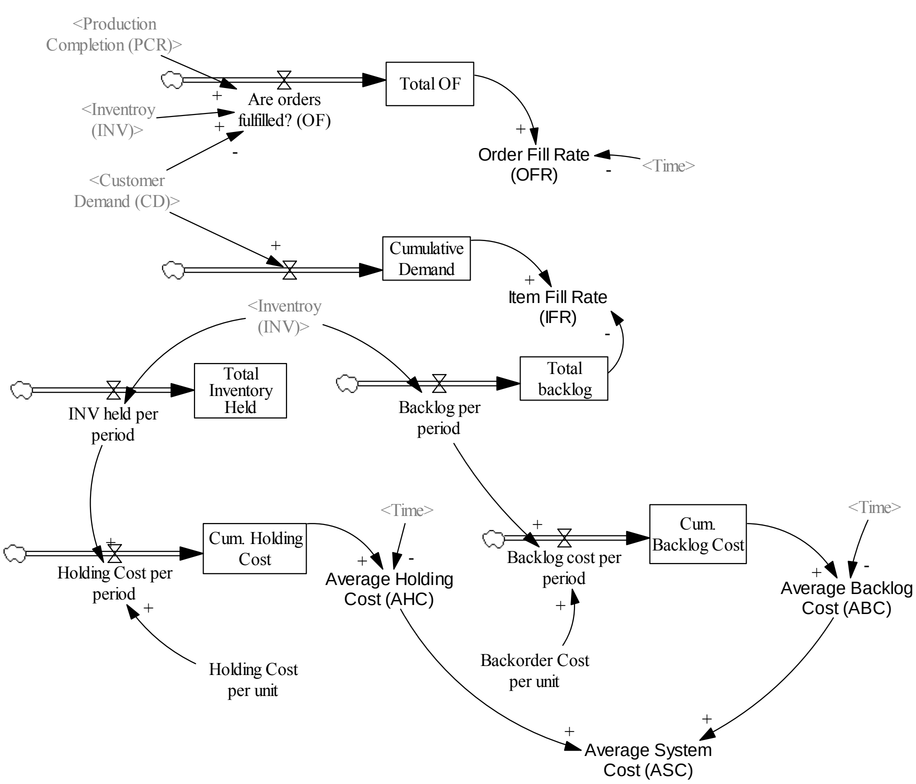
As mentioned earlier, the demand (CD_n) in each period *n* is independent and Normally distributed with mean μ_D and standard deviation σ_D. A stock out is said to occur if the sum of the inventory at the start of a period and production in that period is less than the demand in that period. Suppose the desired inventory (*DINV*) is set as 0, then the demand in the period is almost entirely met by that period's production. Now, the expected production rate tends to the mean demand at steady state. Therefore the probability of stock out will be ~0.5. The probability that the order is fully met from stock, the order fill rate (*OFR*), then becomes 1 – *Pr{stock out}* ≈ 0.5. Hence the production–inventory system presented in Section 2 can be expected to give OFR of about 50% when $DINV_n$=0.

Suppose the desired inventory is set as per Equation (9); still, the system may not achieve the desired order fill rate as the system performance is influenced by parameters α, β and ρ. Also, the *DINV* is a function of s_D, the estimated standard deviation of demand (see Equation 9). Hence the accuracy of this estimate will also influence the system performance.

The production–inventory model, as described in this section, is represented in Figure 1 as a stock and flow diagram.



(a) Sub-model of the production-inventory control system



(b) Sub-model of performance measurement

Figure 1: Stock Flow Diagram of the Production-Inventory Control System

3. Region of Stability of the Production-Inventory System

The production inventory system is represented by equations (1) – (13). The conditions for stability are obtained from the transfer function derived from the z-transforms of Equations (1)–(7), (9). Equations (10) – (13) models the performance measures that do not influence the model dynamics, and hence are not required for stability analysis. To derive the system transfer function, the difference equations in time presented in Section 2 are transformed onto the z-domain, presented in Equation (14) – (20). Equation (20) is the transformed equation after combining Equations (7) and (9).

$$FD(z) = \frac{\rho}{z+\rho-1} CD(z) \tag{14}$$

$$INV(z) = \frac{1}{z-1}\left(PCR(z) - CD(z)\right) \tag{15}$$

$$WIP(z) = \frac{1}{z-1}\left(PREL(z) - PCR(z)\right) \tag{16}$$

$$PCR(z) = PREL(z) / z^{L} \tag{17}$$

$$PREL(z) = FD(z) + WIPADJ(z) + INVADJ(z) \tag{18}$$

$$WIPADJ(z) = \alpha \cdot \left(L \cdot FD(z) - WIP(z)\right) \tag{19}$$

$$INVADJ(z) = \beta \cdot (k_{DOFR} \cdot s_{D} \cdot \sqrt{L+1} - INV(z)) \tag{20}$$

The above simultaneous equations in 'z' are solved to obtain the system transfer function between output variable *PREL* and input variable *CD*, as shown in Equation (21).

$$\frac{PREL(z)}{CD(z)} = \frac{z^{L}(z-1)\{(z-1)(1+L\alpha)\rho + (1+k_{DOFR}s_{D}\sqrt{L+1}(z-1))(z+\rho-1)\beta\}}{(z+\rho-1)(z-1)\{z^{L+1} + z^{L}(\alpha-1) - \alpha + \beta\}} \tag{21}$$

Equation (20) shows the general expression of the system with a fixed production delay L, the smoothing factor of demand forecast ρ, fractional adjustment rates α and β, the scaling factor of safety stock (k_{DOFR}) and the estimated standard deviation of demand s_D. It is observed that the denominator polynomial is independent of k_{DOFR} and s_D, which implies that these parameters (and hence desired inventory value) do not affect the stability of the system. However, k_{DOFR} and s_D appear in the numerator polynomial and hence can be expected to affect the dynamics of the system response.

To determine the stability bounds, the denominator polynomial needs to be solved. Since it is not possible to solve for general L (due to presence of z^L), a fixed pipeline delay of L = 3 periods has been assumed. All further discussions in this article pertain to a model with pipeline delay of 3 periods. Equation (22) represents the system transfer function with L = 3.

$$\frac{PREL(z)}{CD(z)} = \frac{z^{3}(z-1)\{(z-1)(1+3\alpha)\rho + (1+2k_{DOFR}s_{D}(z-1))(z+\rho-1)\beta\}}{(z+\rho-1)(z-1)\{z^{4} + z^{3}(\alpha-1) - \alpha + \beta\}} \tag{22}$$

The transfer function system presented in Equation (22) is stable, in the bounded-input bounded-output (BIBO) sense, if the roots of the denominator polynomial are inside the unit circle in the complex plane (Venkateswaran and Son 2007). The conditions for stability in terms of the parameters α, β and ρ have been obtained using Jury's test (1964). Equations (23)–(26) show the 'binding' conditions for stability. Equation (23) represents the expression for the right boundary, Equation (24) represents the expression for the lower boundary and Equation (25) represents the upper boundary. The region of stability defined by these equations in the (α, β) plane is shown in Figure 2. It is further assumed that (α, β and ρ) are non-negative.

$$\alpha \geq \frac{3\beta^2 - 4\beta - 2 - (\beta - 2)\sqrt{4 + \beta^2}}{2(-3 + 2\beta)} \tag{23}$$

$$\alpha \geq \frac{3\beta^2 - 4\beta - 2 + (\beta - 2)\sqrt{4 + \beta^2}}{2(-3 + 2\beta)} \tag{24}$$

$$\alpha \leq \frac{\beta + 2}{2} \tag{25}$$

$$\rho \leq 2 \tag{26}$$

The system guarantees to be stable when the values of α and β are inside the stable region. Parameter selection on the boundary makes the system variables to continue sustained oscillations; while parameter selection outside the boundary causes the system variables to continue oscillations with exponentially increasing amplitude. Preliminary simulation runs with i.i.d. Normal demand revealed that time varying stochastic input to the system does not affect the system's BIBO stability. Also, it has been found that the system performance measures *OFR*, *IFR* and *ASC* deteriorates for unstable parameter selections. Hence, the investigations of system performances presented in this article are limited within the stable region of the parameter setting.

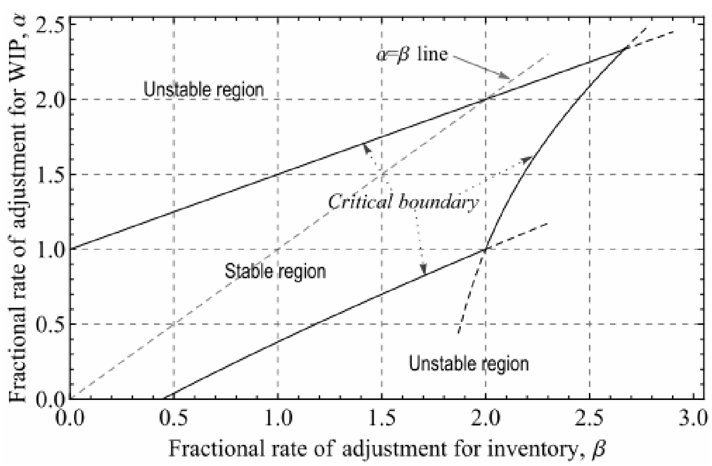


Figure 2: Stability boundary of the production-inventory system with L = 3
(*Source: Bijulal et al. 2011*)

4. Experimental Settings

System performance measures *OFR*, *IFR* and *ASC* (Equation (10)–(13)) for various settings of the parameters α, β and ρ and the methods of computing s_D have been analyzed.

In order to understand the impact of s_D on the system performance, three different settings are taken. In the first case, the estimated standard deviation of demand equals the actual standard deviation of demand ($s_D = \sigma_D$). This captures the scenario where the demand variance is known and hence used in decision making. In the second case, $s_D = \nu FD_n$ where the coefficient of demand variation $\nu = \mu_D/\sigma_D$. This captures the scenario where ν of demand process is known and the demand forecast FD_n is a true estimate of the mean demand

(Makridakis et al. 2005, Axsäter 2000). In the third case, s_D is computed dynamically as the sample standard deviation of the observed demand process, i.e. $s_D = \frac{1}{n-1}\sum_{i=1}^{n}(CD_i - FD_i)^2$. This captures the scenario where the demand process is unknown.

To observe the effect of smoothing factor, ρ, on the system performance, three settings of ρ, have been selected as 0, 0.2 and 1. When $\rho = 0$, the forecasted demand remains unchanged from its initial value throughout the time horizon.

A total of about 335 (α, β) pairs spread throughout the stable region (see Figure 2) have been taken for the simulation study. The points are listed in Table A1 in the Appendix.

Now, it can be expected that the setting ρ=0, α=1, β=1 achieve the desired OFR since at this setting the WIP and inventory discrepancies are always fully accounted for and no noise due to forecasting is part of the ordering process.

4.1 General Simulation Settings

The production lead time L has been fixed as 3 time periods. The values of backorder cost b and holding cost h are both taken as Rs. 2 per item per period. *DOFR* has been selected as 80% which results in $k_{DOFR} = \Phi^{-1}(DOFR) = 0.85$. The initial values of the stocks, FD_0, WIP_0 and INV_0 are set equal to the mean demand, desired WIP and desired inventory values respectively. The demand in each period has been assumed to be *Normal*(1000, 10). It has been ensured that the same random demand pattern is used across multiple simulation runs, which allows for a valid comparison of the results.

The simulation model as described by the Equations (1) – (13) is modeled and analyzed using Powersim® 2.51. The simulation run length is kept as 3650 time periods, with update interval as 1 time period. As it was observed that the system reached steady state in the early part of the run, the replication length of 3650 has been found to be adequate for this study. A round-off error of up to 0.01 was assumed to be acceptable in evaluating the conditional statement in Equation (11).

5. Results and Observations

A preliminary set of experiments were conducted with $DINV_n = 0$ (no safety stock) to determine the impact of random demand on system performance. All other system parameters are as described in Section 4. The results are summarized in Table 2. For both settings ρ=0.2 and ρ=1, the best performance (max *OFR*, max *IFR* and min *ASC*) was obtained at (α, β) = (0, 0). Apart from this particular point, the *OFR* response of all other points was quite close to 0.5, validating our earlier claim. The high values of *IFR* (99%) are attributed to the low standard deviation of demand as compared to the mean demand. Also, it is seen that with ρ=1, the service levels have improved, but the *ASC* has worsened.

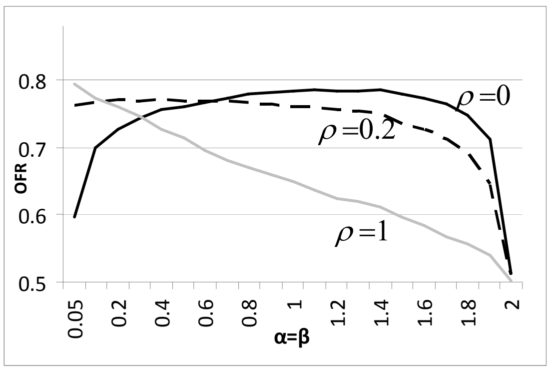
Table 2: System performance with no safety stock

	ρ=0.2					ρ=1			
	Max	**min**	**mean**	**median**		**max**	**min**	**mean**	**Median**
OFR	0.6201	0.4901	0.5046	0.5013		0.6695	0.4934	0.5071	0.5008
IFR	0.9935	0.8979	0.9823	0.9898		0.9955	0.5271	0.9485	0.9798
ASC	408.41	34.45	71.52	40.76		1891.07	35.09	206.90	80.78

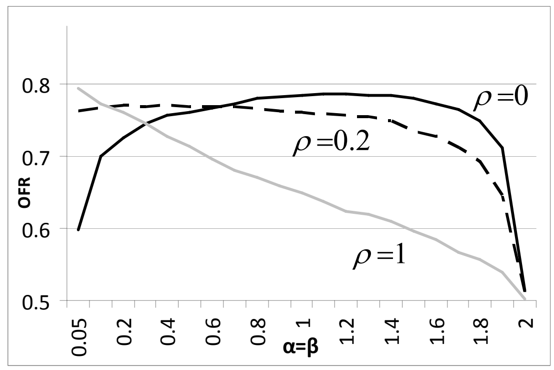
Next, experiments have been conducted by considering safety stock, i.e., $DINV_n$ is set as per Equation (9), with desired order fill rate (*DOFR*) as 80%. The variation of the performance parameters, in response to the different settings of s_D, and ρ over the (α, β) plane is analyzed. A total of 335×3×3=3015 experiments have been conducted.

Recall that the estimated standard deviation of demand, $s_D = \sigma_D = 10$ in the first case, for all settings of ρ. In the second case, $s_D = \nu FD_n = 0.01FD_n = 10$ when ρ=0; the s_D varied between 9.95 and 10.05 when ρ=0.2; and the s_D varied between 9.8 and 10.2 when ρ=1. In the third case, s_D is the sample standard deviation, which stabilized at: 10.03 when ρ=0; 10.55 when ρ=0.2; and 14.00 when ρ=1.

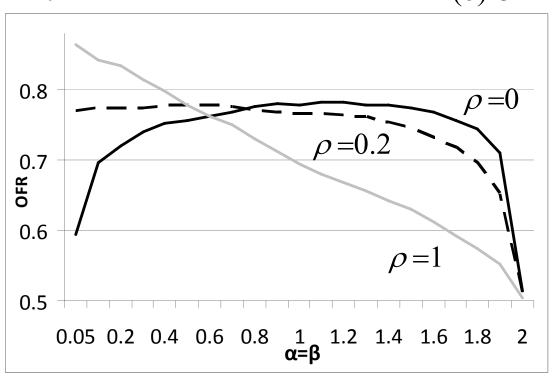
Based on the results, it is observed that the order fill rates obtained are higher and closer to the *DOFR* when safety stock is considered, as expected. For (α, β) points on the stability boundary, the *OFR* obtained were between 0.48–0.53 across all settings of s_D, and ρ. This is again as expected since these critically stable points will cause sustained oscillation, with constant amplitude, of the system output *PREL*.



(a) *OFR* for $s_D = 10$



(b) *OFR* for $s_D = \nu.FD_n$



(c) *OFR* for s_D = sample standard deviation of demand process

Figure 3: *OFR* obtained under different setting s_D and ρ across along D-E line

Figure 3(a)–(c) illustrates the achieved *OFR* for different combinations of s_D and ρ across various α=β settings. The α=β setting indicates equal weightage to both WIP and inventory discrepancy in the ordering scheme, and is commonly referred to as the Deziel-Eilon (D-E) line (Deziel and Eilon 1967). The (α, β) points used to generate the above curves are given in Table A2 in the Appendix. It can be seen from Figure 2 that (α, β) values greater than 2 along the D-E line will result in unstable system response.

Some common patterns of behavior are observed for all three settings of s_D (see Figure 3). When ρ=0, three distinct regions of OFR can be observed as one moves along the D-E line: (i) Initially, there is a steep increase in the *OFR* as the (α, β) value is increased from 0 along the D-E line, with the *OFR* reaching about 0.70 at value 0.2 on the D-E line; (ii) The *OFR* then stabilizes around 0.78 for (α, β) values between 0.7 to 1.6 on the D-E line; (iii) Finally, a steep drop in the *OFR* is observed for (α, β) values beyond 1.8 on the D-E line, with the *OFR* reaching about 0.70 at value 2.0 on the D-E line. Interestingly, when ρ=0.2, low settings on the D-E line (< 0.6) results in high *OFR* of about 0.77; mid settings on D-E line (1.2 to 1.8) achieves a lower *OFR* of about 0.72; and high settings on the D-E line (> 1.9) results in a steep drop in OFR. Now, when ρ=1, it can be seen that *OFR* steadily deteriorates from a high OFR of 0.8 (at α=β=0.05) to a very low *OFR* of 0.5 (at α=β=2.0) as one moves upwards along the D-E line. It is seen that when s_D is computed dynamically as the sample standard deviation of the observed demand process, an *OFR* of about 0.85 is achieved at α=β =0.05 setting. This apparent 'anomaly' could be attributed to the higher value of s_D, and hence *DINV*, in these settings. In general, there seem to be some critical point along the D-E line, when $\rho > 0$, beyond which the OFR measures deteriorates rapidly. Also, it is noted that the setting $\alpha = \beta$=1 and ρ=0 achieves high *OFR*, close to the desired order fill rate, for all s_D settings.

The empirical contour graphs for *OFR* obtained from the simulation results are presented in Figures 4(a) to 4(i). Overall, it is seen that among the *OFR* contours, the 0.75 contour (the contour marking the region with high *OFR*) decreases in area as ρ increases from 0 to 1, for all settings of s_D. Also, there is a marked shift in the region within the (α, β) plane where high *OFR* is obtained for different values of ρ.

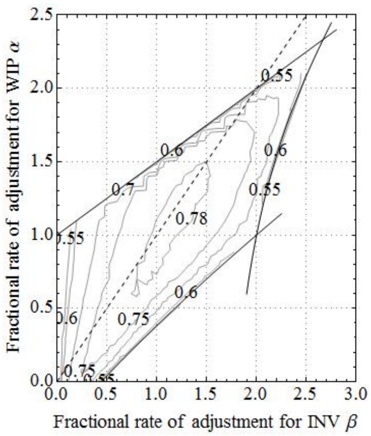
Figures 4(b), 4(e) and 4(h) clearly show that *OFR* improves above the α=β line, i.e. when $\alpha > \beta$ while it reduces below it, i.e. when $\alpha < \beta$. Also, better *OFR* performance is observed for $\alpha \leq 1$ and $\beta \leq 1$ region across any setting of s_D and ρ. The results thus indicate that there exists a well defined operating region within the stability region which can help achieve better system performance, i.e., high *OFR*, under stationary random demand. It is further observed that the scenario α=β=1 (representing the case when the discrepancy in WIP and discrepancy in inventory are completely accounted for in every order) does not result in the highest *OFR* values, except when $\rho = 0$. Even then, it is not the unique best point.

Figures 4(a) to 4(f) shows that the *DOFR* of 80% is achieved neither with s_D= σ, nor with s_D= νFD_n though the achieved *OFR* comes close (~78%) for some parameter settings. However for the case of s_D = sample standard deviation an *OFR* greater than 80% is achieved for $\rho > 0.2$ (see Figures 4(h) and 4(i)). This may be attributed to the higher safety stock carried in the system under this s_D setting.

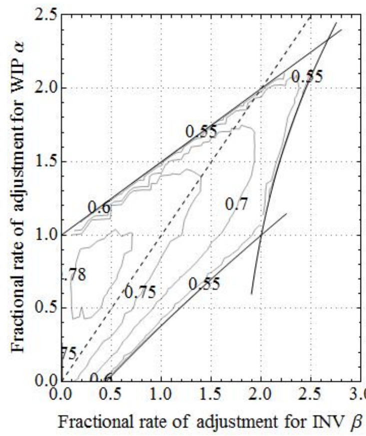
The average system costs (*ASC*) appears to be the lowest at the regions of maximal *OFR*, for a given setting of s_D, and ρ. The median value of *ASC* was found to be about Rs.47 when ρ=0, about Rs.53 when ρ=0.2, and about Rs.89 when ρ=1, across all settings of s_D. Also, for similar values of *OFR*, the *ASC* increases as ρ goes from 0 to 1, for a given s_D. The dominant costs for *ASC* were found to be the holding costs. The holding costs were generally higher when s_D was set as the sample standard deviation of the demand, since in this setting, the *DINV* levels were higher.

The intersection regions of high *OFR* and low *ASC* are illustrated in Figures 5(a) to 5(i). In the plots, the thick black lines denote the *ASC* contours and the thick grey lines denote the *OFR* contours. The desired order fill rate (*DOFR*) is taken as 80%.

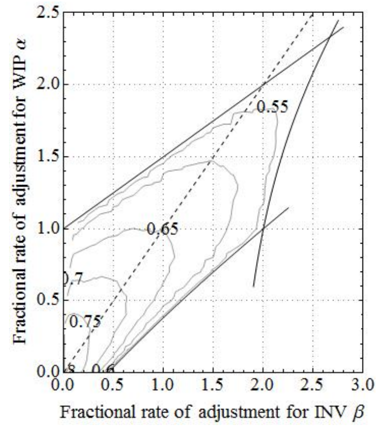
Now, as with *OFR* contours, it is observed that the *ASC* contours also decreases in area as ρ increases from 0 to 1, for all settings of s_D. Also, it is seen that the scenario α=β=1 does not result in the lowest *ASC* values, except when $\rho = 0$. An interesting observation is that in general the region of low *ASC* is much larger than and completely encompasses the region of high *OFR* for all settings of s_D and ρ. That is, for the same system cost *ASC* values, the *OFR* service levels of 75% or 78% can be obtained. This implies that the system parameters can be fine-tuned to obtain higher service levels without increasing system costs.



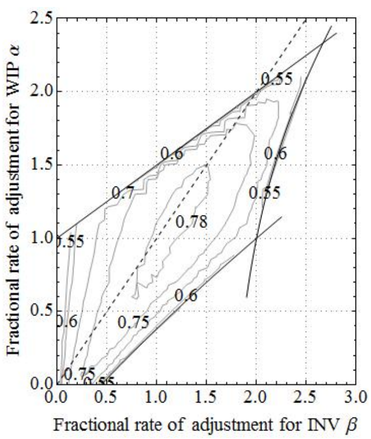
(a) s_D = 10, ρ =0



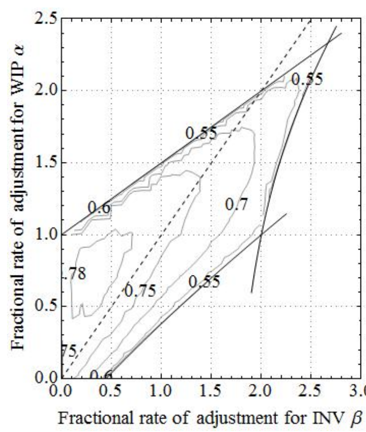
(b) s_D = 10, ρ =0.2



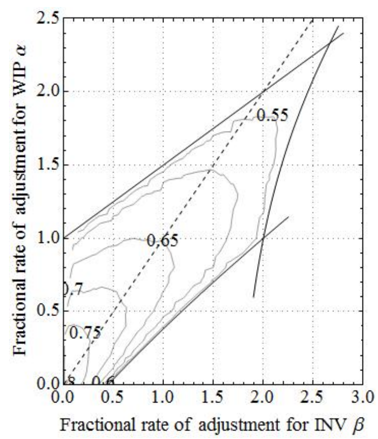
(c) s_D = 10, ρ =1



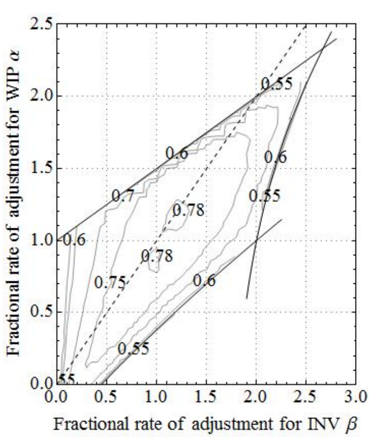
(d) s_D = $v.FD_n$, ρ =0



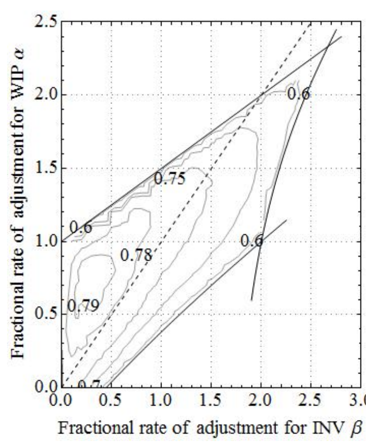
(e) s_D = $v.FD_n$, ρ =0.2



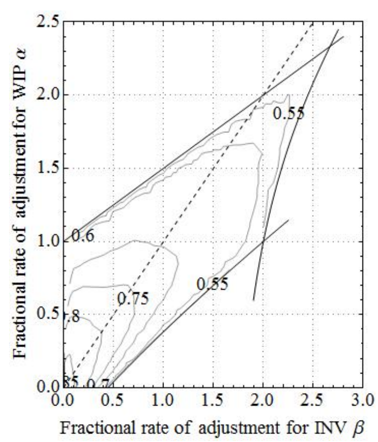
(f) s_D = $v.FD_n$, ρ =1



(g) s_D = sample std.dev, ρ =0



(h) s_D = sample std.dev, ρ =0.2



(i) s_D = sample std.dev, ρ =1

Figure 4: *OFR* contours obtained under different setting s_D and ρ for stable (α, β) points

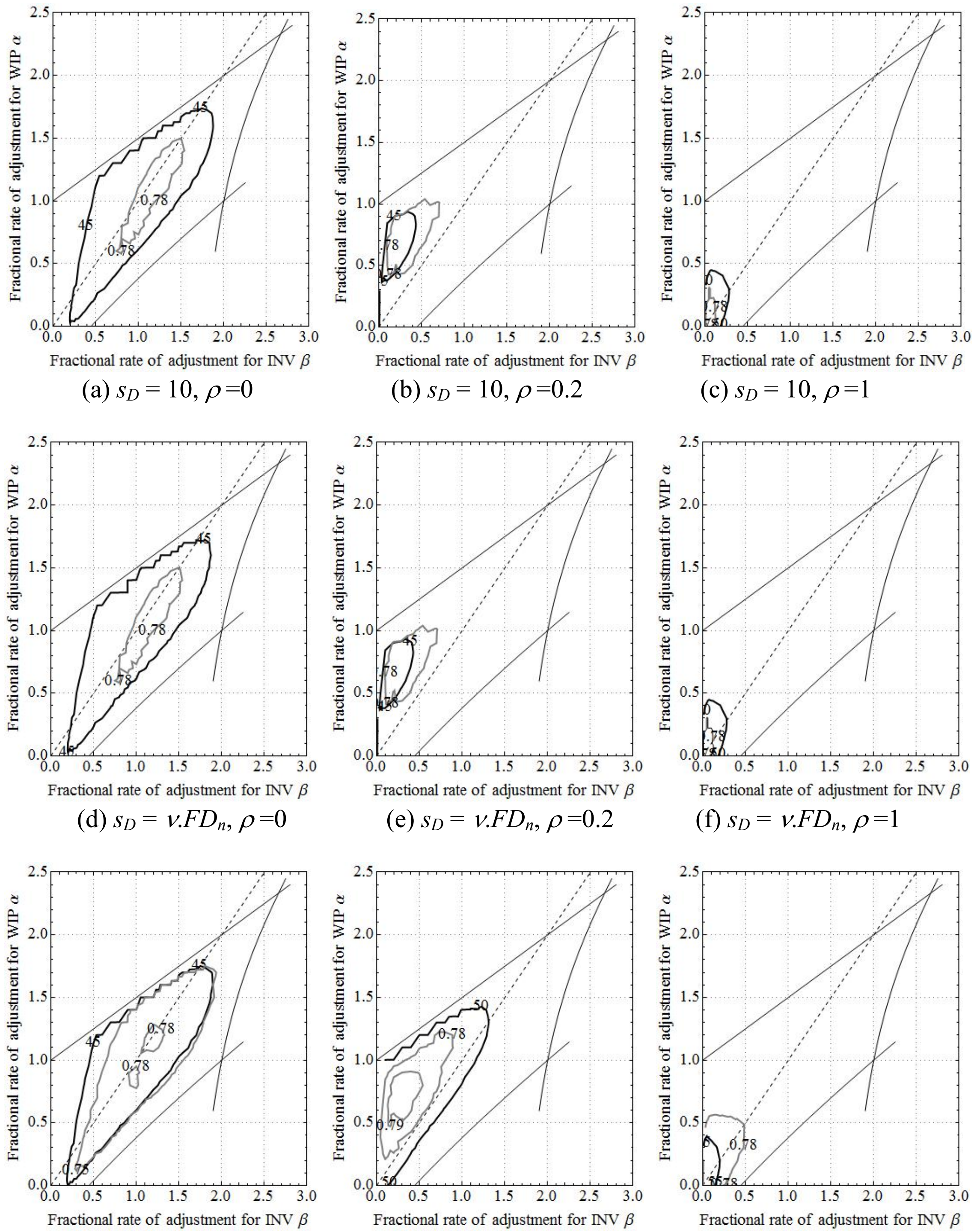


Figure 5: Superimposed *OFR and ASC* contours obtained under different setting s_D and ρ for stable (α, β) points

6. Discussions and Future Work

The production-inventory control system based on the classical industrial dynamics model has been modeled and analyzed for its service level and cost performance. The control parameters for the system are: α, the fractional rate of adjustment for WIP, β, the fractional rate of adjustment for inventory, ρ, the smoothing constant used in forecasting, and s_D, the standard deviation of demand used to determine the safety stock levels. Experiments are conducted with various control parameter settings in order to quantify the system performances, and the results are presented in Section 5.

The results obtained do offer some insights, which could be beneficial to enterprise management. Firstly, holding of additional safety stock is essential for achieving higher service levels. Under the base case of not having any safety stock, the system was able to achieve an order fill rate of about 50% only. Secondly, setting of control parameters inside the stability region is required, even with safety stock, to achieve higher order fill rates. (α, β) points on the stability boundary result in *OFR* of about 50% only, while *OFR* rapidly deteriorates for points outside the stability region. Unfortunately, the *DOFR* of 80% is never achieved under most configurations though the *OFR* does come close (~78%) for some parameter settings. Thirdly, it is observed that the contour marking the region with high *OFR* decreases in area as ρ increases from 0 to 1, for all settings of s_D. Fourthly, the results seem to indicate that under stationary demand conditions, the WIP discrepancies are to be adjusted at a higher rate than inventory discrepancies, that is, $\alpha \geq \beta$ is desired. These make intuitive sense since WIP is purely internal to the production system while the inventory discrepancies are triggered by external (unknown) demand. Under all combinations of s_D, and ρ, the results indicate that the region enclosed by $\alpha \leq 1$, $\beta \leq 1$ and $\alpha \geq \beta$ has comparatively better performances than other regions. Hence it is desirable for the production system to adjust WIP discrepancies at the same or higher rate than inventory discrepancies. Fifthly, completely accounting for the discrepancies, i.e. $\alpha = \beta = 1$, may not always be the best option. $\alpha = \beta = 1$ means that the system tries to adjust the discrepancies fully in each period which may not be required under stationary demand conditions. Sixthly, the system parameters can be fine-tuned to obtain higher service levels without increasing the system costs. That is, for the same system cost *ASC* values, higher *OFR* service levels can be obtained by fine tuning the control parameters.

As future work, the transfer function in *z*-domain is to be explored to seek analytical insights into the amplitudes and settling times of the system response. Further, appropriate procedures to determine the optimal safety stock required to achieve the desired order fill rate based on the control parameter settings can be investigated.

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Appendix

Table A1: (α, β) points used in the simulation runs

No.	**α**	**β**	*No.*	**α**	**β**	*No.*	**α**	**β**	*No.*	**α**	**β**	*No.*	**α**	**β**
1	0	0	*21*	0.02	0.03	*41*	0.05	0.2	*61*	0.1	0.3	*81*	0.2	0.25
2	0	0.01	*22*	0.02	0.05	*42*	0.05	0.3	*62*	0.1	0.35	*82*	0.2	0.3
3	0	0.02	*23*	0.02	0.1	*43*	0.05	0.4	*63*	0.1	0.4	*83*	0.2	0.35
4	0	0.05	*24*	0.02	0.15	*44*	0.05	0.45	*64*	0.1	0.45	*84*	0.2	0.4
5	0	0.1	*25*	0.02	0.2	*45*	0.07	0.05	*65*	0.1	0.5	*85*	0.2	0.45
6	0	0.2	*26*	0.02	0.3	*46*	0.07	0.1	*66*	0.1	0.55	*86*	0.2	0.5
7	0	0.3	*27*	0.02	0.4	*47*	0.07	0.15	*67*	0.15	0.1	*87*	0.2	0.55
8	0	0.4	*28*	0.02	0.45	*48*	0.07	0.2	*68*	0.15	0.15	*88*	0.2	0.6
9	0	0.45	*29*	0.03	0.02	*49*	0.07	0.25	*69*	0.15	0.2	*89*	0.2	0.65
10	0.01	0.01	*30*	0.03	0.01	*50*	0.07	0.3	*70*	0.15	0.3	*90*	0.2	0.7
11	0.01	0.02	*31*	0.03	0.03	*51*	0.07	0.35	*71*	0.15	0.4	*91*	0.3	0.01
12	0.01	0.05	*32*	0.03	0.05	*52*	0.07	0.4	*72*	0.15	0.5	*92*	0.3	0.05
13	0.01	0.1	*33*	0.03	0.1	*53*	0.07	0.45	*73*	0.15	0.6	*93*	0.3	0.1
14	0.01	0.2	*34*	0.03	0.15	*54*	0.07	0.5	*74*	0.15	0.55	*94*	0.3	0.15
15	0.01	0.25	*35*	0.03	0.2	*55*	0.07	0.55	*75*	0.15	0.65	*95*	0.3	0.2
16	0.01	0.3	*36*	0.03	0.3	*56*	0.1	0.05	*76*	0.2	0.01	*96*	0.3	0.25
17	0.01	0.35	*37*	0.03	0.45	*57*	0.1	0.1	*77*	0.2	0.05	*97*	0.3	0.3
18	0.01	0.4	*38*	0.05	0.05	*58*	0.1	0.15	*78*	0.2	0.1	*98*	0.3	0.4
19	0.01	0.45	*39*	0.05	0.1	*59*	0.1	0.2	*79*	0.2	0.15	*99*	0.3	0.5
20	0.02	0.02	*40*	0.05	0.15	*60*	0.1	0.25	*80*	0.2	0.2	*100*	0.3	0.6
101	0.3	0.7	*121*	0.5	0.6	*141*	0.6	1.1	*161*	0.8	0.4	*181*	0.9	0.7
102	0.3	0.8	*122*	0.5	0.7	*142*	0.6	1.2	*162*	0.8	0.5	*182*	0.9	0.8
103	0.3	0.85	*123*	0.5	0.8	*143*	0.6	1.3	*163*	0.8	0.6	*183*	0.9	0.9
104	0.4	0.05	*124*	0.5	0.9	*144*	0.7	0.1	*164*	0.8	0.7	*184*	0.9	1
105	0.4	0.1	*125*	0.5	1	*145*	0.7	0.2	*165*	0.8	0.8	*185*	0.9	1.2
106	0.4	0.2	*126*	0.5	1.1	*146*	0.7	0.3	*166*	0.8	0.9	*186*	0.9	1.4
107	0.4	0.4	*127*	0.5	1.15	*147*	0.7	0.4	*167*	0.8	1	*187*	0.9	1.5
108	0.4	0.6	*128*	0.6	0.1	*148*	0.7	0.6	*168*	0.8	1.1	*188*	0.9	1.6
109	0.4	0.7	*129*	0.6	0.15	*149*	0.7	0.7	*169*	0.8	1.2	*189*	0.9	1.7
110	0.4	0.8	*130*	0.6	0.2	*150*	0.7	0.8	*170*	0.8	1.3	*190*	0.9	1.8
111	0.4	0.85	*131*	0.6	0.25	*151*	0.7	0.9	*171*	0.8	1.4	*191*	1	0.1
112	0.4	0.9	*132*	0.6	0.3	*152*	0.7	1	*172*	0.8	1.5	*192*	1	0.2
113	0.4	0.95	*133*	0.6	0.35	*153*	0.7	1.1	*173*	0.8	1.6	*193*	1	0.4
114	0.4	1	*134*	0.6	0.4	*154*	0.7	1.2	*174*	0.8	1.65	*194*	1	0.5
115	0.5	0.1	*135*	0.6	0.5	*155*	0.7	1.3	*175*	0.9	0.1	*195*	1	0.7
116	0.5	0.15	*136*	0.6	0.6	*156*	0.7	1.4	*176*	0.9	0.2	*196*	1	1
117	0.5	0.2	*137*	0.6	0.7	*157*	0.7	1.5	*177*	0.9	0.3	*197*	1	1.2
118	0.5	0.3	*138*	0.6	0.8	*158*	0.8	0.1	*178*	0.9	0.4	*198*	1	1.4
119	0.5	0.4	*139*	0.6	0.9	*159*	0.8	0.2	*179*	0.9	0.5	*199*	1	1.6
120	0.5	0.5	*140*	0.6	1	*160*	0.8	0.3	*180*	0.9	0.6	*200*	1	1.8
201	1	1.9	*221*	1.1	2	*241*	1.3	0.7	*261*	1.4	1.3	*281*	1.5	2
202	1	1.95	*222*	1.2	0.4	*242*	1.3	0.8	*262*	1.4	1.4	*282*	1.5	2.1
203	1.1	0.2	*223*	1.2	0.5	*243*	1.3	0.9	*263*	1.4	1.5	*283*	1.5	2.15
204	1.1	0.3	*224*	1.2	0.6	*244*	1.3	1	*264*	1.4	1.6	*284*	1.6	1.2
205	1.1	0.4	*225*	1.2	0.7	*245*	1.3	1.1	*265*	1.4	1.7	*285*	1.6	1.25
206	1.1	0.5	*226*	1.2	0.8	*246*	1.3	1.2	*266*	1.4	1.8	*286*	1.6	1.3
207	1.1	0.6	*227*	1.2	0.9	*247*	1.3	1.3	*267*	1.4	1.9	*287*	1.6	1.4

No.	α	β	No.	α	β	No.	α	β	No.	α	β	No.	α	β
208	1.1	0.7	228	1.2	1	248	1.3	1.4	268	1.4	2	288	1.6	1.5
209	1.1	0.8	229	1.2	1.1	249	1.3	1.5	269	1.4	2.1	289	1.6	1.6
210	1.1	0.9	230	1.2	1.2	250	1.3	1.6	270	1.5	1	290	1.6	1.7
211	1.1	1	231	1.2	1.3	251	1.3	1.7	271	1.5	1.1	291	1.6	1.8
212	1.1	1.1	232	1.2	1.4	252	1.3	1.8	272	1.5	1.05	292	1.6	1.9
213	1.1	1.2	233	1.2	1.5	253	1.3	1.9	273	1.5	1.2	293	1.6	2
214	1.1	1.3	234	1.2	1.6	254	1.3	2	274	1.5	1.3	294	1.6	2.1
215	1.1	1.4	235	1.2	1.7	255	1.3	2.1	275	1.5	1.4	295	1.6	2.2
216	1.1	1.5	236	1.2	1.8	256	1.4	0.8	276	1.5	1.5	296	1.7	1.4
217	1.1	1.6	237	1.2	1.9	257	1.4	0.9	277	1.5	1.6	297	1.7	1.45
218	1.1	1.7	238	1.2	2	258	1.4	1	278	1.5	1.7	298	1.7	1.5
219	1.1	1.8	239	1.2	2.05	259	1.4	1.1	279	1.5	1.8	299	1.7	1.6
220	1.1	1.9	240	1.3	0.6	260	1.4	1.2	280	1.5	1.9	300	1.7	1.7
301	1.7	1.8	311	1.8	2	321	1.9	1.8	331	2.1	2.35			
302	1.7	1.9	312	1.8	2.1	322	2	2.05	332	2.1	2.3			
303	1.7	2	313	1.8	2.2	323	2	2	333	2.1	2.2			
304	1.7	2.1	314	1.8	2.3	324	2	2.1	334	2.1	2.25			
305	1.7	2.2	315	1.9	2.35	325	2	2.2	335	2.2	2.5			
306	1.7	2.25	316	1.9	2.3	326	2	2.3						
307	1.8	1.6	317	1.9	2.2	327	2	2.25						
308	1.8	1.7	318	1.9	2.1	328	2	2.35						
309	1.8	1.8	319	1.9	2	329	2	2.4						
310	1.8	1.9	320	1.9	1.9	330	2.1	2.4						

Table A2: Points used along the D-E line, i.e. (α = β)

S.No	1	2	3	4	5	6	7	8	9	10
α = β	0	0.01	0.02	0.03	0.05	0.15	0.2	0.3	0.4	0.5
S.No	11	12	13	14	15	16	17	18	19	20
α = β	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5
S.No	21	22	23	24	25					
α = β	1.6	1.7	1.8	1.9	2					