A Non Smooth Model of National Energy Market for the Regional Energy Integration

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Abstract

In this paper a novel non-smooth model for a national energy market and its extension to n-countries is proposed, showing several differences from the traditional smooth models. The study begins with the classical treatment of system dynamics theory and "jumps" to non linear dynamical systems theory finding mathematical results about its complexity. Such results are important in creating rules for regional integration among countries, as for example in the latinamerican case, because trough of bifurcations diagrams (from dynamical systems theory) is possible to know what are the all possible scenarios of the system giving robustness to the decision making.

keywords: energy market, non smooth, energy policy, dynamical systems, bifurcations, regional integration, decision making in complex scenarios.

Contents

1 Introduction or Motivation

Energy is essential for economic and industrial development of society, seen as the correlation between a nation´s energy consumption and its gross domestic product (GDP). Energy is also the base for many of the most important activities of our society and its prices have significant impact on the costs of many manufactured goods [1].

While at the beginning of the twentieth century, energy markets were seen as natural monopolies and there were disagreements by trying to consider it as competitive market [2], for no more than three decades the vision of these markets changed, giving rise to processes of deregulation and liberalization of energy markets [3, 4, 5].

1.1 Panorama Prior to the Deregulation

Many governments structured their electricity markets as natural monopolies in order to protect and control the prices paid by users, which led to advantages such as economies of scale by building large power plants that dramatically reduce generation costs [2], gains in system efficiency due to the centralized operation of the generation system that would not be achieved when plants are operated independently [2] and facilities for the attention of the transmission and distribution costs, which have usually been very high to be served by private companies [6].

However, these monopoly schemes also produce significant disadvantages as evidenced crisis caused by rationing, and lack of investment in generation capacity, because the companies operate inefficiently with cost overruns that are paid by the final consumer in detriment of the national economy [7, 8, 9, 10, 11]. These inefficiencies are mainly related to political decisions that ignore, largely, criteria such as efficiency and quality of service [12].

Regulated markets for its part, in which there is really no competition, they also present major problems as explained [1], because in the short run, the regulator should provide incentives for generators to operate as cheaply as would happen in the markets competitive, and in the long run, the controller must minimize the average cost of generation of the system. These objectives are antagonistic, and it is impossible for the regulator to meet them simultaneously [2].

1.2 Deregulation and Liberalization

For these reasons, in the last two decades have been implemented the deregulation and liberalization of markets, creating new unique features that differ greatly from traditional financial markets. The model of reform (liberalization) is based on competition and independent regulation. It is expected that increased competition will result in greater efficiency, reliability, lower prices and furthermore that promote economic growth and development [4]. In accordance with this standard applies a package of reforms that include the following [3, 4, 5, 14]

- The creation of electricity markets that operate, among others, under the criteria of economic efficiency scheme introduced by the competition.

- The vertical desintegration of industry, encouraging the participation of private operators as generators, operators, transmitters and distributors. This means that companies streamline their operations and produce economic benefits represented in decreasing tariffs for the consumer. But it also means that the state releases the resources used to support the state monopoly, increasing its capacity to invest in other sectors.

- It introduces mechanisms that oblige to decrease the costs of generation in the short long, while promove the decrease in the average of the costs in generation in the long run.

- Privatization of the different entities resulting from the disintegration

- Market competition in the wholesale and retail.

- Regulated or negotiated access for third parties to transmission and distribution sectors.

Here the state acts as regulator responsible for ensuring adequate provision of service, maintenance of clear market rules that encourage the participation of private actors in the electricity sector and to maintain the necessary conditions for free competition [1].

In this scheme, generating firms try to recover their operating costs and investment returns, they adjust their offerings and prices according to their business strategies [15]. The companies owners of transmission networks makes available its assets to carry electricity from generating units to consumers, their service is paid from regulated tariffs [1, 14]. Distributors and large consumers try to negotiate beneficial bilateral contracts, and operate efficiently in the short term market [16].

Strategic units of each agent must define the objectives and long-term strategies, while operational units must take action and implement plans to achieve those goals. However, the decisions are imperfect, so that their partial results must be monitored in order to correct the actions taken and implement new actions that allow that each agent reach its organizational objectives [1].

Decision-making of agents is very difficult because the deregulated electricity markets can be considered as complex environments that change rapidly [17].

The regulator sets the rules for the operation of the market in pursuit of economic efficiency seeking to reduce the possibility of arbitration, and promoting competition among firms [2].

In the supply chain segments involving the existence of a network, i.e., systems of high voltage transmission and local distribution systems that carry power from generating companies to the users, the competition is not able to be made it because requires that the network must be duplicated.

On the other hand, allow all generators and marketers access to transport networks increases the opportunities for competition, because it offers to end users the opportunity to choose their supplier. This encourages generation companies to try to produce cheaper and compete for retailers, while suppliers are encourages to compete for end customers.

The political risks of liberalization are large, because energy is a very important strategy for development policy of a country. Liberalization can lead to increased prices. Privatization does not usually contribute to achieving the objectives of rural electrification and increasing the efficiency of former state-owned utilities may result in lost of many jobs.

In this new competitive scenario, has recognized the need to understand how market characteristics and special conditions affecting prices, and how agents can capitalize on this knowledge to make better decisions [1]. Such decisions are primarily concerned with the formulation of marketing strategies and investment in the short, medium and long term [1].

1.3 Special features of energy markets

The characteristics that make up the electricity market as a very special market are presented below [4]:

- Inability to economically store: demand must be cleared with "just-in-time" production from generating capacity continuously at every location on the network[1]. This implies that the market must have enough generation assets to meet peak consumption during the day, while some of these assets remain inactive when consumption is served basis. It also implies a high volatility in price behaviour.

- Very low short-run demand elasticity and supply becomes very inelastic at high demand levels as capacity constraints are approached. The low elasticity can not contribute substantially to the price reduction and mitigation of its fluctuations.

- The inability to store electricity, coupled with constant changes of a very inelastic demand, the possible weaknesses in the generation and transmission equipment, and the need for adjustment between supply and demand in all parts of the network, does have reserve generators that can respond quickly to changes in supply-demand relationship.

In the Colombian case, due to our market can fully satisfy the demand from hydro power, the weather plays an important role [14]. Dry seasons induce high prices because demand must be attended by thermal stations which are very expensive, while the rainy season reduces costs by eliminating the need for thermal generators. The cost of electricity also depends on the technological diversity of assets, as well as advances in the use of renewable and / or alternatives energy sources.

Demand is characterized by the presence of many cyclical factors associated with seasons, holidays and work days, hours of sunlight, temperature, etc.

1.4 Problem Approach

In this complex overview of the energy markets is the Colombian national context, where since 1995 the market was deregulated [14]. For Colombia, we consider four activities (generation, transmission, distribution and marketing), with each one of them under specific regulation [14].

The generation and marketing segments are defined as competitive (or potentially competitive), while transmission and distribution activities were defined as monopolies subject to regulation. It was established as a general rule the introduction of competition where be possible and regulation of monopolistic activities [14].

In this framework, and under the creation of transmission lines between Colombia-Ecuador, Colombia-Venezuela, Ecuador-Peru, Peru-Bolivia and the unfinished Colombia-Panama line, which would promote regional energy integration, raising many questions about the ability installed generation and transmission capacity in each nation for attend the demand today, also the capacity in generation and transmission that must be built to attend the energy demand tomorrow and how electricity prices would behave in each country after completion of the integration.

In order to satisfy this need was built in this paper a model that represent a national market as a start point for model the regional integration, and the scheme (causal diagram) for the integration among two countries with the generalization to $n-$countries showed in a differential system.

The simulation model for a single set of parameters represents a scenario of the problem, but we wish to see all the possible scenarios of the system. That is why we use the bifurcation diagrams of dynamical systems theory, because in them you can see the set of all possible scenarios and, therefore, ensure that decisions are made about the knowledge of all possibilities.

[1] Network's congestion, combined with nonstorability, may limit significantly the geographic expanse of competition by constraining the ability of remote suppliers to compete, further enhancing market power problems.

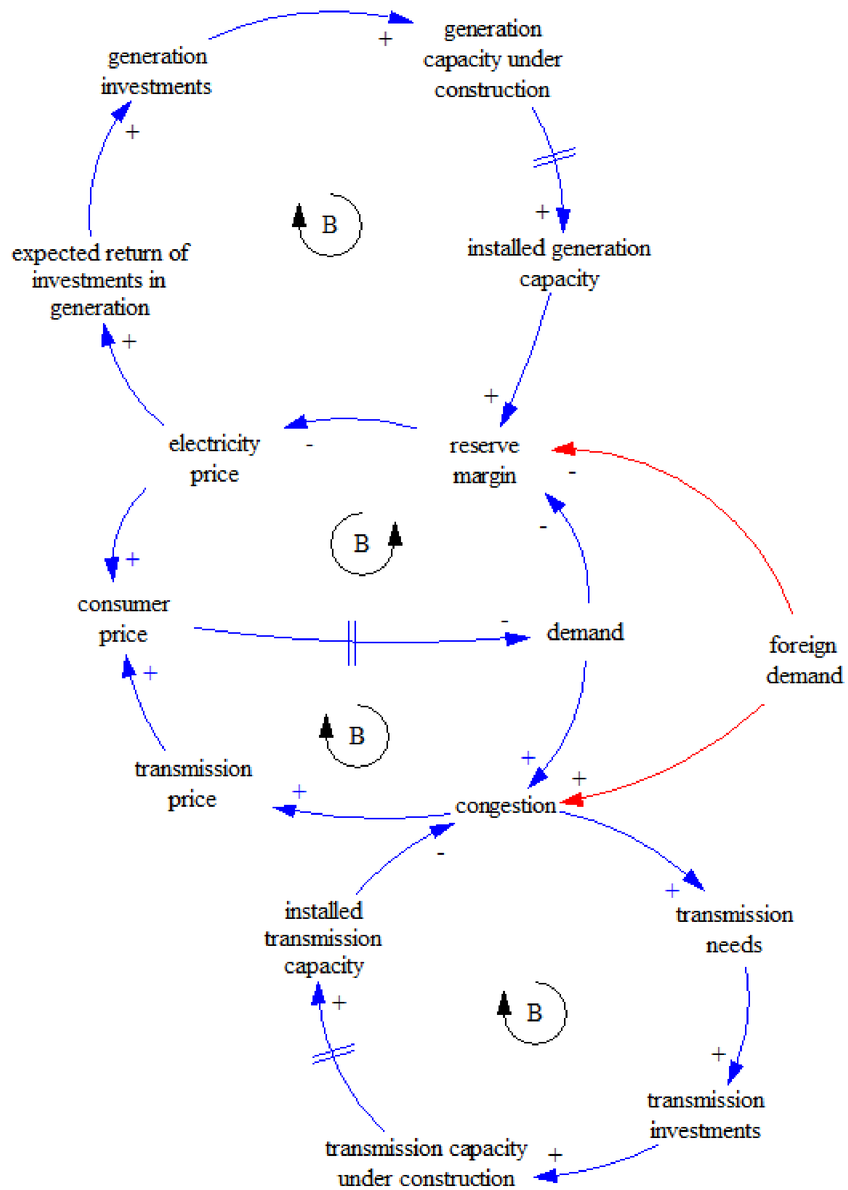

Figure 1: Causal diagram of the energy market dynamics.
Source: Dyner, I., Olivar, G., Redondo, J.M.

An excellent compilation of the main techniques that have been used for modelling electricity markets is presented by [13]. The pricing model was made using processes of reversion of the mean and Poisson [16, 18, 19, 20, 21, 22, 23], using neural networks [24, 25, 26, 27, 28], computational intelligence models [29], fuzzy systems [30] and recurrent neural networks [31].

In this paper we built a model from the system dynamics that was analysed from the perspective of dynamic systems.

2 Model for national market

In this section is presented the causal diagram, the level and flow diagram and the equations of our model.

2.1 Causal diagram for national market

With the system's reserve margin (RM), estimated as the percentage difference between supply (IG)and demand (D)is established the generation price of electricity (GP); if the reserve margin increase then decreases the generation price of electricity, which leads to increasing the expected return on investmentin generation (ER), with the consequent increase in generation investment ($GInv$). When the generation investments are realized, become generation capacity under construction (GC), after a delay, will become installed generation capacity (IG). By increasing installed generation capacity then reserve margin increases, thereby closing the first loop of balance.

The increase in generation prices (GP) means an increase in consumer prices (CP), which, after a delay, decrease demand (D). If the demand will increase the reserve margin would be directly affected decreasing. When pairing, as already mentioned, the reserve margin to the generation price of electricity, we would close a second loop of balance.

With the increased demand (D) will increase congestion (C) on electricity transmission networks. This increase in congestion increases the cost of transmission, which in turn will increase the price to the consumer (CP), who after a delay will decrease demand. This closes a third cycle of balance.

Increasing congestion (C) on transmission lines establishes a need for increased transmission (TN). This increases the transmission investment ($TInv$), which to materialize become transmission capacity under construction (TC), after a delay, was becomes installed transmission capacity (IT), which decrease with increasing congestion on transmission lines. This closes the last balance loop .

In the causal diagram we consider the possible existence of an external demand to the system, which directly affect the reserve margin and the congestion of transmission lines in the exporting country. However, this attribute was not considered for the simulations. Figure 1 shows the causal diagram of this system.

2.2 Model Formalism

The levels and flow diagram is show in Figure 2. Here we present new elements that are not displayed on the causal diagram, such as depreciation of the generating plants and transmission lines. Furthermore we also takes into account the elasticity of demand, elasticity of capacity margin and the elasticity of congestion. The model can change its dynamic behaviour as defined according to is redefined the investment function. This can be taken from economic theory or from investment rules of the markets.

As shown in the diagram of levels and flows, the electricity market can be represented by a five dimensional coupled system of ordinary differential equations. The construction is as follows:

$$\frac{d}{dt}GC = GInv - FP$$

$$\frac{d}{dt}IG = FP - DepG$$

$$\frac{d}{dt}TC = TInv - FL$$

$$\frac{d}{dt}IT = FL - DepT$$

$$\frac{dCP}{dt} = PC$$

Where GC is generation capacity under construction, IG is installed generation capacity, TC is transmission capacity under construction, IT is installed transmission capacity and CP is consumer's price.

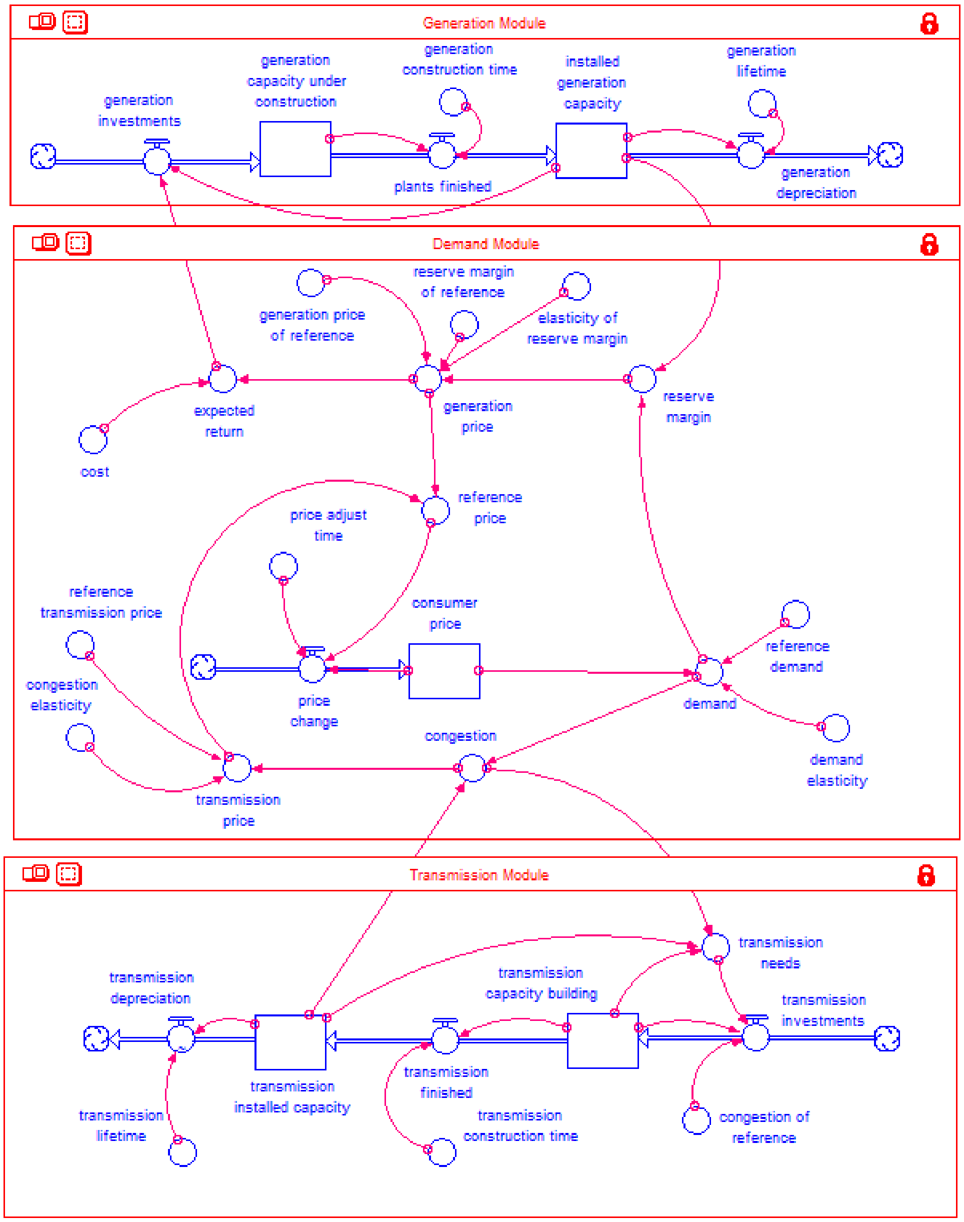

Figure 2: Levels and flows diagram of the energy markets dynamic. **Source**: Dyner, I., Olivar, G., Redondo, J.M.

The investment in generation capacity ($GInv$) is a function of expected return (ER) and installed generation capacity (IG):

$$GInv = max\{0, ER \cdot IG\}$$

with $ER = GP - Cost$, where GP is the generation price and $Cost$ is the generation cost.

The finished plants (FP) and depreciation in installed generation capacity $(DepG)$ are estimated as follows:

$$FP = \frac{GC}{CTG}$$

$$DepG = \frac{IG}{LTG}$$

where CTG is the construction time for generation capacity and LTG is the lifetime in generation capacity. The transmission investment is a function of installed transmission capacity and the transmission needs TN:

$$TInv = max\{C_{ref}, TN \cdot IT\}$$

where C_{ref} is a reference congestion.

The finished lines (FL) and transmission depreciation $DepT$ are defined as follows:

$$FL = \frac{TC}{TCT}$$

$$DepT = \frac{IT}{LTT}$$

where TCT is the transmission construction time and LTT is the transmission lifetime.

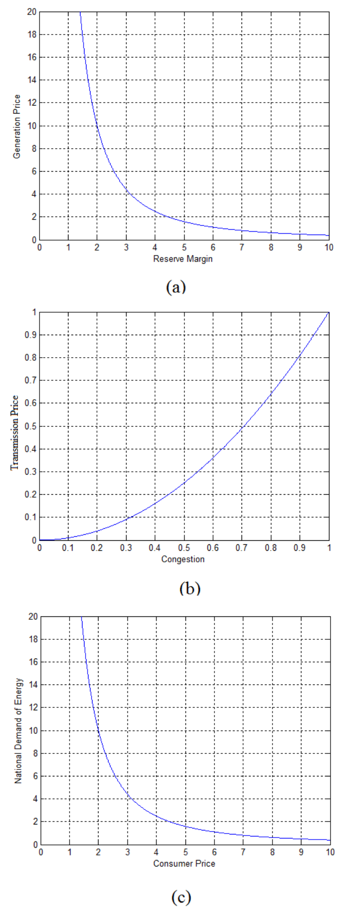

Figure 3: Functions of constant elasticities (a) (b) (c)
Source: Dyner, I., Olivar, G., Redondo, J.M.

The price change (PC) is a first order process, then:

$$PC = \frac{CP - RP}{PAT}$$

where RP is a consumer reference price and PAT is the price adjustment time. The generation price (GP), transmission price (TP) and demand (D) are estimated as function of constant elasticities follows the behaviour in figure 3.

$$GP = GP_{ref}\left(\frac{RM}{RM_{ref}}\right)^{-\beta}$$

$$TP = TP_{ref} \, (C)^{\omega}$$

$$D = D_{ref}(CP)^{-\epsilon}$$

where the capacity margin elasticity (β) and the demand elasticity (ε) are positive real numbers, while the congestion elasticity (ω) is a positive integer; GP_{ref} is the reference generation price, RM_{ref} is the reference reserve margin, TP_{ref} is the reference transmission price and D_{ref} is the reference demand. The reserve margin RM is the percentage difference between supply IG and demand D:

$$RM = \frac{IG - D}{D} = \frac{IG}{D} - 1$$

The congestion (C) is defined as the difference among demand (D) and installed transmission (IT):

$$C = D - IT$$

And the transmission needs (TN(as the difference among congestion (C) and the installed and building transmission sum:

$$NT = C - (IT + TC)$$

Finally, the consumers reference price (RP) is defined as the sum among generation price (GP) and transmission price (TP):

$$RP = GP + TP$$

In summary our system is:

$$\frac{dGC}{dt} = max\{0, \left[GP_{ref}\left(\frac{RM_{ref}D_{ref}}{IG(CP)^{\varepsilon} - D_{ref}}\right)^{\beta} - cost\right] \cdot IG\} - \frac{GC}{CTG}$$

$$\frac{dIG}{dt} = \frac{GC}{CTG} - \frac{IG}{LTG}$$

$$\frac{dTC}{dt} = max\{C_{ref}, \left[D_{ref}(CP)^{-\varepsilon} - (TC + 2IT)\right] \cdot IT\} - \frac{TC}{CTT}$$

$$\frac{dIT}{dt} = \frac{TC}{CTT} - \frac{IT}{TLT}$$

$$\frac{dCP}{dt} = \frac{1}{PAT}\left[GP_{ref}\left(\frac{RM_{ref}D_{ref}}{IG(CP)^{\varepsilon} - D_{ref}}\right)^{\beta} + TP_{ref}\left(D_{ref}(CP)^{-\varepsilon} - IT\right)^{\omega} - CP\right]$$

2.3 Simulation using iThink

We do a simulation using "iThink" (figure 4). Here we look a logical behaviour among state variables (levels). For example the generation under construction and installed generation capacity oscillate periodically with a time lag that corresponds to that expected from the system. Talking about the transmission we see that the increment in installed transmission means a decrement in transmission under construction.

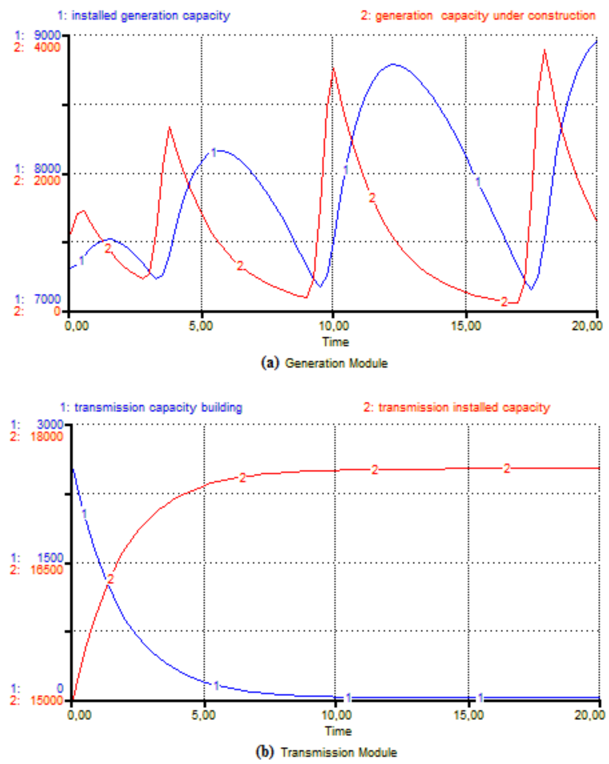

Figure 4: Simulation with the next elasticities values: $\beta = 0.1$, $\varepsilon = 0.1$ and $\omega = 1$. (a) Simulation for the generation module. (b) Simulation for the transmission module. **Source**: Dyner, I., Olivar, G., Redondo, J.M.

3 Model Analysis

In the differential equation system showed above the expressions $max\{0, ER \cdot IG\}$ and $max\{C_{ref}, TN \cdot IT\}$ means that we have a non trivial possibility when $ER \cdot IG > 0$ and/or $TN \cdot IT > C_{ref}$. This conditions means that there exists a commutation regions in the 5-dimensional phase space defined as follows:

$$\Sigma_1 = IG = K_1 \cdot (CP)^{-\varepsilon}$$

$$\Sigma_2 = TC = D_{ref}(CP)^{-\varepsilon} - 2IT - \frac{C_{ref}}{IT}$$

where K_1 is an scalar defined as follows

$$K_1 = RM_{ref}D_{ref}\left(\frac{CP_{ref}}{Cost}\right)^{1/\beta} + D_{ref}$$

In terms of dynamical systems we have a non smooth dynamical system where Σ_i, $i = 1, 2$, are the commutation regions of the system. It divides the phase space in a non computed number of regions, each one with a different vector field defined.

We propose for the study of the model to take the possible scenarios generated from the elasticities interpretation. This interpretation could allows the model study.

3.1 Elasticities

Elasticity is an economic concept that is used to quantify the variation in a variable while another variable is changing. In our work we have three elasticities: the elasticity of demand to consumer's price, the elasticity of price to margin reserve and the elasticity of price to the congestion.

3.1.1 Elasticity of demand to consumer's price

While some power exchange allows the presence of demand, generally in the power exchange the demand don't have active participation, i.e., consumers can not make decisions on quantities and prices in the short term [14]. The reason for this is that, given the complexity of the sector, many experts doubted that a system based in a market could be function properly in real time [14, 37].

This scheme, as explain [14], has as main disadvantage that the electricity in the short term is an inelastic good because prices are not immediately move to the end user, so that a price change will not affect the instantaneous demand. This approach leads to two scenarios: short-term and long term, taking as short term a time period less than three months.

3.1.2 Elasticity of price to capacity margin

The margin reserve is a length to the relation supply-demand defined as the percent difference between the installed generation capacity and the demand:

$$CM = \frac{IC - D}{D}$$

The supply depends largely of the availability of natural resources. In the case of hydroelectric plants, water shortages in times of macro climate phenomena such as "El Niño" stressed the energy system (Colombian case).

These environmental considerations invite us to consider two scenarios, one in which exists availability of resources (water, coal, gas, etc.) so that is possible offer without problems and the other in which climatic variations and shortage of resources makes that the supply be constrained.

3.1.3 Elasticity of price to the congestion

This increased congestion in the short term would increase the price, as in the Peruvian case [38], but it depends of the availability in the transmission.

With the above possibilities in the elasticities are considered eight cases as is presented in the next table:

Cases vs. possible scenarios	short run	long run	resource availability	climate variability	transmission availability	insufficient transmission
1. $\varepsilon = \beta = \omega = 0$	x			x	x	
2. $\varepsilon = \beta = 0$; $\omega \neq 0$	x			x		x
3. $\varepsilon = \omega = 0$; $\beta \neq 0$	x		x		x	
4. $\beta = \omega = 0$; $\varepsilon \neq 0$		x		x	x	
5. $\varepsilon = 0$; $\beta \neq 0$; $\omega \neq 0$	x		x			x
6. $\beta = 0$; $\varepsilon \neq 0$; $\omega \neq 0$		x		x		x
7. $\omega = 0$; $\varepsilon \neq 0$; $\beta \neq 0$		x	x		x	
8. $\varepsilon \neq 0$; $\beta \neq 0$; $\omega \neq 0$		x	x			x

Table 1: Cases vs. possible scenarios

3.2 Inelastic Margin Capacity

In the cases where the capacity margin is ineslatic, we mean $\beta = 0$, we find the next 2-dimensional linear system desacoupled to the other 3 equations:

$$\begin{aligned} \dot{x_1} &= max\{0, (a-p)\cdot x_2\} - \frac{x_1}{q} \\ \dot{x_2} &= \frac{1}{q}x_1 - \frac{1}{r}x_2 \end{aligned} \tag{1}$$

This means that the generation capacity behaviour in this scenario does not depends to the transmission and price-demand behaviour.

Taking this planar system from two possibilities, when $max = 0$ and when $max \neq 0$, we find that if the $max = 0$ the system have an equilibria point in the origin type sink (figure 5).

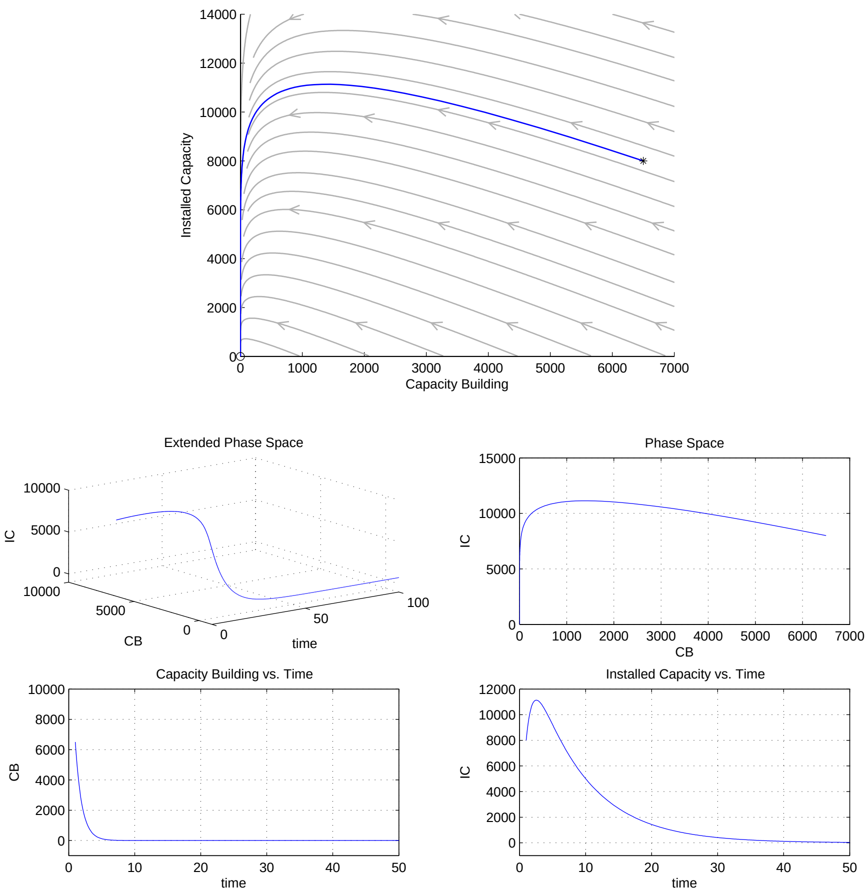

Figure 5: The values of the the graph are: $x(0) = 6500$; $y(0) = 8000$;$q = 1$; $r = 8$.

and when the $max \neq 0$ the system must satisfied $a > p$ with the origin again as equilibrium but type saddle (figure 6).

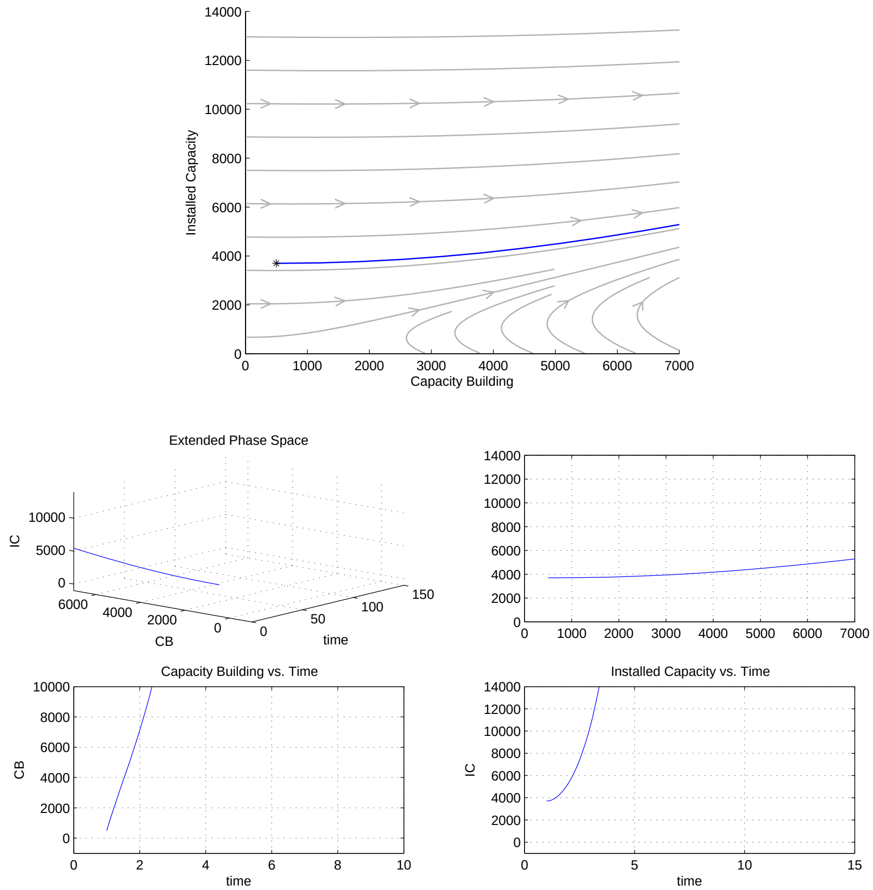

Figure 6: The values of the graph are: $x(0) = 500$; $y(0) = 3700$; $a = 128$; $p = 126$; $q = 2$; $r = 16$.

If we take $a - p = k$ we can do the next bifurcation diagram (figure 7) for the system, following the change in the equilibria stability, with a point of bifurcation when $k = 1/r$.

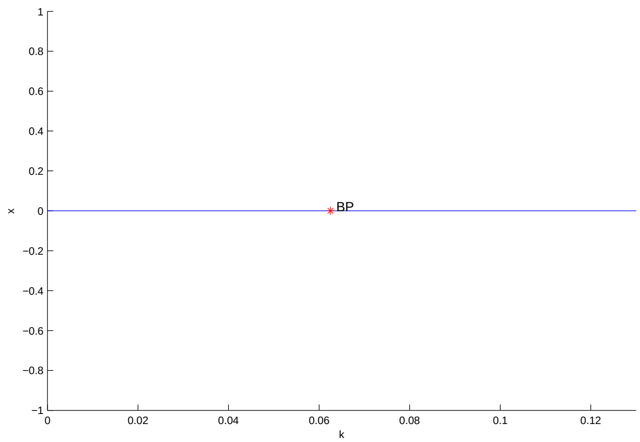

Figure 7: the bifurcation point (BP) show us the saddle-node bifurcation. In the left hand to BP we find stable behaviour and in the right hand the behaviour is unstable.

This tells us that if the utilities don't exceed the ratio $1/lifetime$, the built and the installed capacity tends to disappear in the time. But if the state is achieved, the built capacity and the installed capacity will increase slowly.

4 Model for the integration

Now the challenge is the integration between $n-$countries. Each new country means five new state variables in the system. In this last section we show the energy markets integration as a $5n-$dimensional non smooth dynamical system with $2n-$regions of commutation.

For two countries the causal diagram is presented in the figure 8.

The Dynamical System for $n-$countries is:

$$\frac{dCB_i}{dt} = max\{0, \left[P^E_{ref_i}\left(\frac{IC_i}{MC_{ref_i}\sum_{k=1}^{n} D_{ref_k}(PD_k)^{\varepsilon_k}} - \frac{1}{MC_{ref_i}}\right)^{\beta_i} - c_i\right] \cdot IC_i\} - \frac{CB_i}{CT_i}$$

$$\frac{dIC_i}{dt} = \frac{CB_i}{CT_i} - \frac{IC_i}{LT_i}$$

$$\frac{dTB_i}{dt} = max\{congestion_{ref}, \left[\sum_{k=1}^{n} D_{ref_k}(PD_k)^{\varepsilon_k} - (2IT + TB)\right] \cdot IT\} - \frac{TB}{TCT}$$

$$\frac{dIT_i}{dt} = \frac{TB_i}{TCT_i} - \frac{IT_i}{TLT_i}$$

$$\frac{dPD_i}{dt} = \frac{1}{TAP_i}\left[P^E_{ref_i}\left(\frac{IC_i}{MC_{ref_i}\sum_{k=1}^{n} D_{ref_k}(PD_k)^{\varepsilon}_k} - \frac{1}{MC_{ref_i}}\right)^{\beta_i} + P^T_{ref_i}\left(\sum_{k=1}^{n} D_{ref_k}(PD_k)^{\varepsilon_k} - IT_i\right)^{\omega_i} - PD_i\right]$$

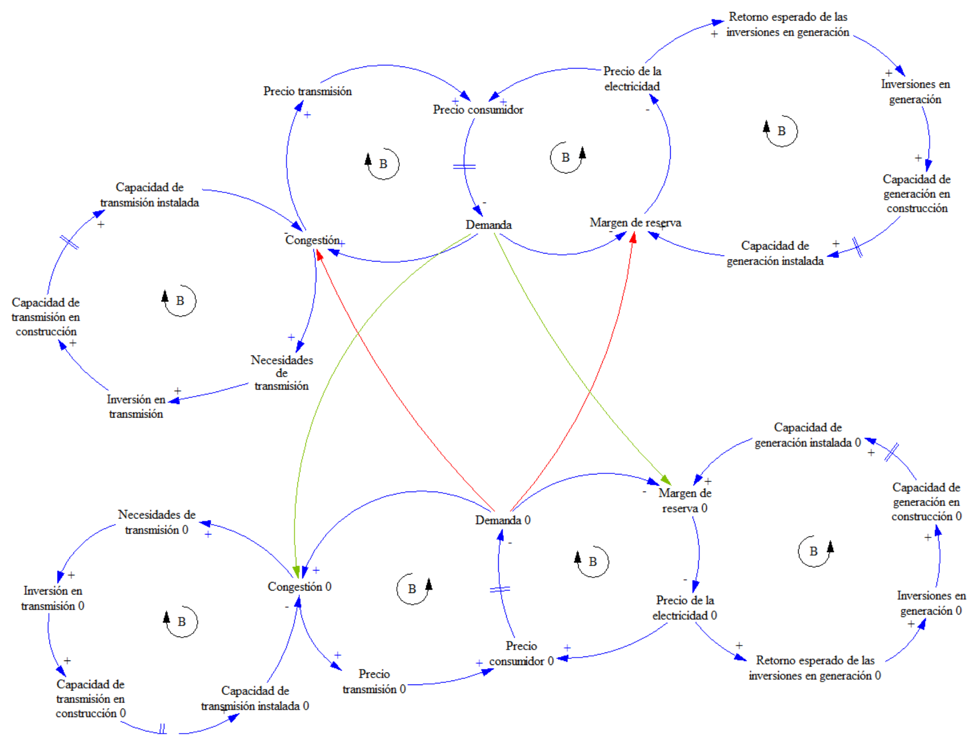

Figure 8: Causal for the integration between two countries.

5 Conclusions

About non smooth dynamical systems perspective, is sure that we will find so interesting results but is not easy to work with the differential equation in this big phase space. However is necessary to say that when we did the simulations in the usual system dynamics software appear problems with the integrator, the reason is that such software understand smooth model but not non smooth models. Our work goes in the way to find the non trivial behaviour of energy markets taking account its commutation regions.

About the model we are thinking in modified the equations of generation price and transmission price because its expressions use powers (called for us elasticities) that have made so heavy the work from dynamical systems such how we want. It not means that the system will be smooth.

The behaviour showed in the simulations for the special cases presented here say us that the model really represents an energy market.

References

[1] Velásquez, J. (2008). Construcción de Escenarios de Pronóstico del Precio de Eléctricidad en Mercados de Corto Plazo. Investigación doctoral. Universidad Nacional de Colombia, Colección facultad de minas: Medellín.

[2] Stoft, S. (2002). Power System Economics,Wiley Interscience.

[3] Armstrong, M., Cowan, S. y Vickers, J. (1994). Regulatory Reform: Economic Analysis and British Experience, The MIT Press, Cambridge.

[4] UNCTAD (2007). Competition in Energy Markets. TRADE AND DEVELOPMENT BOARD Commission on Investment, Technology and Related Financial Issues, Intergovernmental Group of Experts on Competition Law and Policy, Eighth Session Geneva. Item 3(i) of the provisional agenda.

[5] Bunn, D. (1998), Reflections on the progress of electricity restructuring, privatisation and regulation in the uk during 1988 to 1998 in XXXIII Senior Management Meeting C.I.E.R.

[6] Beggs, C. (2002), Energy Management, Supply and Conservation, Oxford: Butterworth Heinemann.

[7] Dyner, I. (1998). El mercado eléctrico colombiano: Resultados, problemas y perspectivas, Energética.

[8] del Sol, P. (2002), Responses to electricity liberalization: the regional strategy of a chilean generator, Energy Policy 30(5).

[9] Jaccard, M. (1995). Oscillating currents, the changing rationale for goverment intervention in the electricity industry, Energy Policy 23(7).

[10] Sanclemente, C. (1993), Desarrollo y Crisis del Sector Eléctrico Colombiano 1890 a 1993, Empresa Editorial Universidad Nacional.

[11] OLADE (1991), Situación Energética de América Latina y el Carible: Transición hacia el Siglo XXI, OLADE.

[12] Newbery, D. (2002). European deregulation problems of liberalising the electricity industry, European Economic Review .

[13] Ventosa, M., Baillo, A., Ramos, A. y Rivier, M. (2005). Electricity market modeling trends, Energy Policy 33.

[14] Dyner, I., Franco C.J. and Arango, S. (2008). El Mercado Mayorista de Electricidad Colombiano. Universidad Nacional de Colombia, Colección facultad de minas 120 años: Medellín.

[15] Hong, Y. y Lee,C. (2005). Aneuro-fuzzy price forecasting approach in deregulated electricitymarkets, Electric Power Systems Research 73.

[16] Angelus, A. (2001). Electricity price forecasting in deregulated markets, The Electricity Journal.

[17] Sterman John D. (2000). Business Dynamics, Systems thinking and modelling for a complex world, Mc Graw Hill.

[18] Deng, S. (2000a), Princing electricity derivatives under alternative stochastic spot price models, in Preceedings of the 33rd Hawaii International Conference on System Sciences.

[19] Deng, S. (2000b), Stochastic models of energy commodity prices and their applications: Mean reversion with jumps and spikes, Technical report, Program on Workable Energy Regulation (POWER) working paper 073.

[20] Ethier, R. (1999), Valuing electricity assets in deregulated markets: A real options model with mean reverting and jumps, Technical report, Cornell University.

[21] Ethier, R. y Mount, T. (1998), Estimating the volatility of spot prices in restructured electricitymarkets and the implications for options values, Technical report, Cornell University.

[22] Knittel, C. y Roberts, M. (2001), An empirical examination of derregulated electricity prices, Technical report, Program on Workable Energy Regulation (POWER) University of California, Energy Institute.

[23] Silva, B., Teixeria, J. y Gomes, L. (2001), Previsão de preços spot e avaliação de projetos de geração termelétrica, in IX Semiário de Planejamento Econômico Financeiro do Setor Elétrico (SEPEF).

[24] Velásquez, J. D. y Dyner, I. (2001), Pronóstico de precios de bolsa de electricidad usando un modelo de redes neuronales artificiales, in EITI-2001, Universidad Nacional de Colombia.

[25] Pulgarín, A., Smith, R. y Poveda, G. (2001), Predicción del precio de la energía eléctrica con un modelo de redes neuronales y usando variables macroclimáticas, in XIV Seminario Nacional de Hidráulica e Hidrología, Colombia.

[26] Conejo, A., Contreras, J., Espinosa, R. y Plazas,M. (2005). Forecasting electricity prices for a day-ahead pool-based electricity energy market, International Journal of Forecasting 21.

[27] Ramsay, B. y Wang, A. (1998). A neural network based estimator for electricity spot-pricing with particular reference to weekend and public holidays, Neurocomputing (23).

[28] Szkuta, B., Sanabria, L. y Dillon, T. (1998). Electricity price short-term forecasting using artificial neural networks, IEEE Transactions on Power Systems 14(3).

[29] Souza, R. (2002). Modelling the brazilian spot price series, in IFORS 2002.

[30] Medeiros, L. (2003). Previsão do Preço Spot no Mercado de Energia Elétrica, PhD thesis, Pontificia Universidade Católica do Rio de Janeiro.

[31] Hong, Y. y Hsiao, C. (2001), Locational marginal price forecasting in deregulated electricity markets using a recurrent neural network, in IEEE Power Engineergin Society Winter Meeting.

[32] Hong, Y. y Hsiao, C. (2002), Locational marginal price forecasting in deregulated electricity markets using artificial inteligence, in IEEE Proceedings of Generation, Transmition and Distribution, Vol. 149.

[33] Besant-Jones, J.E. (2006). Reforming Power Markets in Developing Countries: What Have We Learned? Energy and Mining Sector Board Discussion Paper No. 19. IBRD/World Bank.

[34] Hall, D. (1999). Electricity restructuring, privatization and liberalization: Some international experiences. PSIRU Reports. 9910-E-U-Prob.doc.

[35] Kwoka, J. (2005). Post-PUHCA consolidation of electricity: Five economic facts that should make us somewhat uncomfortable. Paper delivered at the Institute for Public Utilities Regulatory Policy Conference. Richmond, Virginia.

[36] International energy agency (2005). Lessons from Liberalised Electricity Markets. OECD/IEA: Paris.

[37] Bunn, D. (1995). Review of the electricity pool of England and Wales from a demand-side perspective. G. MacKerron and P. Pearson (eds). The UK energy experience. A model or a warning?. Imperial College Press.

[38] Luyo, J. (2008). Reducción del crecimiento económico por crisis en el sector eléctrico en Perú. Conimera: Congreso Nacional de Ingeniería Mecánica, Eléctrica y Ramas Afines. CIP-CDL : Consejo Departamental de Lima ,del Colegio de Ingenieros del Perú.

[39] Aracil Javier, Gordillo Francisco (1997). Dinámica de Sistemas, Alianza Editorial S.A., Madrid.

[40] Wiggins, S.(1990). Introduction to Applied Nonlinear Dynamical Systems and Chaos. Springer-Verlag: New York.

[41] Kuznetsov, Y.A. (1998). Elements of Applied Bifurcation Theory. Second Edition. Springer-Verlag: New York.

[42] Arnold, V. (1973). Ordinary Differential Equations. MIT Press: Cambridge, MA.

[43] Hirsch, M. and Smale, S. (1974). Differential equations, dynamical systems, and linear algebra. Academic Press: New York.

[44] Di Bernardo, M., Budd, C.J., Champneys, A.R. and Kowalczyk, P. (2008). Piecewise-smooth Dynamical Systems,Theory and Applications, Springer-Verlag: New York.

[45] Guckenheimer, J. and Holmes, P. (1983). Nonlinear oscillations, dynamical systems, and bifurcations of vector fields. Springer-Verlag: New York.

[46] Lynch, S. (2004). Dynamical Systems with Applications Using Matlab. Birkhäuser: Boston.

[47] Piiroinen, P. and Kuznetsov, Y. (2005). An event-driven method to simulate Filippov systems with accurate computing of sliding motions. EU FP5 Project SICONOS (Grant no. IST-2001-37172).

[48] Dercole, F. and Kuznetsov, Y. (2005). Slidecont: An auto97 driver for bifurcation analysis of Filippov systems. ACM transactions on mathematical software, Vol. 31, No.1.