Abstract

Recent literature in economic growth suggests that it is taking place a theoretical research convergence among historical studies and neo-growth theory. We argue in this paper that system dynamics, due to be a flexible methodology, may be an instrument for helping to bridge those two strands of thoughts. The paper begins for presenting a stylized historical background for endogenous growth theory which is perhaps one the more appealing modern interpretation of growth process available. A simple endogenous growth model based on this historical background, which explains modern growth without recurring to the hypothesis that a historical singularity has been actually necessary for triggering the process, is then provided and re-written in system dynamics language. An enlarged version of the basic growth model is next presented and it is shown how to assess the strength of the different feedback loops involved in the process of economic growth. The conclusive section of the paper finally  suggests that system dynamics can be an important complementary tool for understanding and perhaps enhancing economic growth in less developed countries.
1 - Introduction

The last decades have witnessed the renaissance in the interest in economic growth but there remain some puzzles in the existing research. For example, why did no country or region experienced sustained intensive growth before eighteenth century? What led to the Industrial Revolution and was it really a turning point in economic history? What caused the increasing divergence in living standards across the world during the last 250 years? The three questions are of course inter-related and attempting to answer them seems to be a necessary step to understand why many countries and regions of the world are still underdeveloped.

The more appealing explanations for modern economic growth suppose in a way or another that a historical singularity would have happened in England in the XVIII century and unleashed a self-reinforcing growth process which has commanded the dynamics of the world economy afterwards. The Industrial revolution thus would have indeed been a turning point in economic history. The classical interpretation by Douglass North on why it occurred in England and in that time, elaborated later by Daron Acemoglu and colleagues (Acemoglu et al. 2005), for example, states that a key pre-condition for Industrial Revolution was the previous Glorious Revolution. That revolution, in reducing the power of the king in expropriating citizens by the creation of new laws or taxes, would have solved the time-inconsistency problem. This problem meant that citizens could not trust the king would keep his commitment of not expropriating citizens because that would go against the own king’s interests. Other interpretations, as the ones by Joel Mokyr and David Landes, take for granted that Industrial Revolution was actually a technological revolution which, while requiring previous social and economic pre-conditions as pointed by North, triggered a self-reinforcing process much based on technological spillovers first through the British economy and later to several other countries.

One important question that arises from those interpretations is if the process of modern growth was more a product of accident than a necessary outcome of capitalist development. In the first case, must the less developed countries today wait for a particularly lucky configuration of factors to eventually overcome backwardness?

Modern growth theory (fortunately) has a plausible explanation for growth which does not depend on radical changes in institutional or technological structures of
countries or regions. That explanation lies more on the capacity of generation and diffusion of new ideas and technologies. But the fact that capacity is also an outcome of countries’ institutional structures suggests that there might be presently taking place a theoretical research convergence among historical studies and neo-growth theory. We will argue in this paper that system dynamics, due to be a flexible methodology, may be an instrument for helping to bridge those two strands of thoughts.

The paper is divided in four parts besides this introduction. In section two, we present a stylized historical background for endogenous growth theory which is perhaps one the more appealing modern interpretation of growth process available. In section three, a simple endogenous growth model is presented and re-written in system dynamics language, which explains modern growth without recurring to the hypothesis that a historical singularity has been actually necessary for triggering the process. Section four enlarges the simple version of the growth model presented in section three and show how to assess the strength of the different feedback loops involved in the process of economic growth. Section five concludes suggesting that system dynamics can be an important complementary tool for understanding and perhaps enhancing economic growth in less developed countries.

2 – Historical background: Joel Mokyr´s interpretation of economic growth

According to Mokyr (2004), growth after 1800 has been firmly grounded in which is known as useful knowledge, a concept that describes two types of knowledge. One is knowledge ‘what’ or propositional knowledge – $\Omega$ – about natural phenomena and regularities. Such knowledge can be applied to create knowledge ‘how’, that is new techniques, which we may call prescriptive knowledge ($\lambda$-knowledge). The core of Mokyr’s argument is that much techniques before 1800 lack a solid epistemic base, that is were based in $\lambda$-knowledge, and therefore rarely led to continued improvements. The widening of epistemic base after 1800 signals a phase transition in the dynamics of useful knowledge in which a positive feedback loop started to dominate the dynamics of the system. Growth in epistemic base led to enlargements of the $\lambda$-knowledge and thus to the creation of new techniques, but $\lambda$-knowledge also produced a feedback into $\Omega$ leading to further expansions of the epistemic base.
Still according to Mokyr (pp. 20-21):

“Positive feedback from \( \lambda \) to \( \Omega \), then, can lead to virtuous cycles much more powerful that can be explained by technological progress separately. The process is self-sustaining because the two types of knowledge are complementary in the technical sense that a growth in one increases the marginal product of the other… If there is sufficient complementary between an upstream process (\( \Omega \)) and a downstream process (\( \lambda \)) in the system, persistent, self-reinforcing economic change can occur even without increasing returns. It should be added that \( \lambda \) itself can also show persistent dynamics, in that new technology leads directly to further inventions that introduce local improvements and ‘debug’ the techniques. Without a corresponding growth in the epistemic base, however, such episodes have tended in the past to converge to a higher level of technology but did not lead to a self-sustained cumulative growth in which knowledge spins out of control.”

The causal diagram in figure 1 below depicts the overall idea.
Figure 1: The Mokyr’s positive feedback loop of the knowledge

The phase transition between the old and the new growth regimes took place obviously in the XVIII century with the Industrial revolution (p. 33):

“Useful knowledge increased by feeding on itself, spinning out of control as it were, whereas before the Industrial revolution it had always been limited by its epistemic base and suppressed by economic and social factors. Eventually positive feedback became so powerful that it became self-sustaining. The positive feedback effects between $\Omega$-knowledge and $\lambda$-knowledge thus produced a self-reinforcing spiral of knowledge augmentation that was impossible in earlier days of engineering without mechanics, iron-making without metallurgy, farming without organic chemistry, and medical practice without microbiology. The changes in social environment in which useful knowledge was created and disseminated led not only to an increase in the size of $\Omega$ (through discovery) but also to higher density (through diffusion).”

By the author, however, the full explanation for why the positive growth loop became dominant exactly in that time is ultimately exogenous (p.287):
“An evolutionary approach to the history of knowledge implies that we cannot ‘explain’ why modern economic growth happened after 1800 much better than we can explain why homo sapiens emerged when it did, and not, say, 30 million years earlier in the middle of the Oligocene. We can show, however, how it evolved from earlier intellectual developments, such as the Renaissance, the scientific revolution, and the Enlightenment.”

Does that mean that less developed countries have to wait for a unique configuration of factors to have their own phase transition? Or there might be some endogenous mechanisms capable to trigger virtuous growth feedback processes such the one described above? In particular, is it possible that incremental changes (and not necessarily an institutional or technological revolution) could lead less developed countries to overcome certain thresholds or tipping points beyond which growth can become a endogenous feature of those systems?

The new growth (or endogenous growth) theory has increasingly provided support to the second alternative above. But while emphasizing the utmost importance of ideas to growth it, even if implicitly, places institutions in the center of the stage, in suggesting some fundamental question about growth like what are institutional configurations that allow the more vigorous pace of knowledge accumulation? The state of the art in growth theory thus points out to a theoretical convergence among institutional-oriented and economic theory-oriented growth theories, in which factors traditionally contemplated in economic models are considered as proximate and institutions are considered as fundamental determinants of economic growth.

Rodrik (2003) has provided a integrated framework for highlighting the relations between proximate and deep determinants of economic growth as depicted in figure below.
Figure 2: Proximate and deep determinants of economic growth

The total output of an economy is a function of its resource endowment or factors of production and the productivity with which these factors are deployed to produce output. If we express this relationship in the form of an aggregated production function, the growth of per capita output can be expressed in terms of three proximate determinants; a) physical capital deepening; b) human capital accumulation; and c) productivity growth. To the extent that growth is driven by other fundamental determinants, however, the causality might well run backwards, for instance from growth to factor accumulation. So it is best to think of accumulation and productivity change as proximate determinants of growth. The deep determinants of growth would be then geography, integration (trade) and institutions. Source: Rodrik (2003)

The central question in growth analysis is: which of the relationships in the figure matters most? The causal interrelationships between the variables indicated by the two-way direction of the arrows in the lower part of the diagram suggest that there are complex feedbacks at work. Which, due to the problem of endogeneity, makes empirical work based only in cross-country regressions among those variables inadequate to reach more sound conclusions. In order to assess the strength of the feedback loops involved it seems we need tools like the algorithm for detecting loop dominance shifts that we present in next section. It needs to be emphasized, however, that system dynamics studies, economic modeling and cross-national econometrics are not substitutes for each other. They can be used in a complementary fashion with traditional economic modeling generating novel hypothesis about the inner working of economic growth, system dynamics models providing hints to assess the strength of the feedback loops involved and econometrics suggesting new cross-national tests. In the next section we suggest how this could be done.
The great advances in modern economic growth theory have taken place mainly after the fundamental problem of modeling Solow technological residual was solved. This is the key contribution of the modern endogenous growth theory which explains economic growth mainly as an outcome of knowledge accumulation which is subject to increasing returns. Examples of goods capable to generate increasing returns are softwares, a patent, a mechanical drawing and a blueprint. While conventional economic goods are both rivalrous and excludable, and so can be privately provided and traded in competitive markets, two fundamental attributes of knowledge are (partial) non-rivalry and non-excludability.

Nonrivalry means, first, that goods that exhibit this feature can be accumulated without bound on a per capita-basis and, second, that those goods are capable to generate spill-overs through the economy.

To see this more formally, suppose the production function $F$ below, where $A$ represents investments in non-rival and $X$, investments in rival inputs. It follows that:

$$F (\omega A, \omega X) > F (A, \omega X) = \omega F (A, X)$$

Because of the properties of homogenous functions, it also follows that a firm subject to these kinds of production possibilities would not survive as a price taker. The reason is that if the product is sold by its marginal cost, and technology is freely available to all producers, even to firms that do not invest in knowledge, prices will be lower than factor payments in innovative firms and there will be no incentives to invest in knowledge in the long term. For investment in knowledge happen, then, it is necessary technology to be at least partly excludable, by patents or other institutional mechanism.

If those mechanisms are provided, aggregate output will grow more than proportionally to the input use, since investment in knowledge for one firm increases the general stock of knowledge, generating spill-overs to other firms, even if they do not invest in knowledge themselves. The argument is much more complicated involving the use of relatively sophisticated models, but intuition suggests that if investment in knowledge is an increasing function of profits and the best institutional arrangements...
for gaining access to the knowledge that already exists in the world are provided, growth may then become endogenous. An interesting model by Kremer (1993) gives us some fundamental insights about the basic logic of endogenous growth models.

The model states that the long-run history of population growth and technological change is consistent with the population implications of models of endogenous technological change. Based on very simplified assumptions about how technology affects the growth rate of technology and how population affects the growth rate of technology, it builds an interpretation which fits actual data surprisingly well.

The starkest version of the model is as follows

\[ Y = A p^{α} T^{1-α} \quad (1) \]

Y is output, A is the level of technology, p is population and T is land, which is normalized to 1. Per capita income, therefore, equals \( A p^{α-1} \).

Assuming diminishing returns to labor imply that a unique level of population, \( p^* \), generates the steady state equilibrium level of per capita income \( y^* \):

\[ p^* = \left( \frac{y^*}{A} \right)^{1/(α-1)} \quad (2) \]

The last assumption is that if each person’s chance of inventing something – g – is independent of the population size and if A affects research output linearly, the growth rate of technology will be:

\[ \frac{\dot{A}}{A} = pg \quad (3) \]

Taking logarithm of the equation (2) and differentiating it with respect to time, we have:

\[ \frac{\dot{p}}{p} = \frac{1}{1-α} \frac{\dot{A}}{A} \quad (4) \]

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\(^{1}\) Romer (1994: 21)
Substituting in the expression for the growth rate of technology from 3, we get:

\[
\frac{p}{p} = \frac{g}{1-\alpha} p 
\]  

(5)

Which gives the testable proposition that the growth rate of population will be proportional to the level of population. Econometric tests performed by the author indicate that the models fits data from one million B.C to 1990 surprising well, without having to recur to any exogenous further explanation.

The basic model can be expressed in the system dynamics language straightforwardly as below.
Running the model for the period 1 AD- 2000 with parameters $g/(1-\alpha) = 2.87e^{-006}$ (value calibrated in simulations), $\alpha = 0.67$, and initial population = 170 million of people, we get the trajectory depicted in figure 3. The simulation as it is easy to check matches the pattern of population growth data surprising well (population data is taken from Kremer, op. cit., p.683).

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2 The equations of the model are:
Population Growth Rate = Population Flux = Population* $g/(1-\alpha)$
$g/(1-\alpha) = 2.87e-006$
Population = INTEG (Population Flux,170)
Notice that the model does not obviously capture exogenous effects as the “black death” epidemic in the middle ages. We can however easily include those kind of effects in the model using function Pulse. In Figure 5 below we model the assumption that the epidemic has killed 1/3 of world population in the year of 1350. Run “Black Death” displays this scenario, showing that such strong isolated effect may have affected permanently population dynamics.
Yet the basic model is obviously too simplified, for not taking in account basic facts of growth, as the increase in research productivity after 1750 and the demographic transition which took place in developed countries in the last 50 years or so.
An enlarged version of the basic model and a procedure for identifying loop dominance shifts

In order to correct those shortcomings, the author tested a number of different specifications, which however did not change the core conclusions of the basic model. Such alternative specifications, using system dynamics language, are showed in figure 12 below,

Figure 6: Kremer’s model is system dynamics language
The three self-reinforcing loops on the top of the diagram imply increase in research productivity, that is in the capacity of population having new ideas capable to be transformed in new technologies, which would shift upward the world production function. The first loop – which we have called “the knowledge spiral” – models the positive effect of the knowledge on the generation of new ideas, as proposed by Mokyr. This effect predicts that the higher the technological level, the higher the capacity of population to create new ideas, that is the larger the value of g. The second self-reinforcing loop – called “Smith spiral” - models the likely positive effect of population growth on g, reflecting possible agglomeration effects, such the ones brought about for division and specialization of work deepening over research productivity. The third self-reinforcing loop – “technological development spiral” – specifies the positive effect of economic growth - which in theory requires better institutions such as secure property rights – on research productivity. The inclusion of those loops makes the model consistent with data in the last decades, such as the absence of technological convergence among the most populated countries and the stability of world average technological growth rate. Yet the augmented model generates predictions for population growth that are qualitatively similar to those from basic model and, therefore, it also does not explain the recent leveling of population growth (Kremer, p. 692).

The last (negative) feedback loop included in the model – “the fecundity loop” - specifies the negative effect of high levels of income on fecundity. We model this relation using a lookup variable as below:

Population Growth Rate = g/(1-alfa)*População* Income Impact on Fecundity
Income Impact on Fecundity = Income Effect on Fecundity (Relative Per Capita Income)
Income Effect on Fecundity = [(0,0)(3,2), [(0,1),(1,1),(1.1,0.7),(1.5,0.3),(1.8,0.2),(2,0),(3,0)]
Relative Per Capita Income = Per Capita Income/ Income Tipping Point

In simulation below, we assumed the Income Tipping Point is 4000 unities of wages, the value of per capita income around 1970\(^3\), when world population growth rate

\(^3\) The value of per capita income is given in terms of unities of wages per million of persons, that is monetary value of the wages equals 1 for million of persons. To compute the monetary value of income in each year we must multiplying the value in unities of wage for the value of average world wage in that year. Per capita income of 10700 in 2000, for instance, corresponds to a monetary income of US 7000, if
started to fall (Kremer, p. 683), suggesting that the Income Effect on fecundity has become operative.

Figures 7 and 8 below depict simulations using the formulation above. Notice that population now presents a clear tendency to stabilize at some point after 2100. Income per capita, however, will continue to grow beyond this point as far as knowledge continuous to be accumulated. In the long term, eventually, as far as less developed countries experience their own demographic transitions, world population will level of and so do technological growth and income per capita.

Figure 7: Population Dynamics in the Kremer’s augmented model

we assume a average world wage of US$ 4000,. This figure is consistent with estimation usually accepted in studies on economic growth; see for instance Barro e e Sala-i-Martin (2004).
A simple procedure to identify tipping points like the one in which the impact of income on fecundity starts to dominate system dynamics is proposed by Ford (1999). That procedure consists basically in activating and de-activating loops and identifying the threshold beyond which the dynamics of the variable of interest changes (see appendix). The simpler way to de-activate the fecundity loop is establishing the income- tipping point at a value high enough not to be reached by the system, say at 50000 unities of wages; doing that means that the loop will be inactive along all simulation time. Figure 9 shows that dynamics of population changes around 1980, year beyond which the fecundity loop starts dominating the system dynamics. That is of course the inflection point of population curve with fecundity loop activated, but that is simple to realize just because there is the only loop actually simulated in the model. In more complex models, where for instance the three self-reinforcing loops were included in simulation, we should necessarily apply a procedure as the one above mentioned in order to identify loop dominance shifts.
5 - Conclusion

In the beginning of this paper we proposed that a theoretical convergence is taking place in the field of economic growth theory. On the one hand institutional factors have increasingly been acknowledged as very important or even decisive for growth. On the other hand, economists have highlighted the accumulation of knowledge as the engine of growth. The link between the two strands of thoughts is obviously that we should look for causes of economic backwardness of less developed countries in the elements of their institutional matrix which impair the development or diffusion of new ideas and technologies through the economic system. That is hardly an original idea; since at least the classical works of Schumpeter it is well known that innovation is a powerful engine of growth. The specific contribution of the so called new growth theory is that we today have formally identified the crucial features of growth process. In particular, thanks to the works of economists like Paul Romer, we are aware of the importance of institutional protection to intellectual property rights to the process.
Yet models of endogenous growth generate very complicated dynamics which usually cannot be solved analytically, which has forced researchers to use for instance phase diagrams analysis in order to understand the main properties of their dynamics. Thus an old criticism to system dynamics models – namely that they would not present analytical and therefore more general solutions- are presently outdated. On the contrary, for its own nature, system dynamics models are more flexible and therefore capable to provide more straightforward formulations than analytical models based on differential or difference equations. Due to be more flexible, they for instance allow easily modeling the impact of different institutional configuration on the generation of knowledge process and therefore on economic growth itself.

We seem therefore entitled to predict that traditional modeling will likely continue to be important for understanding the basic mechanics of economic processes while system dynamics modeling will probably be more important for helping to identify the specifics of those processes. For instance, as soon as we have reached the conclusion that the lack of technical progress is a crucial factor explaining low growth in a particular less developed country, we could deploy system dynamics methodology to understand, using tools such as sensitivity analysis and calibration procedures, what institutional configuration would produce the best incentives to innovation and knowledge accumulation and so the higher sustainable log-term growth rate.

The exercise performed in this paper attempted to show that formal modeling and econometric testing continue to be indispensable in so far they can lead to highly counter-intuitive conclusions not obtainable by other means. For instance to the astounding suggestion that it is not really necessary to suppose the occurrence of any institutional or technological revolutions at some point in the past for explaining modern economic growth. That essential feature of modern world – namely continued growth - could have been instead produced by the accumulation of very gradualist changes which at some point would have triggered a self-reinforcing growth loop. Historians may be somewhat uncomfortable with this type of argument, because it apparently would make their work irrelevant. But that would be a mistake.

System dynamics by allowing taking in account the effects of different institutional configuration on economic growth, if anything, seems rather to open a wider space for historical studies in economic research in so far system dynamics models only can fit data if they are based on reliable parameters. While growth economists focused mainly on analytical models, having accurate parameters could be
not that decisive. But since we presently have so a powerful methodology as system
dynamics to study the more remote long-term implications upon economic system of
slightly different initial conditions, history can become more important for economic
studies than ever.

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Appendix

The algorithm proposed by Ford (1999) to identify loop dominance shifts is the
following:

1) Identify the variable of interest that will determine feedback loop dominance
   and simulate the behavior of that variable over time.
2) Identify as a time interval which the variable of interest display only one
   atomic behavior pattern\(^4\), that is the time interval in which the trajectory
   overtime presents the same second derivative. This is the reference time
   interval.
3) Identify the candidate loops, that is the feedback loops that may influence the
   variable of interest.
4) Identify or create a control variable in each loop that is not a variable in other
   feedback loops and can vary the gain of the candidate loop. Use the variable
   to deactivate each loop
5) Simulate the variable of interest over the reference time interval with each
   loop deactivated and identify the atomic behavior pattern of the variable of
   interest during the time interval
6) If the atomic behavior pattern is different than the reference pattern
   identified in step 2, the loop tested dominates the behavior of the variable of
   interest under the conditions during that time interval. If the atomic behavior
   pattern is the same and there are no shadow feedback structures involved the
   loop does not dominate system dynamics in that time interval\(^5\).
7) Repeat steps three trough 6 for the remaining loops

\(^4\) There are three basic behavior patterns based on the net rates of change of the variable of interest: a)
   linear behavior, when the absolute value of the net rate of change of a system variable is constant, b)
   exponential growth or decay, when the absolute value of the net rate of change of a system increases over
time and c) logarithmic growth or decay, when the absolute value of the net rate of change decreases over
time.

\(^5\) Shadow feedback structures occur when two or more loops jointly dominate the dynamics of a system;
in that case we should test for loop dominance deactivating all the linked loops at the same time. For the
purposes of this work, we will consider only the simplest case where there are no shadows structures
involved. For more details on how to identify shadow structures see Ford, 1999, pp. 18-23.
By applying the procedure detailed in the last subsection to the Kremer’s model, we can identify the interval in which the fecundity loop spiral dominates population dynamics:

1) the variable of interest is the population level and we simulate its dynamics over a period of 2000 years beginning in the year 1 AD.
2) the reference time interval is given by the period comprehending the years 1980 through 2000 of the simulation in which the system presents a clear logarithmic atomic behavior, that is where \( \frac{d^2 x}{dt^2} < 0 \).
3) “fecundity loop” is chosen as the candidate loop.
4) “fecundity loop” is deactivated for equaling income tipping point to 500000
5) The behavior of the variable of interest over the reference time is simulated with “fecundity loop” deactivated. Behavior of the variable population with candidate loop activated and deactivated is depicted in figure 9
6) the dynamics of the variable of interest in the reference time interval changes from exponential to logarithmic (the atomic pattern changes from a positive to a negative value), indicating that the “fecundity loop” dominates the behavior of the variable of interest from year 1980 to year 2000.
7) the next step would be to repeat steps 3 through 6 for the remaining loops which is beyond the scope of this work

References