A System Dynamics View of the Phillips Machine

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Abstract
Although system dynamics practitioners appeal to a bathtub analogy to explain the basic concepts of system dynamics, few physical water systems have been used for this purpose. The celebrated Phillips machine, a hydraulic macroeconomics model built in 1949, illustrates how this might be done for a serious system dynamics model. This paper reviews the physical details of the Phillips machine, relates them to standard system dynamics components, and proposes two similar machines to represent basic structures in system dynamics instruction.

Recent experimental evidence shows that many people find the basic system dynamics notions of accumulations, information feedback, and delays difficult to incorporate into their customary ways of thinking. (Cronin *et. al*. 2008; Sterman and Booth Sweeny 2007; Booth Sweeny and Sterman 2000) To overcome this barrier, system dynamicists have appealed to a water analogy where stocks become tanks or bathtubs, outflows become drains, and controls become faucet valves. (Sterman 2000; Morecroft 2007) The analogy invokes the past experiences and knowledge that people already have from filling physical bathtubs or controlling the temperature of real showers, but the analogy remains in the conceptual realm and is used only as a thought aid to comprehend the foundation concepts of system dynamics. Further recent experimental evidence shows that computer models of dynamic systems, in conjunction with causal loop diagrams, are a more effective way of teaching macroeconomic principles than the traditional methods of manipulating static graphs. (Wheat 2007) It is not difficult to conjecture that physical dynamic models based on the hydraulic analogy and built of real tanks, pipes and sluices would be powerful instructional tools not only for demonstrating the first principles of system dynamics but also for explaining the subtleties of complex topics such as the dynamics of macroeconomics.

In fact, this conjecture has been tested already, albeit unscientifically. The legendary Phillips hydraulic economic model demonstrated the dynamics of contemporary macroeconomic theory to generations of graduate students in economics at the London School of Economics, Cambridge University, and elsewhere. (Leeson 2000; Moghadam 1989; Bissel 2007) Phillips built the first machine in 1949 to better understand macroeconomics as an undergraduate student himself. Upon demonstration to faculty in November of 1949, Phillips' garage-built system became the prototype for improved hydraulic machines that served as mainstay economics lecture apparatus for more than two decades. (Newlyn in Leeson 2000) Today, the sole operational Phillips machine resides at Cambridge University where it performs as a celebrity for special events. The machine seems almost alive as it noisily adjusts its variables in response to the operator's manipulations. The sensory impact alone guarantees a memorable experience, and the physical manifestation of abstract variables, all visible, all changing, all obviously connected, offers a conceptual bridge for people unfamiliar with system dynamics -- and a physical metaphor for those unfamiliar with macroeconomics.[1]

The Phillips machine contains all the main elements of a modern system dynamics computer simulation, including stocks, flows, auxiliary variables, information feedback, non-linear functions, time graphs, and exogenous controls modifiable during simulation. Because he designed the machine a decade before Forrester developed modern system dynamics notation (Forrester 1959), Phillips worked directly with plumbing concepts to model the relevant macroeconomic concepts. All modern references to the Phillips machine describe its operation in terms of physical tanks, cascades, troughs and sluices. This paper supplies a system dynamics interpretation of the design principles employed in the Phillips machine. It is motivated by the desire to construct other hydraulic physical models of other dynamic systems as lecture aids for explaining basic system dynamics concepts. The effectiveness of such physical models as teaching aids cannot be evaluated experimentally until they are constructed. Here, we draw design ideas from a system that worked. In so doing, we also translate the original hydraulic design into modern system dynamic model components, and suggest plumbing designs for other similar machines.

Unfortunately, physical plumbing diagrams and stock and flow diagrams are not quite isomorphic. The analogy between water in physical tanks and abstract stock and flow diagrams unravels in at least two key areas. First, the flows of system dynamics contain no "stuff". System dynamics model implementations only contain stuff in the stocks. Stuff is magically transported from stock to stock at known rates without being present in flows except as a stuff-per-time measure. Water, by contrast, has to flow through pipes, troughs, or sluices to get from one tank to the next. This means that a pipe can be regarded as a flow for some purposes and a stock for others. The answer to the question, "How many stocks are there in the Phillips machine?" depends on the granularity with which we model it with our system dynamics tools. The second problematic area concerns outflows and flow measurement. The Bernoulli equation from fluid mechanics implies that water will escape from a small hole in a tank at a rate proportional to the one-half power of the height of the water above the hole. (Feynman 1964) This nonlinear relationship means that, unlike the standard system dynamics analogies, real bathtubs do not drain in linear proportion to the amount of water they contain. The nonlinear relationship is fixed by physics and requires extraordinary means to circumvent.

Phillips' ingenious resolutions of these problems created much of the physical structure of his machine. This paper first expresses various physical parts of the Phillips machine plumbing in standard system dynamics stock and flow notation. We next offer a stock and flow model of the complete machine that has been simulated using a standard system dynamics software package. This requires that we analyze the Phillips machine as a model itself and ascribe modeling intent to its various parts. The paper then suggests two plumbing designs for elementary lecture apparatus using the same design methods evident in the Phillips machine.

Valves

The valves in the Phillips machine are all sliding gates whose horizontal positions set the rate of water escape from a small reservoir of constant head (height of water) just above the gate. There is typically a left gate and a right gate whose positions represent two separate valve control variables. The resulting flow is proportional to the sum of the two variables. When both variables assume their extreme values to diminish the flow, the gate is closed and no water escapes the reservoir. Significantly, the gates never control a free-flowing cascade or the outflow from a stock directly, but only the flow from constant head reservoirs. Generally, the constant head reservoirs in the Phillips machine are not stocks, as they always contain the same fixed head of water and do not contribute to the dynamic behavior of the machine. They will not appear in our stock and flow translations of the physical water flow structure. A single exception to this rule, discussed later, occurs where two outflows share the same constant head reservoir.

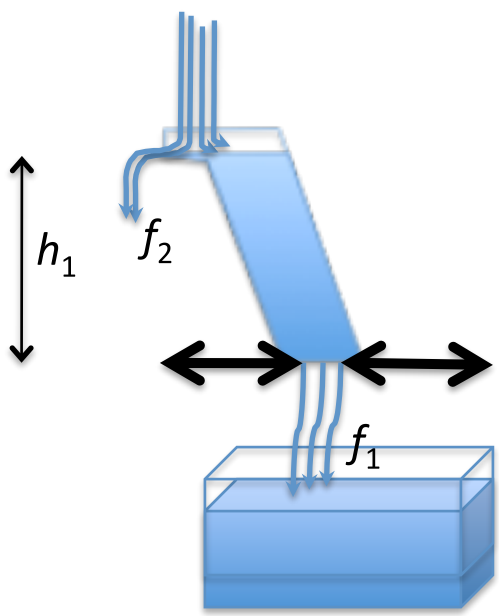
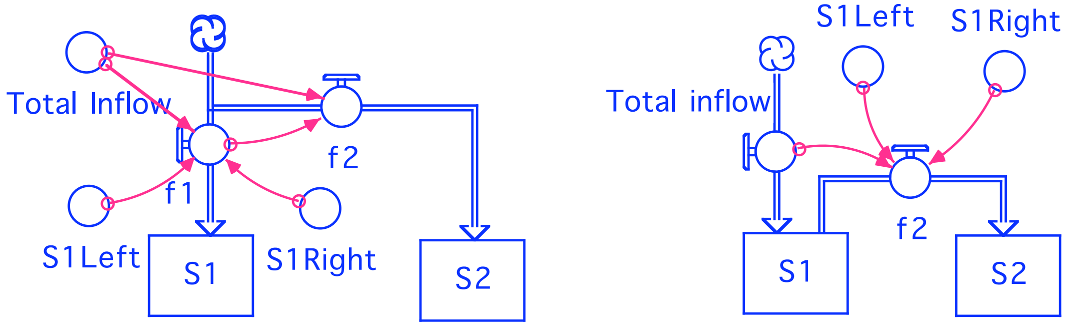


Figure 1: Split Flow Constant Head

Figure 1 shows one of two methods Phillips employed to realize a constant head reservoir, in this case to control the inflow to a stock. The gates, shown as horizontal double arrows, determine f_1, the outflow of a fixed head reservoir. By assumption, the total inflow to the reservoir must exceed f_1, so a residual flow, f_2, is also created. The height of the reservoir, h_1, will be the head of water above the gates. Thus, for an inflow greater than f_1, the gates control f_1 directly and f_2 implicitly. The machine uses this scheme to obtain precisely metered inflows for all of its stocks. Either or both of the flows f_1 and f_2 can be used to fill stocks, depending on the modeler's intent. Phillips always used both flows.



$f1$ = Total Inflow * (S1Left + S1Right)/2

$f2$ = Total Inflow – $f1$

Figure 2: Equivalent Stock and Flow Diagrams for Figure 1

Figure 2 shows the stock and flow notation equivalents to the split flow constant head reservoir. The constant head reservoir itself never appears. S1Left and S1Right are fractions between 0 and 1. If f_2 is not used elsewhere in the model, then, conceptually, it can be absorbed into the cloud.

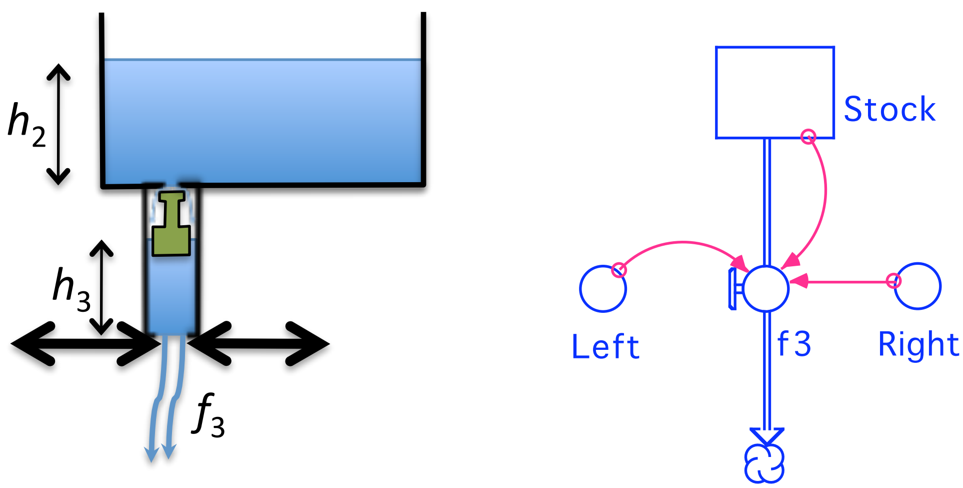


Figure 3: Float Controlled Constant Head

The second method to achieve a constant head reservoir appears in Figure 3. This method was only used for stock outflows. A floating valve releases water into the constant head reservoir when the level below the float falls below the desired head, h_3. As the reservoir level reaches the desired value of h_3, the float-valve closes. Thus, the stock outflow f_3 is decoupled from the head of water in the stock, h_2. Should the stock level, h_2, fall to zero, the small reservoir will rapidly empty and f_3 will go to zero. In the corresponding stock and flow abstraction, $f_3 = \text{MIN}(k_1\sqrt{h_2}, k(\text{Left} + \text{Right}))$ where k, k_1 are constants with appropriate dimensional units. The valve control variables, Left and

Right, are the positions of the two gates and can be arbitrary positive functions of time less than their maximally open values.

Significantly, all constant head reservoirs in the Phillips machine, inflow and outflow alike, have the same head.[2] Thus, all the sliding valve gates operate with the same linear scale factor relating horizontal movement to amount of water released.

Clouds

There are no infinite sources and sinks of water in a freestanding hydraulic machine. The water must recycle continually from a tank at the bottom of the machine to an input drain at the top. Conceptually, any part of the circulating loop of water not used to model some variable is in the cloud. Model components such as stocks and flows merely divert water from this main circulation loop to perform their function, then return it to the loop, possibly via the tank at the bottom of the machine.

In the Phillips machine, the main loop circulation represented aggregate income of the country. Hence, the main circulation was not a "cloud", but an endogenous variable. However, during special modes of operation one or more of the main model stocks were held at nearly fixed levels by water drawn from and delivered to large tanks hidden behind the machine. Even though the unseen tanks modeled real-world economic variables, (Newlyn in Leeson 2000) it is useful to think of them as clouds in understanding the machine's operation.

Phillips employed a water-sensing switch in the machine's bottom tank to energize a pump to recycle the main circulation water to the top of the machine. This arrangement operates in gasping pulses, sending surges into the stock and flow cascade while producing charismatic sloshing sounds and visible turbulence. The prominent tank at the bottom of the machine, marked "National Income", receives the water from the model above and passes it on to a smaller adjacent tank, labeled "Income" through a specially shaped slot or weir. The pump then recycles the water to the top. In system dynamics terms, the "National Income" tank is the stock in a first-order material delay in the circulation of income. The tank's level does double duty as a flow measurement device, indicating the size of the flow of income.

Information and action causality

The flows in system dynamics structures depend on the values of stocks or other flows. The Phillips machine captures these interdependencies by attaching floats in various tanks to lines, pulleys and counterweights that, in turn, control the sliding valve gates of other flows through motion-translating cams. The cams are called "functions" by those familiar with the machine, and operate as graphical variables or nonlinear lookups do in system dynamics simulation tools. As each float moves up and down with the level of the water in its tank, the pulleys and lines transfer that movement to a vertical movement in proximity with the flow to be controlled. The corresponding cam, pulled vertically by the motion of the line, drives a pin on the valve gate in a horizontal movement dictated by the track of a curved slot cut in the sheet of plastic that comprises the cam. Thus, the cam imposes a functional relation between the vertical movement of the float (independent variable) and the horizontal movement of the valve gate (dependent variable). A straight-line track imposes a linear relation while a curved track imposes a nonlinear one. The cam holders and valve gates have measurement scales attached, so that readings of all the variables in the machine can be taken manually as the machine operates. One can remove the slotted plastic sheets (cams) from their holders easily. If a cam is removed, its associated sliding valve gate must be moved manually. This breaks the functional dependence between float and valve and renders the valve variable an exogenous assumption. Some of the valve cams are tied together vertically so that one float variable will drive their common vertical movement.

Measurement of stocks and flows

The levels in water stocks are easily measured with floats. Stocks in the Phillips machine contain large floats. These move with sufficient force to drive the pulley-line-cam-valve linkage described above. Some of them have enough spare force to operate lines connected to the pens on a moving chart recorder, so as to produce a time graph of stock level.

Flows of water are more difficult to measure. Phillips used a refined version of a slot flow meter to obtain accurate estimates of the flows in his machine. A slot flow meter is a small tank with a vertical slot or weir cut in one of its vertical sides. Water in the flow fills the tank from above and escapes through the weir. The shape of the weir determines the functional relationship at equilibrium between the height of the water in the tank and the rate of flow out the weir. Ideally, the outflow will be proportional to the height of the water in the tank. Then the height of water is a direct measure of the flow. Unfortunately, the physics of fluids imposes a one-half power relationship between water height and flow out a hole. Slots with vertical sides, i.e., U- shaped, cause the outflow to be proportional to the three-halves power of the water height. V-shaped slots enforce a five-halves power relation. Phillips' description of the first version of the machine, the Mark I, mentions slots whose width, W(*x*), is given by the formula (Phillips in Leeson 2000)

$$W(x) = k/\sqrt{x}$$

where k is a constant and x is measured vertically upward from an origin just below the bottom of the tank. Hydraulics engineers know that proportional flow weirs such as these slots are obtained with a slightly different curve that does not suffer from the infinitely wide base of the $x^{-1/2}$ slot. Known as Sutro weirs, they have a rectangular base with a nonlinear tapered slot above it. They enforce an affine relationship between the flow and the head

$$\text{flow} = k_1\text{head} + k_2, \qquad k_1 \text{ and } k_2 \text{ being constants.}$$

The curved portion of the Sutro weir is given by the formula $W = b\left(1 - \frac{2}{\pi}\tan^{-1}\sqrt{\frac{x}{a}}\right)$.

Here, W is the curved slot width, x is the height above the rectangular slot base, a is the height of the rectangular base, and b is its width. (Glade 1997)

The slots in the second version of the Phillips machine, the Mark II, resemble Sutro weirs.[3] It is quite likely that Phillips carefully calibrated the slots to produce the proportional relationship.

Some of the flow measurement boxes near the top of the machine experience such turbulent variation in water level that a float in the main box would give hopelessly noisy measurements. Phillips introduced a small measurement tube, a manometer, connected through a passage at the bottom to the flow meter box. A tiny float in this tube follows the first-order average value of the fluctuating water level in the main box. A screw in the passage adjusts the lag time of the average. Being quite small, the float requires mechanical amplification to operate the line-pulley-cam-valve mechanisms. Phillips used a servo motor to drive the shaft of the pulley suspending the float. The servo control switch resides on the float so that the motor will raise the switch when it is submerged and lower it when it is not. Thus the switch will follow the level of the water without actually floating, and the torque at the pulley shaft will operate multiple cam-valve combinations.

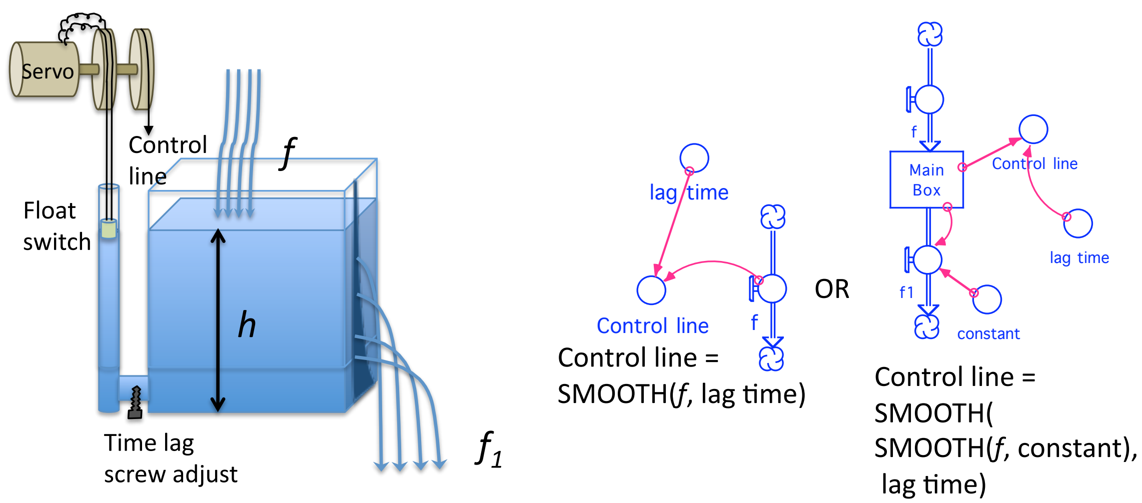


Figure 4: Slot Flow Meter With Proportional Flow Slot

Figure 4 shows the structure of the slot flow meter and two alternative interpretations. The left "flow only" interpretation applies if the water in the main box does not model anything. The formula for the "Control line" variable is SMOOTH(f, lag time). In the right "two flows and a stock" version, the flow meter is recognized as a first order material delay. When the main box is large enough, the control line is operated directly by a large float in the main box rather than by a servo driven by a manometer. In the Phillips machine, the large "National Income" tank at the bottom is functionally a material delay flow meter measuring the main flow, "Income". One adjusts lag time by changing the volume of the main box tank with a moveable wall.

These flow measurement boxes are all constructed of clear plastic. Therefore, the observer can visually monitor the flow estimates at all times and see their time relation to the levels of the stocks they feed or drain. If the observer is close enough, the time delay in the manometer level becomes apparent and allows one to trace the causal effect of the information delay thereby created.

Unusual Plumbing structures --Derivative of a flow and Constant Stock

Phillips was one of the first economists to apply servo control theory to economics. His seminal paper (Phillips 1954) explores what later became known as PID control to stabilize an economic system. (Bissell 2007) In his machine there is a structure, known as the accelerator, that foreshadows this work. A small tank with an adjustable leak hole in its bottom is suspended inside the National Income tank by a control line and tension spring attached further up the machine. If the water level in the National Income tank falls rapidly, the suspended tank will fall with it due to the temporary buoyancy of the small tank. As water leaks out through the hole, the small tank will return to approximately its original position, where the spring tension equals the weight of the small tank alone. The position of the control line thus estimates the smoothed time derivative of the Income flow. This estimate drives one of the "Investing" valve gates to cause a surge of investing when income flow rises quickly.

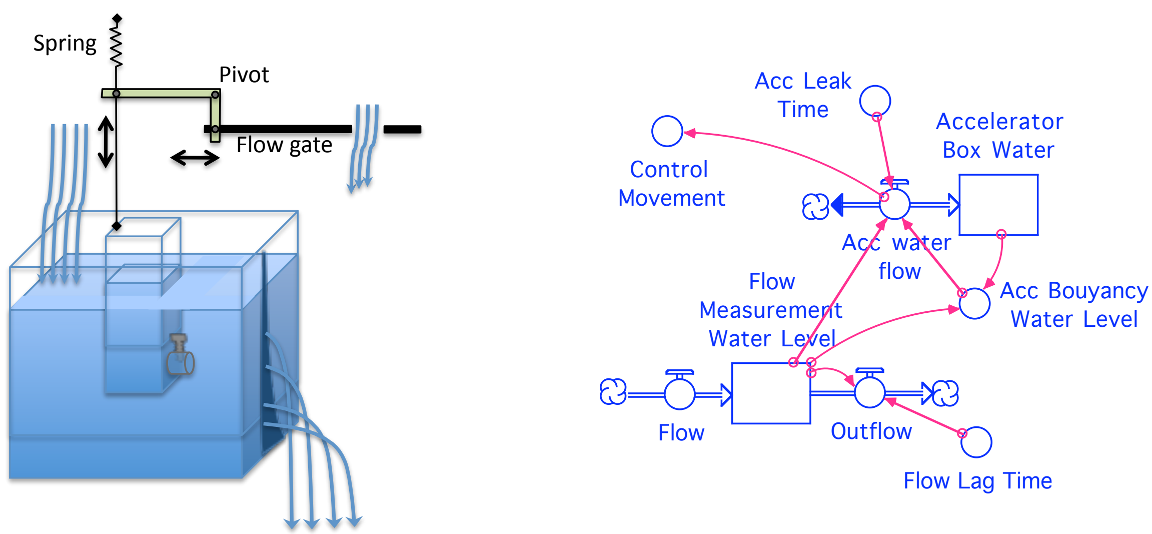


Figure 5: The Accelerator Structure and Stock and Flow Equivalent

Figure 5 shows a system dynamics model for the accelerator. The underlying flow to be measured, Income in this case, appears as large curved arrows in the left hand diagram.

A second unusual structure displays Phillips' efficiency in use of plumbing. The constant head reservoir meant to decouple the water head in the "Surplus Balances" tank from that tank's outflow, actually has two separate gated drains, "Government Exp" and "Investing", and two inflows, the "Taxation" flow and the "Surplus Balances" float-controlled drain. Conceptually, the structure implements a stock that is kept at a constant level by the joint action of its two inflows, the float, and two outflows. The sum of the two outflows must be greater than the "Taxation" inflow so that the inflow from "Surplus Balances" will keep the reservoir at constant head. Physically, this reservoir is quite wide and connects the "Government" side of the machine with the "Banking" side. "Government Exp" (outflow) and "Taxation (inflow) are located at one end of the reservoir while "Investing" (outflow) and "Surplus Balances" (float-controlled inflow) are located at the other. Thus, if the net value of "Taxation" less "Government Exp" is negative (meaning the government is running a deficit) water will flow along the reservoir from the Banking sector to the Government sector to represent Government borrowing to balance the books without raising taxes. If instead the Government runs a surplus, water will flow from the Government end of the reservoir to the Banking end to represent repurchase of government debt. There is a white vane in the reservoir that shows the direction of water movement. Interestingly, this unlabeled constant head reservoir is partially hidden behind the main cascade of the machine. This has led to jokes among economists that the machine accurately models the government concealing its monetary activities.[2] Although this structure does not translate to a single system dynamics building block, we can capture its functionality with a bi-flow driven by the combined effects of the inflows and outflows of the reservoir.

A System Dynamics Model of the Phillips Machine

We next suggest, in Figure 6, a system dynamics interpretation of the complete Phillips machine. The diagram assumes that the sequence of calculations constraints implied by the causality arrows is enforced. Although the simulator produces similar behavior to that described in the referenced articles, it could not be checked against the actual machine. Behavior graphs of simulated output are therefore omitted here.

According to the diagram, there are 7 stocks in the Phillips machine. Two of them, "Consume" and "Domestic Expend" are small and perform flow measurement with little material delay as described above, but do impose an information delay indicated by the "perception lag time" constants. Other physical aspects of the machine require a knowledge of the modeling intent behind the machine to understand their function.

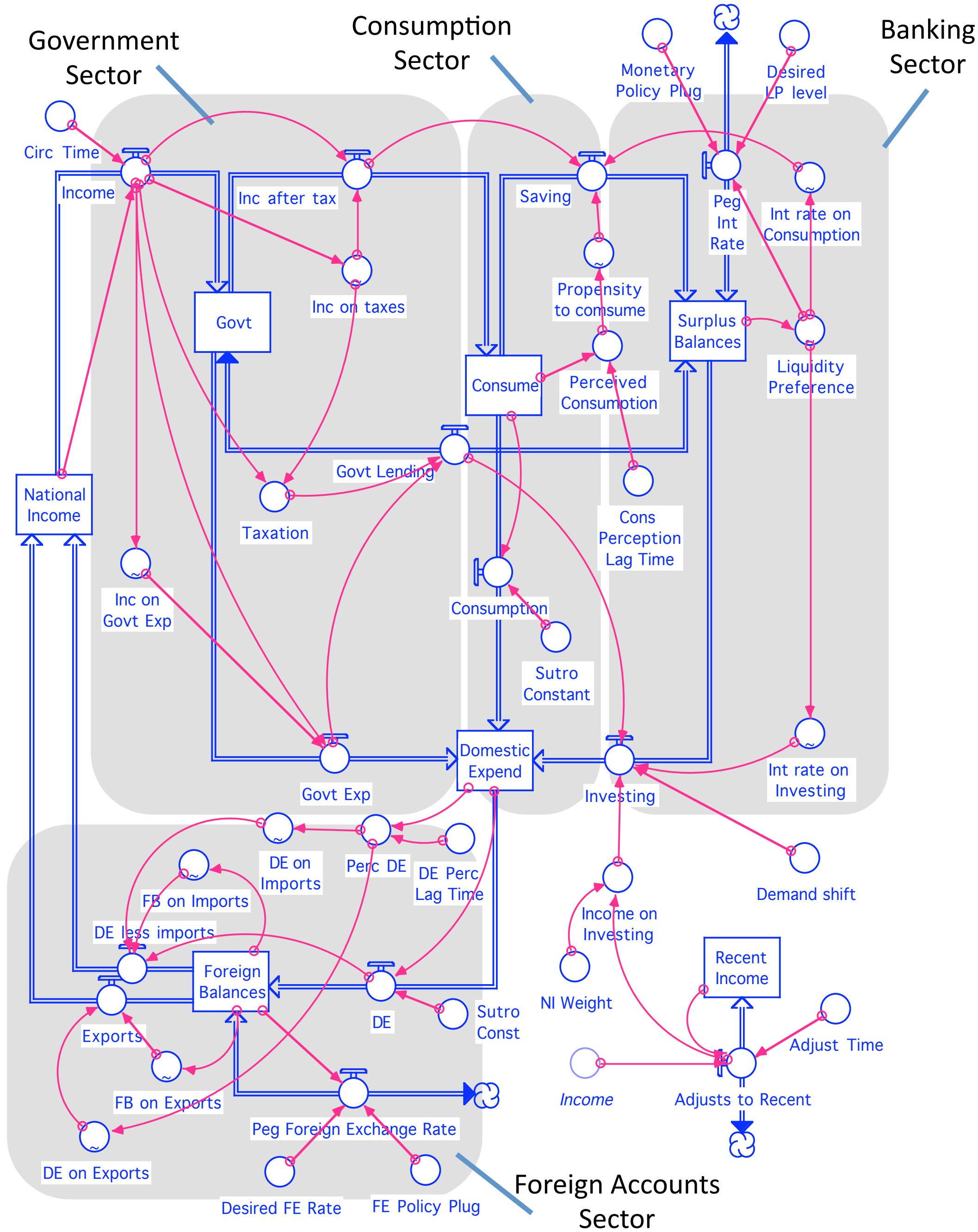


Figure 6: The Phillips Machine as Stocks and Flows

The machine circulates water, representing money, through sectors representing government, consumption, banking, and foreign transactions. Interest rates are inversely proportional to "Surplus Balances" and foreign exchange rates are inversely proportional to "Foreign Balances". Circulation time is determined by the "National Income" and "Income" material delay structure. Exogenous assumptions include the shape of the ten cam graphs, the circulation and perception delays, the accelerator's "Adjust Time" and input time functions such as "Demand Shift" on the diagram. The operator can run the machine in one of four modes determined by the settings of the "FE Policy Plug" and

"Monetary Policy Plug" variables. These freeze the levels of their associated stocks at predetermined values by adding or removing water to or from the main circulation.

The "Govt Lending" biflow between the "Govt" stock and the "Surplus Balances" stock allows the government to lend or borrow money to or from the private sector. Should "Government Exp" exceed revenues from "Taxation", money will flow from "Surplus Balances" to fund the deficit. Conversely, when "Government Expenditure" fails to consume all the tax revenues, the excess will move to the banking sector to be spent as "Investing"

Using Phillips' Designs for System Dynamics Instruction

The suggested effectiveness of physical hydraulic models as education apparatus must remain a conjecture until such devices are built and tested experimentally. Certainly barriers to this end include difficulty of fabrication, limited generality of the machines themselves, finding appropriate models to implement, and, of course, the clear potential that such machines will make a mess on the floor. Nevertheless, Phillips' hydraulic model of a dynamic system exemplifies the potential. It enabled the most advanced economists of its time to understand the importance of dynamic complexity in economics. It evokes nostalgia for the students fortunate enough to have studied with it. It continues to appear as an interest topic in the popular press, and has even influenced at least one recent novel and internationally recognized kinetic sculpture.[4,5] To spur further development, this paper offers two simple designs along the lines of the Phillips machine. These are just concepts and have not been built. Hopefully, they may give the reader a few ideas for other machines.

Figure 7 shows a simple Phillips-like promotion chain. Each of the three stages has two exogenous constants: an average waiting time controlled by the cam graphs driven by the floats, and the hiring rate or pass fraction set manually by the operator. The effects of changing any of these constants while the machine operates can be demonstrated. Once the machine has reached equilibrium, we can see how long it takes to become a Pro by introducing coloring to the water in the Rookies tank. If the overflow tubes are transparent, we can see the magnitude of the drop-out streams relative to each other.

Figure 8 shows a variation on the promotion chain to form a two-stage supply chain. The inflow gates for all stocks except the first are set fully open. Alternatively, they can be set to represent the times of shipping delays, provided the overflow pipes are blocked. The float in the inventory stock feeds back ordering information to the gate controlling initiations to stage 1. Delay times in the chain are determined by the graphs in the cams and the shipping delay times, if used. When the operator simulates a sudden increase in demand by opening the right hand outflow gate on the Inventory tank, we expect that the system will oscillate, limited by the maximum capacity at the input and in the shipping delay tanks.

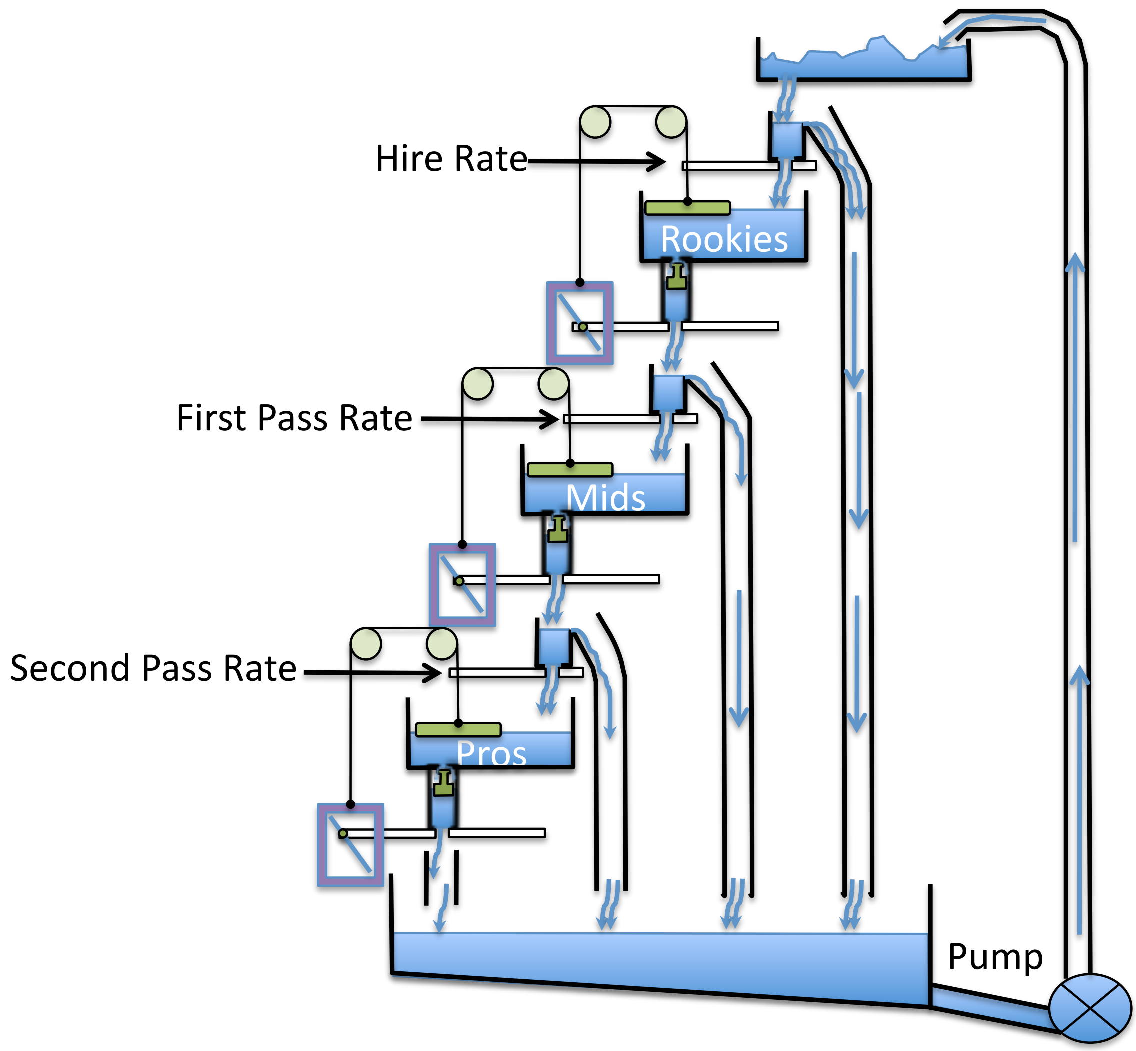


Figure 7: A Hydraulic Promotion Chain

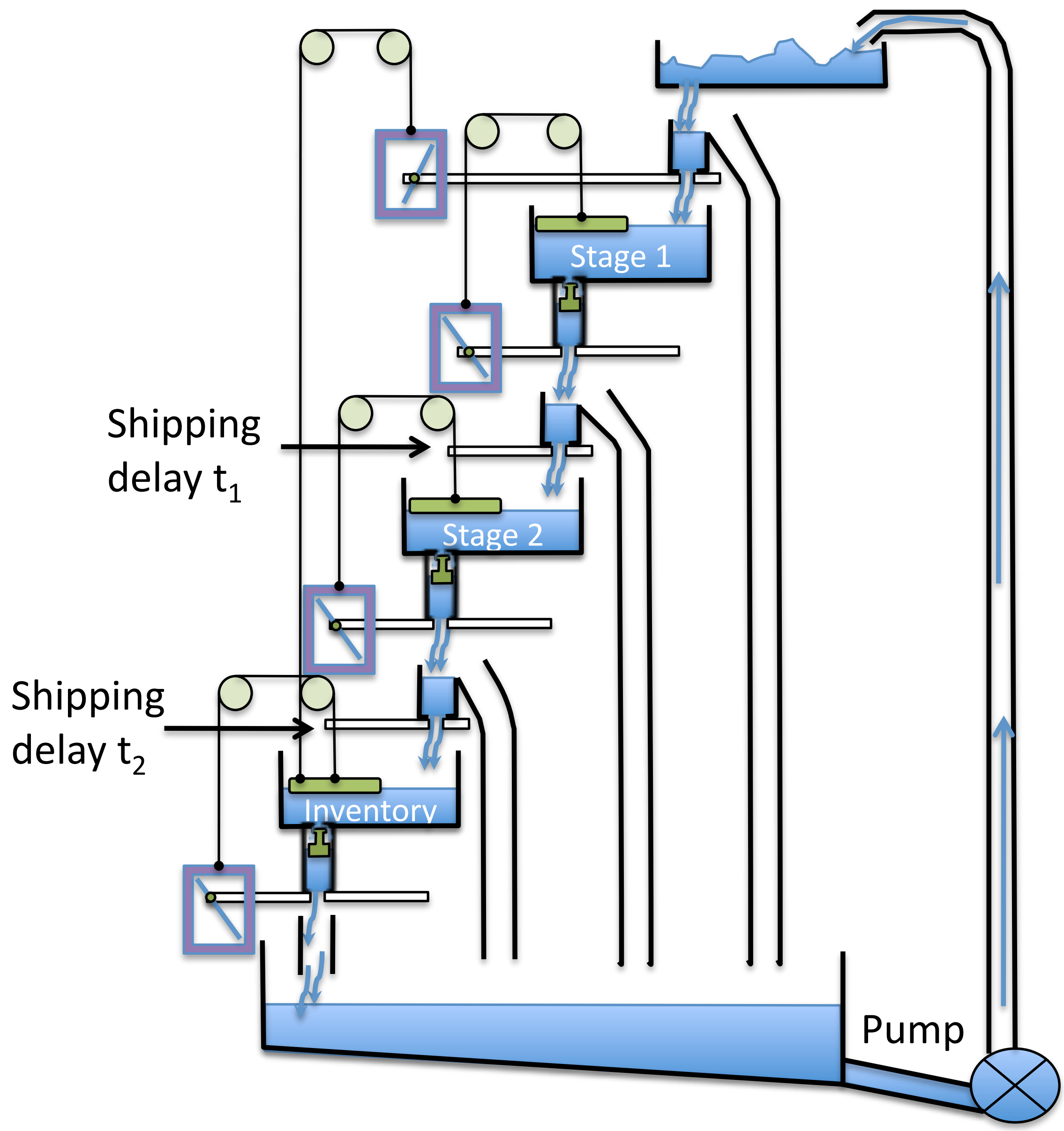


Figure 8: A Hydraulic Supply Chain

Conclusion

The Phillips machine, a serious dynamic model representing a complex subject, was used extensively both as an instructional aid and as a research tool to formulate advanced economic theory. It is the only physical implementation known to the author of the oft-quoted water analogy for guiding students to understand basic system dynamics concepts. Several conclusions seem warranted:

1) The Phillips machine, besides being an historic analog computer, is a system dynamics model in its own right. Moreover, Phillips was doing system dynamics modeling in the early 1950s as he used the machine to gain insights into what

dynamic elements were missing and how the most fundamental parts of macroeconomics worked in dynamic terms. Therefore, the Phillips machine or a representation of it in modern system dynamics notation, deserves special recognition as a stepping stone in the development of system dynamics as a discipline.

2) Because all of the variables are visible as the Phillips machine operates, and because the user interacts with it as it runs, the machine is very useful as a lecture aid. Many observers have described the audience-delighting behavior of the machine and related the deep insights into macroeconomic principles it has inspired. (Vines, D., Barr, N., Brown A., Meade, J., Swade, D., Newlyn, W., Dorrance, G., all in Leeson 2000). We cannot conclude that it is better or worse than computer models for this purpose, only that it has both engaged and inspired many audiences.
3) One can translate, subject to some implementation constraints, from the modern stock and flow notation backwards to plumbing notation to derive possible designs for other hydraulic models. Here we have examined a mapping from the Phillips machine's plumbing to modern system dynamics notation and have applied that mapping in reverse to arrive at putative designs for other instructional apparatus. Although the instructional effectiveness of such apparatus is only conjectural, the historical success of the Phillips machine suggests that similar machines ought to be built and the conjecture tested.

Notes

1. Dr. Allan McRobie of Cambridge University demonstrates the sole working Phillips machine in a brief video clip available on the internet. A link to this video can be found at the site www.mediaplayer.group.cam.ac.uk. The best color photograph of an operating Phillips machine known to the author appeared in a *Fortune* article, "The Moniac", in March 1952. A good scan of this picture can be downloaded from www.fulltable.com/vts/f/fortune/n/m04.jpg. A color diagram sufficient to deduce most of the machine's operations appears at www.fulltable.com/vts/f/fortune/n/m03.jpg
2. The many excellent diagrams of the Phillips machine omit this important point. The standardization of water head must be a feature of any similar machine one would build. The author is grateful to Dr. McRobie for mentioning it, and for explaining the purpose of the directional vane in the partially hidden "Government" tank.
3. The literature on Sutro weirs is quite well developed. They find widespread use in water systems as a simple way to measure flow.
4. A recent novel by British author Terry Pratchett, *Making Money*, (pub. Harper 2008) features a character called "the Glooper" who operates a water driven economics computer in the basement of the Mint in London.
5. In 2006 Michael Stevenson created a working replica of the Phillips machine as an artwork representing economic and social policies gone awry. It was

exhibited in San Francisco, London and Antwerp in 2006, 2007 and 2008, respectively

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