The dynamic models of economic policy in the depiction of System Dynamics

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Abstract:

The study is a scientific and teaching approach of the issues of economic policy in depiction of System Dynamics. There has been presented the dynamic version of the model of aggregated supply - aggregated demand: AS-AD, the model of rational expectations and the model of the economic cycle. Conducted simulations concerning a theoretical economy enable to indicate the general recommendation for decisions-makers.

Key words: macroeconomics, economic policy, aggregated supply, aggregated demand, Keynes, AS-AD model, rational expectations, economic cycles

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1 Introduction

The study is a scientific and teaching approach of the issues of economic policy in depiction of System Dynamics.

The dynamic version of the model of aggregated supply - aggregated demand: AS-AD is presented in the study. There are simulated the following cases:
- the positive demand shock caused by the growth of money supply $M_s$ and government expenses $G$;
- the negative exogenous supply shock;
every time at different assumptions of nominal wages $W$ rigidity.

The effectiveness of demand and supply policy is studied, enabling to specify general instructions for decision-makers.

The AS-AD model is analysed from the point of view of the rational expectations theory. The model that includes forming rational expectations instead of adaptive ones is proposed.

In a separate model there has been carried out the analysis of economic cycles - an approach of structural models researchers, including the co-relation between the delays of real production $Y$.

The simulations concern a theoretical economy, being in at the beginning in the point of balance. The parameters were selected in order to carry on correct calculations. I believe that this article can be the basis of fruitful discussion and scientific co-operation. Empirical verification of conducted simulations will be presented during the conference. Working models in Vensim are available from the author on request.

2 The AS-AD model

The IS-LM model was used in the economic analysis till the second half of the 60s. In that time the economy of the United States suffered stagflation - the phenomenon of recession combined with high inflation. The AS-AD model turned out to be useful in the interpretation of this phenomenon. The model consists of:
- the equations of the aggregated demand (the AD curve), which shows the real production $Y$ in the balance points of the IS-LM model for the average changeable level of prices $P$;
- the equations of the aggregated supply (the AS curve) - the formula presenting the changes of the average level of prices $P$ at different production capacities (different labour $L$ - in short run).

![AS-AD model](image-url)
The AD curve can be derived by side-comparing the equation of the IS-LM model:\(^1\)
- the IS curve:

\[
R = \frac{a + e + G + g - (1 - b(1 - t) + m)Y}{d + n}
\]

(1)

where:
- \(Y\) - real production;
- \(R\) - real interest rate;
- \(a\) - autonomous consumption demand;
- \(b\) - marginal propensity to consume;
- \(d\) - marginal propensity to invest;
- \(e\) - autonomous investments;
- \(G\) - governmental expenses;
- \(g\) - autonomous net export;
- \(m\) - marginal propensity to import;
- \(n\) - sensibility of net export to the changes of interest rate;
- \(t\) - income tax rate;

and
- the LM curve:

\[
R_{\text{nom}} = -\frac{M_S}{hP(\tau)} + \frac{k}{h}Y
\]

(2)

where:
- \(h\) - sensibility of money demand to the changes of the real interest rate \(R\);
- \(k\) - sensibility of money demand to the changes of the real production \(Y\);
- \(P\) - average price level;
- \(R_{\text{nom}}\) - nominal interest rate.

The real interest rate \(R\) equals

\[
R = R_{\text{nom}} - \pi^e_{(t, \tau+1)}
\]

(3)

where:
- \(\pi^e_{(t, \tau+1)}\) - expected inflation between the \(t\) moment and \(t + 1\).

On the basis of equation 2, one can receive the equation of LM curve as a function of the real interest rate \(R\):

\[
R = -\left(\frac{M_S}{hP(\tau)} + \pi^e_{(t, \tau+1)}\right) + \frac{k}{h}Y.
\]

(4)

The formula of the AD curve presents the negative dependence between the real production \(Y\) and the average price level \(P\):

\[
Y = \frac{M_S}{hP(\tau)} + \frac{a + e + G + g}{d + n} + \pi^e_{(t, \tau+1)}
\]

(5)

\[
\left[\frac{1 - b(1 - t) + m}{d + n} + \frac{k}{h}\right].
\]

The second element of the AS-AD model is the AS curve - the dynamic formula describing the values of the average price level \(P\) in the situation of not adjusting the level of

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\(^1\) Detailed introduction to the IS-LM model was omitted here because it is presented almost in every textbook in macroeconomics.
the real production \( Y \) to the level of potential production \( Y^* \). The short-term AS curve allows only the changes of the employment \( L \), caused by balancing the labour demand \( L_D \) and the labour supply \( L_S \) at the given level of real wages \( \frac{W}{P} \).

2.1 The algebraic form of the AS curve

Philips’s curve without inflation expectations is characterised by the equation:

\[
\frac{W_{t+1} - W_t}{W_t} = -\varepsilon (u - u^*)
\]  

(6)

where

\( W \) - nominal wage;

\( u \) - unemployment rate;

\( \varepsilon \) - the indicator of the convertibility of the variability of the unemployment rate \( u \) to the natural unemployment \( u^* \) and the rate of the expected growth of nominal wages.

The wage in the moment \( t + 1 \) amounts to:

\[
W_{t+1} = W_t[1 - \varepsilon (u - u^*)].
\]  

(7)

The expected growth of real wages amounts to

\[
w_{(t)} - \pi_{(t, t+1)}^e\]

(8)

where

\( w_{(t)} \) - nominal wages growth rate.

Philips’s curve with the inflation expectations is equal to

\[
w - \pi_{(t, t+1)}^e = -\varepsilon (u - u^*) + \upsilon
\]  

(9)

where

\( \upsilon \) - long-term exogenous supply shock (\( \upsilon > 0 \) in case of a negative shock, \( \upsilon < 0 \) in case of positive shock).

The relation between the average price level \( P \) and wages \( W \) amounts to

\[
P = \frac{(1+z)}{a} W
\]  

(10)

where

\( z \) - margin per production costs;

\( a \) - the number of product units per labour unit;

\( \frac{W}{a} \) - the cost of labour per product unit.

At the assumption of the constant quotient of the average level of prices \( P \) and the nominal wages \( W \)

\[
\frac{P}{W} = \frac{(1+z)}{a} = \text{const},
\]  

(11)

\[\text{In the study the constant level of the potential production } Y^* \text{ is assumed. This assumption does not distort the results of the analysis. In a long period the full adaptation of the quantity of the production factors is possible. The potential production } Y^* \text{ is calculated on the basis of the form of economic growth, i.e. Solow’s and Kalecki’s growth formula.}\]
the rate of nominal wages equals the rate of inflation
\[ w_t = \pi_t. \] (12)
Then, the modified Philips’s curve has the following form
\[ \pi_t - \pi_{t+1}^e = -\varepsilon (u - u^*). \] (13)
On the basis of Okun’s law
\[ \frac{Y - Y^*}{Y^*} = -\omega (u - u^*), \] (14)
where
\[ \omega \] - the indicator of the convertibility of a product and unemployment;
one can introduce the value of the demand gap to the formula of Philips’s curve and receive
the equation of the AS curve:
\[ \pi_t - \pi_{t+1}^e = f \left( \frac{Y - Y^*}{Y^*} \right), \] (15)
where \( f = \frac{\varepsilon}{\omega} \).
The inflation \( \pi \) is a positive function of not adjusting the real production \( Y \) to the potential production \( Y^* \), corrected by the inflation expectation \( \pi_{t+1}^e \).

### 2.2 Solution of the AS-AD model

Iteration calculations in the AS-AD model are carried out according to the following algorithm:

<table>
<thead>
<tr>
<th>Input variables</th>
<th>Equations</th>
<th>Output variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{(s)} ), ( \pi_{t-1}, \pi_{t-2} )</td>
<td>Expected inflation</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[ \pi_{t+1}^e = s_1 \pi_{t-1}^e + s_2 \pi_{t-2}, ] (16)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( s ) - the weight of inflation from the previous period</td>
<td></td>
</tr>
<tr>
<td></td>
<td>The IS curve</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[ R = \frac{a + e + G + g}{d + n} - \left( \frac{1 - b(1-t) + m}{d + n} \right) Y, ] (17)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>The LM curve</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[ R = -\left( \frac{M_s}{hP_{(s)}} + \pi_{t+1}^e \right) + \frac{k}{h} Y, ] (18)</td>
<td></td>
</tr>
<tr>
<td>( P_{(s)}, \pi_{(s)}, Y, Y^* )</td>
<td>Inflation rate</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[ \pi_{t+1} = \frac{P_{t+1} - P_t}{P_t}, ] (19)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Phillips’s curve</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[ \pi_{t+1} = \pi_{t+1}^e - \varepsilon (u - u^*), ] (20)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Okun’s law</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[ \frac{Y - Y^<em>}{Y^</em>} = -\omega (u - u^*), ] (21)</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Equations in dynamic version of AS-AD model
Algebraic formulas of the AS and AD curves enabled to work out the dynamic version of AS-AD model in Vensim.

![AS-AD model in Vensim](image)

The fixed delays used in the calculations between streams and the resource level in the top part of the model meet the important role. The delays enable to maintain the value of variables in the system. At the beginning of the next iteration the “flow 0” value (corresponding to the \(P_{t+1}\) variable) is moved to the “Pt” reserve, being the basis of the decision process in a next period. This situation is presented in the following example of a situation chart:

<table>
<thead>
<tr>
<th>Time (Year)</th>
<th>&quot;flow0&quot;</th>
<th>&quot;Pt&quot;</th>
<th>&quot;flow1&quot;</th>
<th>&quot;Pt-1&quot;</th>
<th>&quot;flow2&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>17</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>18</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>19</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>20</td>
<td>6</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 2. Calculations between streams and the resource level in time

2.3 The demand shocks AD

The AS-AD model enables to analyse demand shocks, which are caused by the fluctuations of the aggregated demand AD. In case of the positive shock one can observe the phenomenon of expansion: the real production \(Y\) and the average level of prices \(P\) is increased
(the AD curve moves to the right). The negative demand shock causes the opposite situation.

The effect of the expansive demand shock i.e. the growth of the money supply $M_S$ is the increase of the average level of prices $P$. The result of that is the decrease of the real wage $\frac{W}{P}$. The labour $L$ becomes cheaper, which results in employing the greater number of employees and increase the real production $Y$ above the level of the potential production $Y^*$. The growth of the average price level $P$ over the expected average price level $P^e$, causes that the current real wage $\frac{W}{P}$ is lower than the expected real wage $\frac{W}{P^e}$. Employees notice the decline of the real wage $\frac{W}{P}$ and start to demand the higher nominal wage $W$. They start to renegotiate the wage contracts. The increase of wages being the element of a cost structure of running a business causes the movement of the AS curve to the left, which decrease the deviation of the real production $Y$ from the level of the potential production $Y^*$.

![Figure 3. Adjustments of AS curve in case of the positive demand shock](image)

If the shock of the aggregated demand AD is not fully anticipated, the growth of the nominal wage $W$ will not be proportional to the growth of the average level of prices $P$. The bigger surprise will result from the shock of the aggregated demand AD or the less elastic the wages are, the bigger will be the increase of the real $Y$ production in a short period.

Any growth of the average level of prices $P$, which is not caused by the changes of the real values (i.e. increasing the asset capital, the growth of population, etc.) has to be compensated by the equivalent change of the nominal wages $W$. The change has to maintain the real wage $\frac{W}{P}$ at the level providing the long-term balance on the labour market $L^*$. The contentious issue is the time of adaptations. The hypothesis of a partial rigidity of wages seems the most realistic.

### 2.4 The supply shocks AS

The short-term supply shocks signify changes of the real production $Y$ and the average level of prices $P$ (by the AS curve). The most common cause of fluctuations of the aggregated supply AS are the changes of the prices of production resources. Although the prices of raw
materials are not an element of the function of production their increase causes the increase of the costs of running a company. It will cause the increase of prices at the specified fixed margin added to the costs of production.

The relationship between wages and prices modified with the cost of materials has the form of:

\[ P = \frac{(1+z)}{a} W + vP_m \]  

(22)

where

- \( v \) - the quantity of raw materials necessary to produce a single unit of a final product;
- \( P_m \) - the price of materials.

The share of the cost of a raw material \( P_m \) in the price of the product \( P \) is defined by the equation:

\[ p_m = \frac{P_m}{P} \]  

(23)

The price of the product amounts to:

\[ P = \frac{1+z}{1-vp_m} \frac{W}{a} \]  

(24)

for \( vp_m < 1 \).

The increase of the share of a raw material in the price of a product \( p'_m > p_m \) can be inserted to the equation of the Philips’s curve’s formula:

\[ W_{t+1} = W_t[1-\varepsilon(u-u^*)]. \]  

(25)

Then, the equation of the AS curve taking into account the increase of the prices of raw materials has the following form:

\[ P_{t+1} = P_t \frac{(1-vp'_m)a}{1+z} \frac{(1-vp_m)a}{1+z} \cdot [1-\varepsilon(u-u^*)]. \]  

(26)

The new level of prices \( P_{t+1} \) will amount to:

\[ P_{t+1} = P_t \frac{(1-vp'_m)}{(1-vp'_m)} \cdot [1-\varepsilon(u-u^*)]. \]  

(27)

The increase of the cost of the raw material \( P_m \) in the price of the product \( P \) will cause the movement of the AS curve to the left.

The average level of prices \( P \) \( \uparrow \)

\[ AS_2 \]

\[ AS_1 \]

\[ AD \]

\[ \]

\[ P_1 \rightarrow P_2 \]

\[ Y_2 \rightarrow Y^* = Y_1 \]  \( \text{The real production} \ Y \)

Figure 4. The negative supply shock
The economy will be in the new point of a short-term balance at the higher average level of prices $P$ and the lower real production $Y$. The companies decrease the level of employment $L$, in the same time as a result of the growth of prices $P$ - the real wage $\frac{W}{P}$ will decrease.

The growth of production costs can signify specifying the new lower level of the real production $Y$, at the higher unemployment level $u$. This phenomenon will take place when employees want to maintain the real wage $\frac{W}{P}$ at the level from the period before the shock.

The different situation will take place when the growing unemployment forces employees to accept the lower level of nominal wages $W$ or when a cause of a supply shock disappears. Then, the growth of prices $P$ will be limited and the employment will return to the potential level $L^*$. 

3 The changes of aggregated demand AD and aggregated supply AS

The changes of the balance are more often caused by the changes of the aggregated demand AD, which is characterised by the bigger changeability, than the aggregated supply AS.

The following issues were subjects of simulations with the assumption of the different rigidity of nominal wages $W$:
- the positive demand shock (increasing the money demand $M_S$);
- the positive demand shock (increasing the governmental expenses or another autonomous component of the AD curve);
- the negative short-term exogenous supply shock.

The simulations concern a theoretical economy, being at the beginning in the point of balance. The parameters were selected in the way enabling to carry on correct calculations.

In case of every shock one has assumed that the $s_2$ parameter in the equation of the expected inflation $\pi_{e(t+1)}$ (see equation 16) amounts to 0 and $s_1$ has one of the following values: 0.7, 0.5 or 0.1. The lower value of the $s_1$ parameter means the bigger elasticity of the average price level $P$, thanks to which the price and wage adaptations to the long-term level are faster.

3.1 The growth of the money supply $M_S$

Increasing the money supply $M_S$ causes the movement of the AD curve, which results in mutual adaptations of the AS-AD curve.

The money supply $M_S$ (auxiliary variable) is specified by the equation:
$$M_S = 900+\text{STEP}(300, \ 1).$$ (28)

The growth of the money supply $M_S$ does not cause any changes of the real production $Y$ and the real interest rate $R$ in a long term. After initial deviation both variables go back to the level from the period before the shock.
The average level of prices $P$ increases proportionally to the increase of the nominal money supply $M_S$. The most of prices are increased already in the first iterations of the adaptation process.
Because of the change of the nominal money supply $M_S$ (not the real money supply $\frac{M_S}{P}$) the rate of inflation $\pi$ in a long run goes back to the level from the period before the shock.

The dynamics of the rate of inflation $\pi$ is strongly related to the nature of the adaptations of the average price level $P$. The rate of inflation $\pi$ increases significantly at the beginning of the analysis, then, it begins to decrease gradually to the level from the period before the shock.
Figure 9. The rate of inflation $\pi$ in case of the growth of the nominal money supply $M_S$

In case of the most elastic wage and price adaptations ($s_1 = 0.1$), on the basis of the chart of the average price level $P$ - the real production $Y$, one can notice almost direct transition to the point of a long-term balance (from the level of the real production $Y^*$ for $P = 1$ to the same level of the real production at $P = 1.33$). In other cases ($s_1 = 0.5$ or $0.7$) the adaptations occur with delays.

Figure 10. The chart of the average price level $P$ - the real production $Y$

in case of the growth of the nominal money supply $M_S$

The consequence of the changes of the quantity of production is the oscillating character of adaptations within the confines of relations between unemployment $u$ and inflation $\pi$. In each case of the simulation the long-term balance corresponds with the level from the period before the shock.
Figure 11. The chart of the rate of inflation $\pi$ in case of the growth of the nominal money supply $M_S$

3.2 The increase of the governmental expenses $G$

The increase of the governmental expenses (or the exogenous component of the aggregated demand $AD$) means, similarly like in case of the growth of the money supply $M_S$, the movement of the AD curve, after which the adaptations of the AS and AD curve take place.

Governmental expenses are characterised by the equation:

$$G = 1200 + \text{STEP}(300, 1).$$  \hfill (29)

The increase of the governmental expenses $G$ does not cause the changes of the real production value $Y$ in a long-term.

Figure 12. The real production $Y$ in case of the increase of the governmental expenses $G$

The increase of the governmental expenses $G$ causes the increase of the real interest rate
\( R \) in a long-term. This phenomenon is described in macroeconomics as “crowding-out effect”.

Figure 13. The real interest rate \( R \) in case of the increase of the governmental expenses \( G \)

The average level of prices \( P \) increases as a result of the increase of the governmental expenses \( G \).

Because the increase of the average level of prices \( P \) is not caused by the increase of the nominal money supply \( M_S \), the real money supply \( \frac{M_S}{P} \) is decreased.
Figure 15. The real money supply in case of the increase of the governmental expenses $G$

The rate of inflation $\pi$ in a long term stays at the level corresponding to the value from the period before the shock.

Figure 16. The rate of inflation $\pi$ in case of the increase of the governmental expenses $G$

On the chart presenting the average level of prices $P$ and the real production $Y$ one can observe the gradually disappearing oscillations of the real production $Y$ around the potential production $Y^\ast$. The only difference in comparison to the initial state is the different average level of prices $P$, increasing from $P = 1$ to $P = 1.15$. 
3.3 The short-term negative supply shock

The negative shock causes the growth of the average level of prices $P$ and the decrease of the real production $Y$. The AS curve moves to the left because of the increase of the costs of business activity. With the assumption of a short-term character of the negative supply shock:

- the cause of the supply shock must disappear. In case of the growth of the prices of raw materials the prices should return to the level from the period before the shock;
or
- employees have to agree to decrease the real wage to the level enabling the return of the unemployment $u$ to the natural level $u^*$.

The short-term supply shock described by the formula:

\[
\text{supply shock} = \text{STEP}(0.2, 5)-\text{STEP}(0.2, 6),
\]

was inserted in the “flow 0” equation:

\[
\text{flow 0} = \text{Pt}*(f*(Yt-"Y\star")/"Y\star"+1+\text{INFe t}+\text{supply shock}).
\]

The short-term supply shock causes the permanent decrease of the real production $Y$ and the growth of the average price level $P$.

![Figure 19. The real production $Y$ in case of the short-term negative supply shock](image1)

![Figure 20. The average level of prices $P$ in case of the short-term negative supply shock](image2)

The rate of inflation $\pi$, after initial increase, comes to deflation, enabling the return of the average level of prices $P$ to the value from the period before the shock.
Figure 21. The rate of inflation $\pi$ in case of the short-term negative supply shock

The increase (decrease) of the average level of prices $P$ causes the decrease (increase) of the real money supply $\frac{M_S}{P}$, which results in increasing (decreasing) the real interest rate $R$.

In a long period the real money supply $\frac{M_S}{P}$ and the real interest rate $R$ reach the level from the period before the shock.

Figure 22. The real money supply in case of the short-term negative supply shock
Figure 23. The real interest rate $R$ in case of the short-term negative supply shock

The movements of the AS and AD curves representing the changes of the average level of prices $P$ and the real production $Y$ show the oscillating character of fluctuations during the return of the economy to the state from the period before the shock.

Figure 24. The chart of the average price level $P$ - the real production $Y$ in case of the short-term negative supply shock

The influence of the demand gap (the gap of the real production $Y$) on the changes of the unemployment $u$ causes the oscillating fluctuations on the chart presenting the relation between the inflation (rate) $\pi$ and the unemployment (rate) $u$. 
4 The long-term aggregated supply AS curve

The long-term AS curve shows the potential level of production $Y^*$, which is created in the conditions of the natural unemployment $u^*$ at the full usage of the production capacities. The long-term AS curve has the shape of a vertical line on the chart showing the average level of prices $P$ and the real production $Y$, because prices have no influence on the production capacities of an economy in a long term. Moving the long-term AS curve represents the increase of the potential production $Y^*$ and - in a longer perspective - the decrease of the average level of prices $P$.

The causes of the movements of the long-term AS curve include the changes of real values (i.e. increasing the quantity of production resources, improvements of technologies, productivity increase, long-term supply shocks, etc.) This phenomenon took place in the USA in the 90s, when the significant technological progress took place.
The long-term negative supply shock was simulated. The shock causes - as a result of the permanent increase of business costs - moving the long-term AS curve to the left. The supply shock described by the formula:

\[ \text{supply shock} = \text{STEP}(0.2, 5), \]  \hspace{0.5cm} (32)

was introduces to the "flow 0" equation:

\[ \text{flow 0} = \text{Pt} \ast (f \ast (\text{Yt} - \text{Y}^{*}) / \text{Y}^{*} + 1 + \text{INFe} \ast \text{t} + \text{supply shock}). \]  \hspace{0.5cm} (33)

The long-term supply shock causes the decrease of the real production \( Y \) and the increase of the average price level \( P \).

![Figure 27. The real production \( Y \) in case of the long-term negative supply shock](image)

![Figure 28. The average level of prices \( P \) in case of the long-term negative supply shock](image)
The dynamics of the rate of inflation $\pi$, after the initial growth, gradually decreases causing smaller and smaller increase of the average level of prices $P$.

![Figure 29. The rate of inflation $\pi$ in case of the long-term negative supply shock](image)

The growth of the average level of prices $P$ causes the decrease of the real money supply $\frac{M_s}{P}$, which causes the increase of the real interest rate $R$.

![Figure 30. The real money supply in case of the long-term negative supply shock](image)
Figure 31. The real interest rate $R$ in case of the long-term negative supply shock

The crossing point of the curves AS and AD goes in the form of oscillating fluctuations from the right bottom to the middle part of the chart.

Figure 32. The chart of the average price level $P$ - the real production $Y$ in case of the long-term negative supply shock

The balance points on the chart presenting the inflation $\pi$ and the unemployment $u$ move in an oscillating way from the left to the middle part of the chart.
5 The policy of a state during the negative long-term supply shock

As a result of a permanent increase of production costs there may appear the social pressure on the short-term increase of the fluctuations of real production $Y$. There was presented the analysis of the efficiency of the economic policy in the situation of the negative exogenous supply shock, created (like in the point 4 in equation 32) by the formula:

$$\text{supply shock} = \text{STEP}(0.2, 5),$$

inserted to the "flow 0" equation:

$$\text{flow 0} = \text{Pt}*(f^*(Y_t-Y^*)/Y^*+1+\text{INF}_t+\text{supply shock}).$$

The case of the negative supply shock without taking up a policy is presented in the one of the analysed situations.

There were used the instruments of an economic policy:
- stimulation of the aggregated demand $AD$ by the growth of:
  - the money supply $M_S$:
    $$M_s = 900+\text{STEP}(400, 15);$$
  - the governmental expenses $G$:
    $$G = 1200+\text{STEP}(400, 15);$$
- the stimulation of the positive supply shock as a result of modification of the supply shock equation:
  $$\text{supply shock} = \text{STEP}(0.2, 5)-\text{STEP}(0.2, 15).$$

The initial supply shock takes place in the moment $t = 5$, the realisation of every policy takes place in the moment $t = 15$. In every situation there are assumed fast wage and price adaptations ($s_i = 0.1$).

The simulations present the inefficiency of the supply policy increasing the money supply $M_S$ or the governmental expenses $G$ in limiting the decrease of the real production $Y$. The
monetary policy is neutral for the real interest rate $R$ and the fiscal policy causes its change in a long period. The supply policy exerts a positive influence on the both mentioned variables causing their return to the level from the period before the negative supply shock.

![Figure 34. The real production $Y$ in case of different policies](image1)

![Figure 35. The real interest rate $R$ in case of different policies](image2)

The demand policy causes the inflation pressure and the average level of prices $P$ is increasing. The effect of the supply policy is different - there is the deflation pressure and the average level of prices $P$ goes back to the level from the period before the shock.
The monetary policy does not influence the real money demand $\frac{M_S}{P}$, the fiscal policy causes its decrease and the supply policy enables this variable to go back to the level from the period before the shock.
The chart of the average level of prices $P$ and the real production $Y$ presents the complexity of the correcting mechanism. The initial situation without any policy is presented as the blue line starting from the right side of the chart and ending on its left side. The supply policy causes the *turn back* of the real production $Y$ from the initial target point to the initial level, which is located on the right side of the chart. In case of the demand policy (specified by the red and green lines) there occurs the parallel movement of the balance point to the right, almost to the middle of the chart, and transition to the balance point on the left side, at the new higher average price level $P$.

Similarly like in the earlier chart, the chart of inflation $\pi$ and the unemployment $u$ seems to be complex, thus, almost unclear. The initial situation, without any policy, is presented as
the blue line, starting in the left part of the chart at the inflation $\pi$ amounting to zero. Then, the inflation increases to the level 0.2 and from this moment starts to decrease gradually, going through to the right side of the chart. The demand policy causes a little distortion of the changes of the unemployment rate $u$ from the specified track, however, the new balance corresponds to the long-term balance for the case of not realising any policy. The supply policy causes temporary deflation at the high level of the unemployment $u$ and the gradual return to the point of balance from the beginning of the analysis.

![Inflation (Rate of Unemployment)](image)

**Figure 40.** The chart of the rate of inflation $\pi$ in case of different policies

### 6 The theory of rational expectations

Adaptive expectations (i.e. $\pi_{t(t+1)} = s_1 \cdot \pi_{t(t-1)}$) assume the persistence of inflation expectations. The basis of forming rational expectations is all available information. The expectations of inflation are inseparably connected with a state policy. The anticipation of activities considers especially a fiscal policy, because the character of that policy depends on the governing coalition and the realisation of that policy (i.e. passing a budget) is characterised by a few month delay. One can discuss the anticipation of decisions in a monetary policy, however, there is the need for transparency and credibility of the institutions’ activities in this sphere.

The expected level of prices $P^e$ is a positive function of not adjusting the real production $Y$ to the potential production $Y^*$, increased by the value of a random error $\varepsilon_i$ - which is made at estimating the state policy:

$$P^e_i = P_{t-1} + \beta_i(Y^*_{t-1} - Y_{t-1}) + \varepsilon_i.$$  \hspace{1cm} (39)

Rational expectations assume the correctness of the prognoses of the average price level $P$:

$$P_i - P^e_i = \varepsilon_i,$$  \hspace{1cm} (40)

thus the real production $Y$ is equal to:

$$Y_i = Y^*_i + \alpha \varepsilon_i.$$  \hspace{1cm} (41)
The inflation expectations $\pi^e$ are rational, when the random error $\varepsilon_t$ has normal distribution.

Rational inflation expectations assume the ineffectiveness of an economic policy. Systematic and fully predictable interactions have no effect on the real production $Y$.

The demand policy consisting in the increase of the growth rate of the money supply $M_S$ is a special case in the AS-AD model. According to the theory of rational expectations increasing the inflation $\pi$ causes the increase of the nominal wage $W$ to the level enabling to keep the average level of the real wages $\frac{W}{P}$ at a constant level. Assuming the inflation at the constant level during the few previous periods (i.e. 1%) it is rational to have the inflation expectations $\pi^e$ on the similar level for the next period. In case of the growth of the inflation $\pi$ to the level of i.e. 4% employees should demand the higher (by 3%) increase of the average level of nominal wages at the beginning of the next period.

In the AS-AD model with rational expectations the real production $Y$ in a long term reaches values exceeding the level of the potential production $Y^*$. It should be emphasised that such a situation occurs when the growth rate of the money supply $M_S$ is bigger than the growth rate of the real production $Y$. On the basis of Philips’s curve of the equation:

$$\pi_{(t, t+1)} - \pi_{(t, t+1)}^e = f\left(\frac{Y - Y^*}{Y^*}\right),$$

(42)

at the assumption of adaptive inflation expectations:

$$\pi_{(t, t+1)}^a = s_1 \cdot \pi_{(t, t+1)},$$

(43)

the gap of the real production $Y$ and the inflation $\pi_{(t, t+1)}$ meet the following equality in a long term:

$$\frac{Y - Y^*}{\pi_{(t, t+1)}} = \frac{1 - s_1}{f}.$$

(44)

Because for the different values of the parameter $s_1$ the level of inflation $\pi$ increases by the same value the deviation of the real production $Y$ from the potential level $Y^*$ depends positively on the coefficient $f$. The increase of the inflation $\pi$ causes the underestimation of the inflation expectations $\pi^e$ and initial restraining of phenomena causing inflation. That is the reason why the wage and price adaptations cannot keep up with the growth of the money supply $M_S$.

The study of the AS-AD model with rational expectations requires modifying the model of adaptive expectations. There is introduced the condition, according to which the rate of inflation has the value of the growth rate of the money supply $M_S$. The scheme of the AS-AD model with rational expectations is presented below.
The increase of the rate of the growth of the money supply $M_S$ from 1% in the given period to the level of 4% occurs in the moment $t = 20$. The money supply $M_S$ is specified as the level variable, and it is characterised by the equation:

$$M_s = M_s(0.01 + \text{IF THEN ELSE}(\text{Time}<20, 0, 0.03)).$$

(45)

The track of the growth of the money supply $M_S$ is presented in the following chart. The rate of the growth of the money supply $M_S$ is represented by the inclination of a tangent to the curve in a given point.

Figure 42. The nominal money supply in AS-AD model with rational (red line) and adaptive expectations (blue line)
The AS-AD model was put to the simulation with adaptive and rational expectations. In case of the model with adaptive expectations there occurs the growth of the real production $Y$ over the level of the potential production $Y^*$. In the model of rational expectations the real production $Y$ goes back to the point corresponding to the potential production $Y^*$. Changes of real production $Y$ in case of the model with rational expectations in the moment $t = 1$ are due to initial values variables of variables.

![Graph 43](image1.png)

**Figure 43.** The real production $Y$ in AS-AD model with rational and adaptive expectations

The increase of the money supply $M_S$ causes in both cases the increase of the rate of inflation $\pi$.

![Graph 44](image2.png)

**Figure 44.** The rate of inflation $\pi$ in AS-AD model with rational and adaptive expectations
The significant influence on the size of the real money supply is exerted by the speed of the partially rigid price level $P$. It is the speed of the adaptation of the average price level $P$ to the money supply $M_S$ that determines the final level of the real money supply $\frac{M_S}{P}$ in a long term.

The increase (decrease) of the real money supply $\frac{M_S}{P}$ is accompanied by the decrease (increase) of the nominal interest rate $R_{nom}$. 
The real interest rate $R$ is equal to the nominal interest rate $R_{\text{nom}}$ diminished by the inflation expectations $\pi^\circ$. In case of adaptive expectations the real interest rate $R$ decreases gradually to the new long-term level. In case of rational expectations the real interest rate $R$ stays at the initial level because the decrease of nominal interest rate $R_{\text{nom}}$ is accompanied by the increase of average price level $P$.

The dynamics of changes is clearly presented on the chart of the average price level $P$ and the real production $Y$ as well as on the chart of the inflation $\pi$ and the unemployment $u$. In case of adaptive expectations one can observe the increase of the real production $Y$ in comparison to its initial value. Next, the real production $Y$ in the rational model of expectations is equal to the potential production $Y^*$ from the beginning of the analysis. The
patterns of adaptations are easier to analyse if one notices that in the simulation the average price level $P$ is increasing from period to period. The iterations for the following higher values of the average price level $P$ represent later and later simulations.

In case of adaptive expectations there occurs the decrease of the rate of unemployment $u$. In the model of rational expectations unemployment goes back to the point of the long-term balance $u^*$. 

Figure 49. The chart of the average price level $P$ - the real production $Y$ in AS-AD model with rational and adaptive expectations

Figure 50. The chart of the rate of inflation $\pi$ in AS-AD model with rational and adaptive expectations
7 The model of the economic cycle

According to the theory of rational expectations, the basis of the real production $Y$ equation

$$ Y_t = Y^*_t + \alpha \varepsilon_t $$

(46)

is the assumption of the random character of the production deviations to the potential level $Y^*$. Econometric analyses indicate that there is the opposite situation: there is the co-relation between the real production $Y$ from different periods. This phenomenon is presented in the equation:

$$ Y_t = Y^* + \beta Y_{t-1} + \varepsilon_t $$

(47)

where $\beta > 0$, even in case of a delay longer than one single period.

The persistence in the form of the positive parameter $\beta$ is the basis of an economic cycle. The $\beta$ parameter limits the range of the changes of the real production $Y_t$ during the transition to the potential level $Y^*$.

The proposal of the model of economic cycle in a closed economy is presented below.

The aggregated demand for goods and services is defined by the equation:

$$ Y_{AD} = C + I + G $$

(48)

The consumption demand of households $C$ is defined by the remaining income being to the disposal of $Y_d$ from the previous period:

$$ C = a + b Y_{d,t-1} = a + b (Y_{t-1} - T_{t-1}) = a + b (1 - t_{t-1}) Y_{t-1}. $$

(49)

Investments are specified by the formula:

$$ I = \gamma I_{t-1} + \chi (Y_{t-1} - Y_{t-2}) - \eta R + z $$

(50)

for $\gamma$ (Lat. gamma) from the range $(0; 1)$, positive $\chi$ (Lat. chi) and $\eta$ (Lat. ni) and the parameter $z$ - the random variable with the normal distribution.

The real money supply $\frac{M_S}{P}$ is defined by the formula:

$$ \frac{M_S}{P} = \frac{1}{l} \frac{Y}{R} $$

(51)

where the positive $l$ defines the liquidity preference.

The level of production capacity utilisation is presented in Okun’s law; the relation between the unemployment $u$ and the average price level $P$ - in Philips’s curve.

The scheme of the model is presented below:
The iterations received on the basis of the simulation model enable the analysis of fluctuations of an economic cycle. On the chart of the real production $Y$ there are visible the economic cycles which last about 15 years. The economy at the beginning of the simulation is in the state of a recession gap.

The periods of expansion are characterised by an increased inflation and vice versa: the recession is characterised by the lower rate of inflation.
The rate of inflation $\pi$ in the model of the economic cycle

The average level of prices oscillates together with the changes of the real production $Y$.

Short-term Philips’s curve is visible on the chart of the unemployment $u$ and inflation $\pi$. 

Figure 53. The rate of inflation $\pi$ in the model of the economic cycle

Figure 54. The chart of the average price level $P$ - the real production $Y$ in the model of the economic cycle
8 The summary

The conducted simulations enable to indicate the general recommendation for decision-makers.

In the situation when the cause of recession is the insufficient aggregated demand $AD$, it is advisable to use an expansive monetary and fiscal policy for the short-term aims. However, it should be noticed that there is the danger of the inflation pressure $\pi$ when the basis of the policy will include the growth of the money supply $MS$.

A special threat to the economy poses the negative supply shock causing the increase of the costs of business. In such a situation one has to choose the problem, which the decision-makers will try to cope with:

- decreasing the rate of unemployment $u$ is carried out by using the expansive demand policy;
- the reduction of the inflation $\pi$ is possible thanks to the restrictive demand policy.

Within the confines of the policy one should try to avoid the situations when shocks, especially the supply shocks, generate the inflation expectations $\pi^e$ and cause the significant decrease of the real production $Y$. It is advisable for decision makers to carry out actions supporting positive supply shocks.

The model of rational expectations and the model of the economic cycle can show some advanced aspects of the analysis of an economic policy.
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