

Extrapolating Expectations: An Explanation for Excess Volatility and Overreaction

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Earnings announcements have always been in the spotlight of the financial world. Whether as a predictor for future price movements, or for the formation of the contemporaneous rational stock price, earnings have been used by analysts, investors and academics. Within the framework of a rational valuation model (Gordon growth model) future earnings determine today's stock price. On the other hand, news on earnings induce a so called post-earnings-announcement drift, as first noted by Ball and Brown (1968), thereby affecting the future stock price.

The last phenomenon has been documented ever since, and today it is mostly known as momentum. Momentum in general refers to the tendency of "winner stocks", i.e. stocks that had positive abnormal holding period returns for 3-12 months relative to a benchmark, to continue to outperform for the subsequent 12 months, while "loser stocks", i.e. stocks that had negative abnormal holding period returns for 3-12 months relative to a benchmark, to continue to underperform for the subsequent 12 months.

Several explanations have been proposed to account for the phenomenon, among them transaction costs, delayed stock reaction to market risk factors and underreaction, or even overreaction. Overreaction appears as a quite popular explanation in the recent finance literature, indicating investors being overly optimistic about future stock returns. They tend to extrapolate past performance into the future, resulting into positive feedback trading and hence into self fulfilling expectations. Shiller (1988) and Frankel and Froot (1988) present interesting evidence to support overreaction.

Momentum strategies are applied in the financial markets, for investors to capture abnormal returns, as high as 25% per year. They are indeed a viable and applicable trading strategy and not a theoretical artifact. In fact, several mutual funds follow those strategies to capture excess returns, in their effort to beat the market.

Yet, the existence of momentum itself, violates one of the cornerstones of modern financial theory, the Efficient Market Hypothesis (EMH), introduced by Eugene Fama (1970). According to the EMH, an efficient market fully reflects all available information, therefore price changes must be unforecastable if they are properly anticipated, i.e. if they fully incorporate the expectations and the information of all market participants. Momentum is in contrast to the EMH, since it implies the opposite, namely the partial incorporation of information into stock prices.

In addition, a counterargument against the EMH is the excess volatility exhibited in financial markets. Under excess volatility we refer to the volatility of stock returns that cannot be explained through the variation in fundamentals (earnings). The debate was first initiated by LeRoy and Porter (1981) and Shiller (1981), who first made the claim that prices move too much to be rational forecasts of future earnings discounted at a constant rate. Hence, excess volatility is equivalent to predictability in stock prices, which is a direct violation of the EMH.

In this paper, we try to explain excess market volatility by means of momentum. Price setting mechanisms are introduced based on demand/supply balance as well as based on mechanical trading strategies of investors. Two types of investors are introduced: fundamental investors who believe that the marketed assets have some intrinsic value and make their trades based on the relative value of the current price relative to that intrinsic value; and momentum traders, who extrapolate past stock performance into the future and act accordingly by buying when the stock price rises and by selling when the price falls. Momentum investors should not be characterized as irrational; they do not completely ignore news about earnings, which are equivalent to fundamentals in this model. Their trades partially reflect the fact that they do take earnings announcements into account while submitting their demand. Incorporating news on earnings in their demand has a direct impact on the market clearing price, making it move volatile, regardless of the nature of the news (good or bad).

An additional feature of the proposed model is that it can actually explain post earnings announcement drift. The presence of momentum traders pushes the stock price further up (down) in the presence of good (bad) news, hence leading to the actual realization of their extrapolated expectations: it is a self fulfilling prophecy. Stock returns seem to be moving even more in the anticipated direction (up or down) therefore producing the observed momentum effect.

The efficient markets hypothesis (EMH) has been the central proposition of finance for nearly thirty years. In his classic statement of this hypothesis, Fama (1970) defined an efficient financial market as one in which security prices always fully reflect the available information. If the theory holds, the market truly knows best and investors are better off holding passive market portfolio. Investors should forget active money management altogether. However, excess volatility, especially recent volatility in NASDAQ average, cannot be fully explained by EMH.

Amongst the literature of most relevance to the whole volatility issue is Robert Shiller's "Market Volatility" (Shiller, 1990). Shiller proposes that investor reactions, due to psychological or sociological beliefs, exert a greater influence on the market than good economic sense arguments. Shiller does not totally disregard the work of economists before him who proposed the Efficient Market Hypothesis (EMH). In fact, he admits that the EMH can be substantiated by statistical data, but he believes that investor attitudes are of great importance in determining price levels. His book provides statistical evidence that excess volatility exists in the stock market and therefore volatility cannot be totally explained by the EMH. Excess volatility is the level of volatility over and above that which is predicted by efficient market theorists.

The model strives to explain market volatility by introducing price setting mechanism based on demand/supply balance as well as introduce psychological factors of investors who trade based on their beliefs about the market. Fundamental investors are value investors. They believe that the market (or an asset) has some intrinsic price. Fundamental investors make their trades based on the relative value of a current price to this intrinsic value. Momentum investors try to chase a trend by buying when the price rises and selling when a price falls. Note, the model assumes only two types of assets: risk-free (cash or Treasury bills) that is not traded on the equity market and equity that is traded on the market. In this model, volatility of the market is the same as the volatility of the risky asset – equity.

A. Time Horizon

The model runs for 2 years. There are exactly 253 trading days per year. Therefore, there are 506 trading days per two years. To round up, the model is run for

500 days. Trading days are chosen instead of calendar days because equities are traded only on the trading days. Earnings are exogenous to the model. They are reported quarterly. Therefore, the minimum time considered was a quarter – 63 days. However, it takes time for fundamental investors to obtain quarterly earnings information and make trading decisions based on it. Therefore, the model should be run for at least a year, given delays in the model. Delays as well as combined behavior of fundamental and momentum investors lead to oscillations in price, volatility, and other variables. However, over time the price comes to equilibrium fundamental value, assuming no further surprises in earnings. Therefore, the model is allowed to be run for 2 years.

B. Core Structure

Model Boundary

The model boundary depends on the model purpose (Sterman, 2000). The purpose of the model is to formulate a dynamics model that explains why markets are excessively volatile. Non-risky and risky assets are modeled. Everything that is not invested in a risky asset (“Equity Invested”) is assumed to be invested in a non-risky asset (“Cash”). “Cash” and “Equity Invested” are modeled in a way to track how much equity was bought and sold. Income and Consumption are introduced in the model; however, they are not explicitly modeled. They are constant. Financial institutions and instruments such as short sales and margin purchases play a role in explaining excess market volatility. However, they are not modeled in order to attain as simple model as possible.

Assumptions

- Earnings are assumed to be reported continuously. In reality, earnings are reported quarterly.
- The model only has two types of investors: fundamental and momentum. In reality, more types of investors exist. For example, Shleifer (2000) introduced noise as well as arbitrage investors. However, even among fundamental and momentum investors, differences in risk preference, age, and family situation

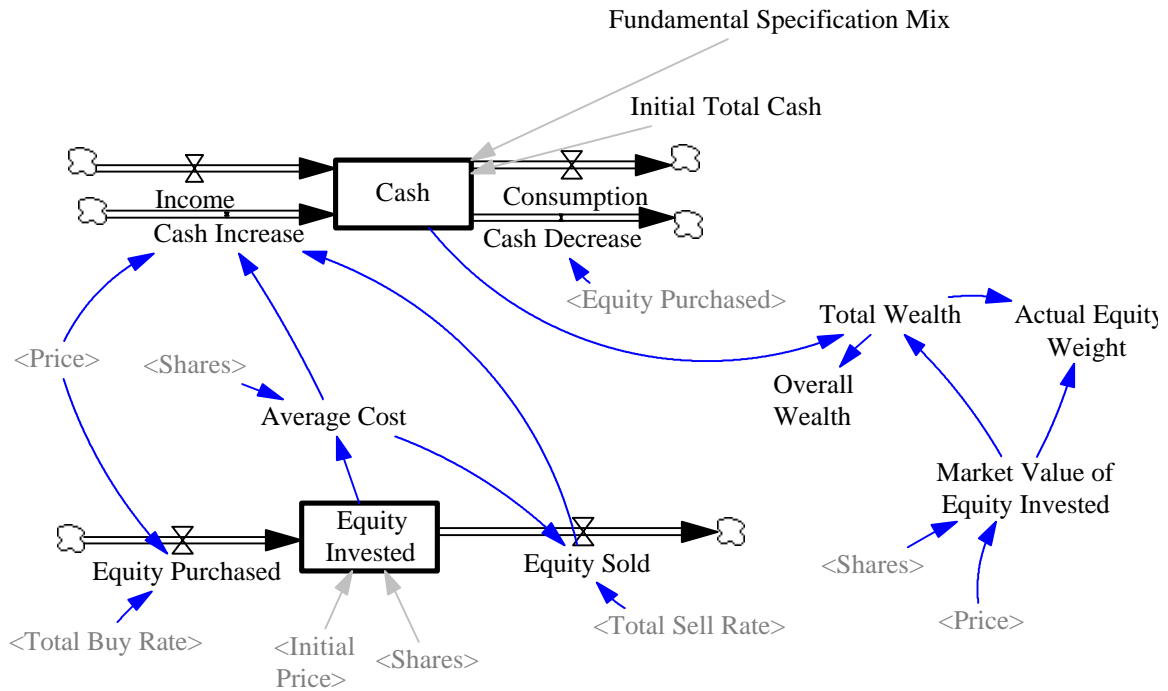
exist. The model presents average representative groups of fundamental and momentum investors.

- The relative balance of fundamental and momentum investors can only be changed exogenously. During the run of the model, investors cannot change their preferences and move between two groups.
- Total number of shares in the market is constant due to the absence of IPOs and share buybacks.
- Fraction of earnings reinvested in more capital is zero.
- Two types of assets exist: Risk-free and risky.

Investor Equity and Cash Holdings

Figure 1 presents stock and flow structure for “Cash” and “Equity Invested”. “Cash” can be increased by “Income” and “Cash Increase,” increase in cash due to selling stocks. Note that the model has an array structure built in. Each stock is modeled both for fundamental and momentum investors (see model documentation). “Actual Equity Weight” for each type of the investors is calculated in order to compare it with the desired equity weight and make further decisions whether to buy or sell a stock. In the question, I was asked to set up the model in such a way that relative balance of fundamental investors and trend chasers can be changed. “Fundamental Specification Mix” in the model serves this purpose.

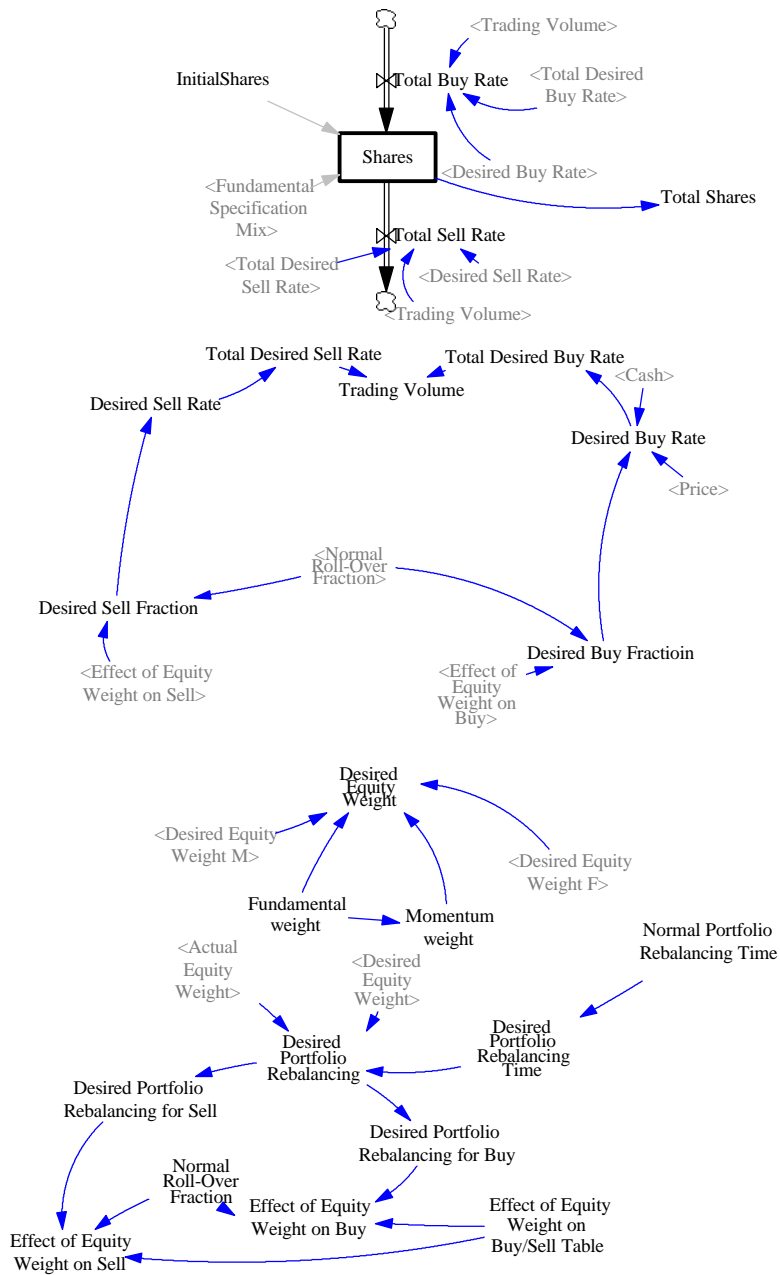
Figure 1. Investor Equity and Cash Holdings



Shares Balance and Calculation of Equity Buy and Sell Fractions

Figure 2 has two purposes. First, based on “Desired Equity Weight,” “Actual Equity Weight,” and “Normal Roll-Over Fraction,” each type of investors decides whether to buy or sell and how much to trade. The second part tracks “Shares.” The total amount of shares should be conserved. Everything that is sold by one type of investor should be purchased by another type.

Figure 2: Shares Balance

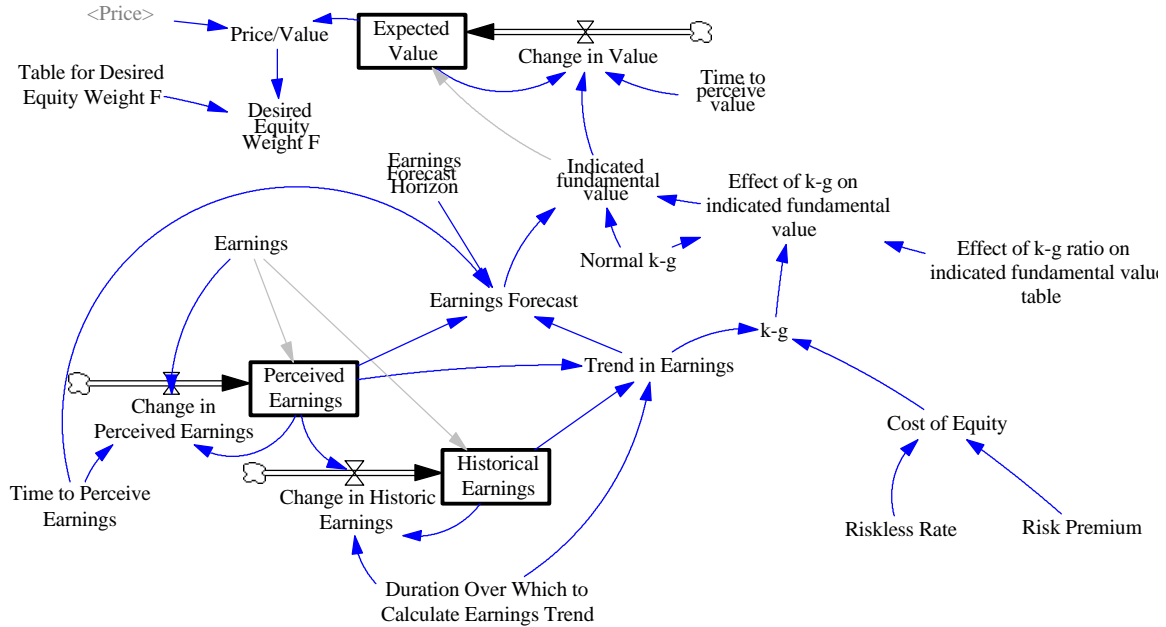


Fundamental Investor Decision

Figure 3 presents the stock and flow diagram for the fundamental investor decision. Fundamental investors buy a stock if they think that a stock is undervalued. They compare current price with the intrinsic value of a stock. The intrinsic value of a

stock is calculated based on the forecast of earnings and the inverse of the cost of equity minus the growth in earnings. They sell if they perceive that the stock is overvalued.

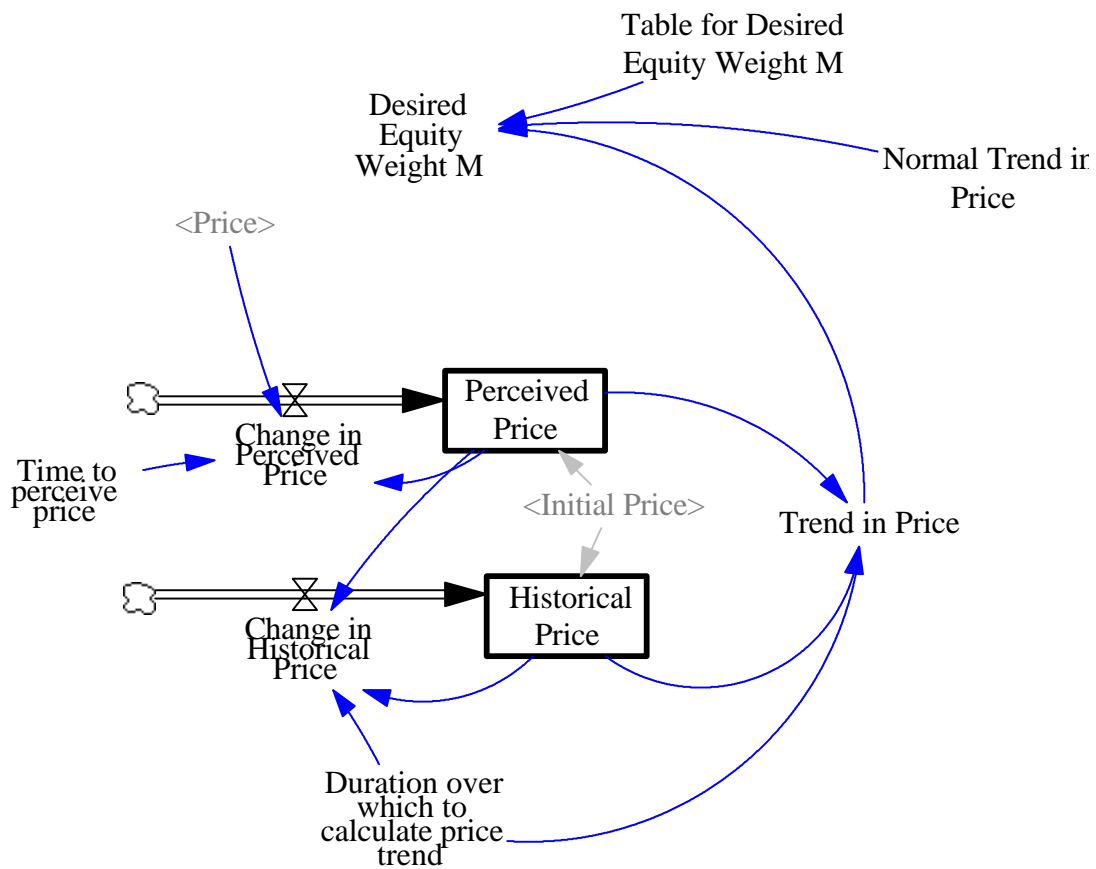
Figure 3: Fundamental Investor Decision



Momentum Investor Decision

Figure 4 depicts the stock and flow structure of the decision taken by momentum investors. Momentum investors only care about the trend of the price in making their decisions. They buy on increasing trends and sell on decreasing trends.

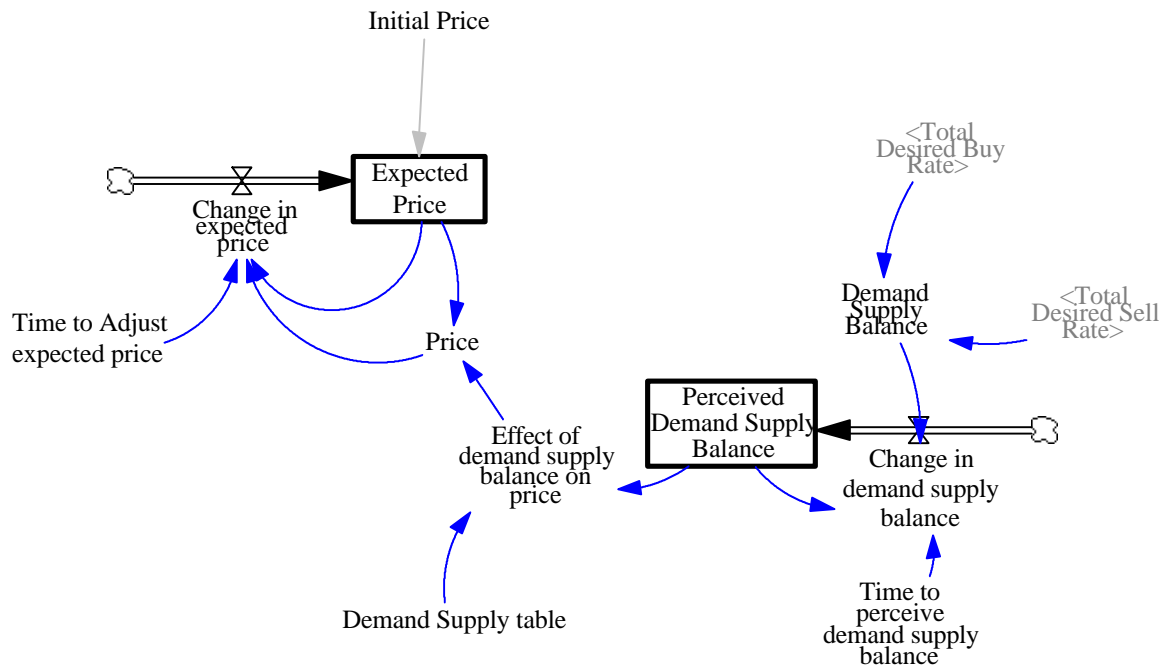
Figure 4: Momentum Investor Decision



Pricing

Figure 5 depicts pricing structure. Price is determined by demand/supply balance and expected price.

Figure 5: Pricing



C. Flow Equations and Decision Rules

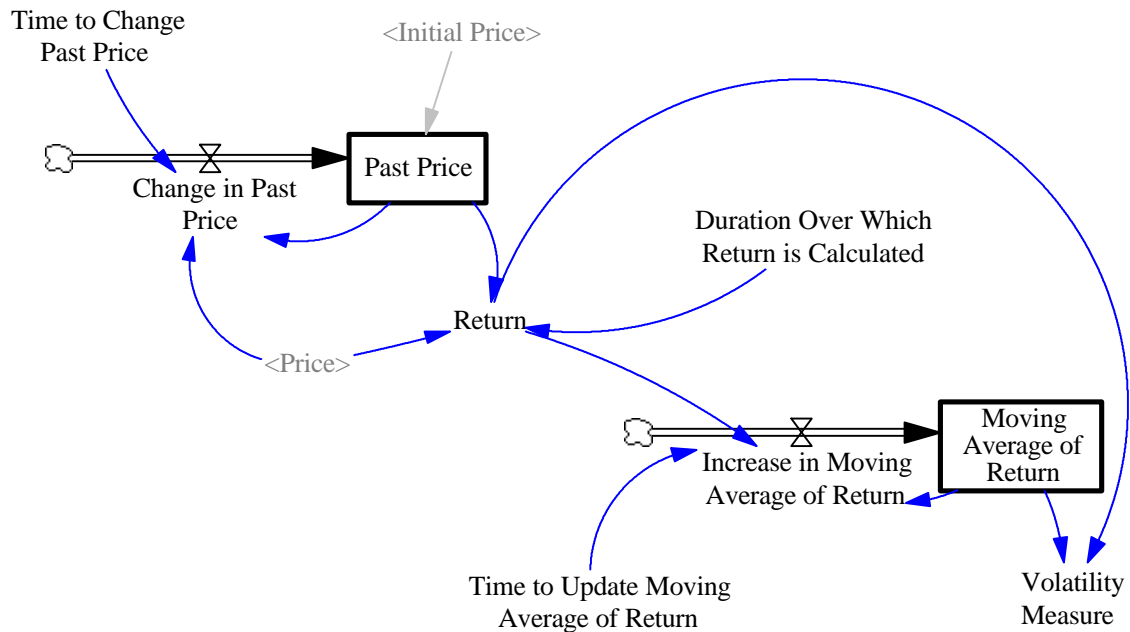
The model equations are carefully documented in Question3.mdl and are attached on a disk. The most important relationships and decision points are listed in this section.

Volatility

Figure 6 describes a stock and flow structure of the way volatility is measured in the model. It should be noted that it is measured in the simplest possible way in order to strive for elegance and simplicity of the model. However, at the same time the formulation captures the time-varying nature of volatility. “Moving Average of Return” measures a weighted average of the current return and past returns with recent returns weighted most heavily. It is assumed that a time constant over which the moving average is calculated is 5 days. An exponential average possesses this quality and is easy to

represent in a simulation model (Forrester 1961, p. 406-411). Volatility is calculated as the squared difference between a current return and the moving average of return.

Figure 6: Volatility



In the model volatility exhibits a time-varying nature. The time varying nature of asset return volatility was first proposed by Fischer Black (Black, 1976). Robert F. Engle proposed to use autoregressive conditional heteroskedasticity (ARCH) to calculate stock volatilities (Engle, 1982). According to ARCH, a natural way to update a variance forecast is to average it with the most recent squared “surprise.” The squared “surprise” can be calculated as the squared deviation of the rate of return from its mean.

Engle also pioneered GARCH, generalized autoregressive conditional heteroskedasticity model. (Engle, 1982). GARCH allows greater flexibility in the specification of how volatility evolves over time compared to ARCH approach. According to GARCH approach, the updated estimate of market-return variance in each period depends on both the previous estimate of variance and the most recent squared residual return on the market. Even though GARCH is the most used and more appropriate to estimate volatility of returns, this model is going to strive for simplicity

and use squared difference between an asset return and a moving average of asset returns with more emphasis on the most recent return as an approximation for stock volatility.

In a discrete case, volatility can be measured as a mean squared error (MSE) of asset returns:

$$s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}$$

where s^2 is the volatility

x_i is an asset return

\bar{x} is an average asset return

n is the number of observations

Note that this measure of volatility is unbiased only in the case of returns distributed normally.

Bounded Rationality of Investors

Decision rules for both fundamental and momentum investors are formulated separately. Each type of investors decides on his own rules that are not altered during the simulation. However, two types of investors interact in the market and are both faced with the same price. The behavior of each type of investors is bounded rational (Morecroft, 1983). Decision rules and bounded rationality of investors lead to the overall oscillations in prices in the market and excess volatility as presented in Part D.

Equity Valuation Model and Fundamental Investors Decision

Fundamental investors make their trading decisions by comparing current price with expected earnings. “Earnings Forecast” is modeled based on TREND function (Sterman, 2000). The function is explicitly modeled (see attached model). The model assumes that fundamental investors make their decisions based on a dividend discount model, pioneered by Myron J. Gordon. According to the model:

$$V_0 = \frac{D_1}{k - g}$$

where V_0 is an intrinsic price of a stock

D_1 is the value of dividends in next period (t=1)

k is the cost of equity

g is the growth rate of dividends

This dividend discount formula relates the P/E multiple to the cost of equity k and the real earning growth rate g . Note, in the derivation of this formula it is assumed that the fraction of earnings reinvested in more capital is zero. Therefore,

$$P_0 = \frac{E_1}{k - g} = \frac{E_1}{r_f + EMRP - g}$$

where P_0 is the intrinsic stock price

E_1 is the value of earnings next period (t=1)

r_f is the risk-free rate

$EMRP$ is the market risk premium

g is the real earnings growth rate

In the model, risk-less rate is assumed to be 3.8%/year and EMRP is 7.4%/year (Salomon Smith Barney, 1999).

Momentum Investors Decision

Momentum investors trade based on the trend in price. If a trend is positive, they buy. If it is negative, they sell. The model is formulated in such a way that the desired equity weight for momentum investors is a function of “Trend in Price” to “Normal Trend in Price.” Initially, the model had “Price Forecast” and compared it to the current price; however, according to Shleifer (2000), momentum traders place a market order today in response to a past price change. They do not formulate any price forecast unlike

fundamental traders who formulate earnings forecast. Therefore, the model was changed to represent this rule. “Trend in Price” is calculated according to a TREND function (Sterman, 2000) that is explicitly modeled (see attached model).

Pricing

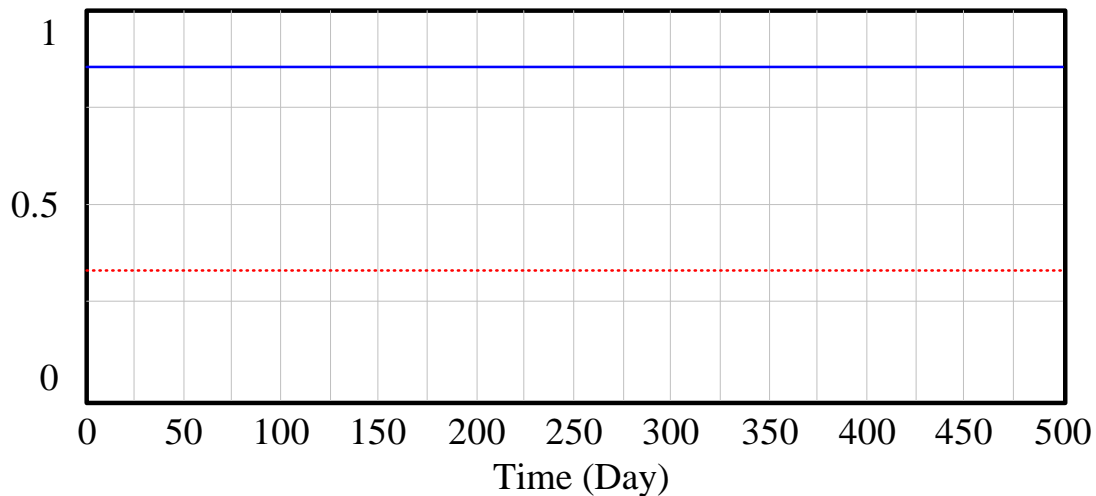
Price setting mechanism is modeled by using an anchoring and adjustment process (Sterman, 2000). The anchor is the “Expected Price” and the cue is the effect of the Demand/Supply balance. If demand exceeds supply, then the price is adjusted upward. In a reverse case, the price is adjusted downward.

D. Model Analysis

Test 1: Fundamental Investors Decision

Earnings were increased by 100% leading to the increase in the desired equity weight by fundamental investors.

Graph for Desired Equity Weight F



Desired Equity Weight F : run2 ————— fraction
 Desired Equity Weight F : run1 fraction

run1: Earnings = 11.2/252 (\$/Share/Day)

run2: Earnings = 2* 11.2/252 (\$/Share/Day)

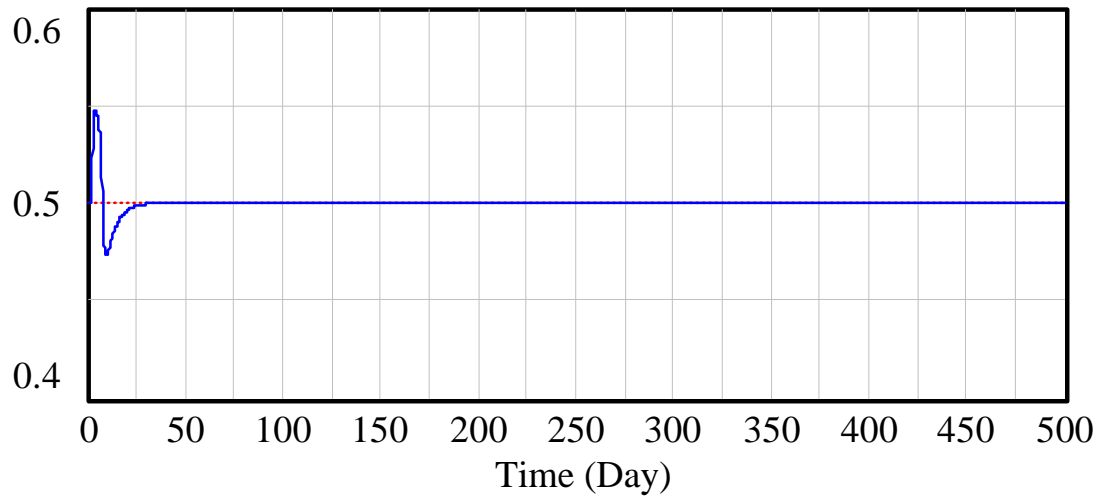
The behavior is as expected.

Test 2: Momentum Investors Decision

A pulse in price was introduced.

Price = $18.97 + \text{PULSE}(2, 5)$ where Initial Price is 18.97 \$/Share

Graph for Desired Equity Weight M



Desired Equity Weight M : run2 ————— fraction
Desired Equity Weight M : run1 fraction

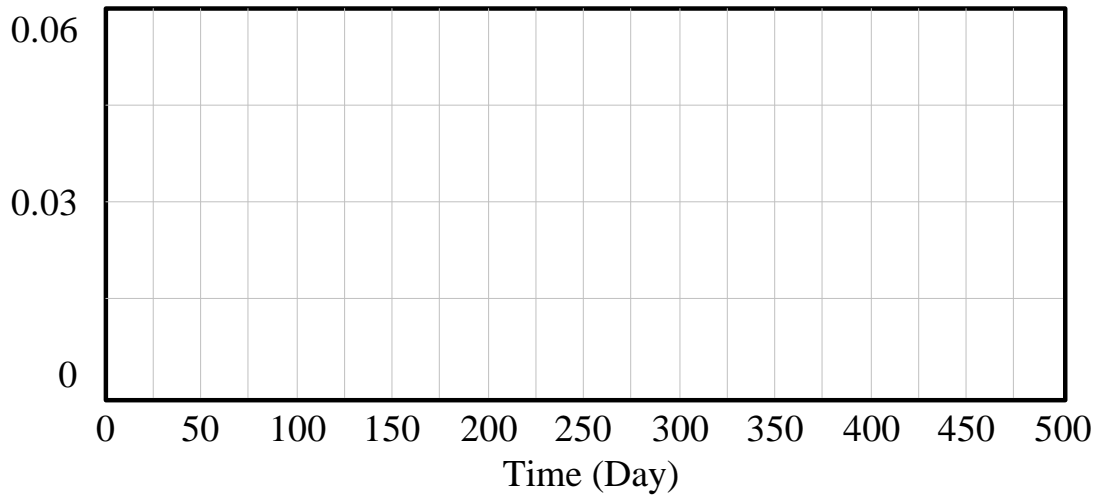
run1: Price = Initial Price

run2: Price = Initial Price + PULSE(2, 5)

As can be seen from the graph, the trend of price increased and then decreased as expected.

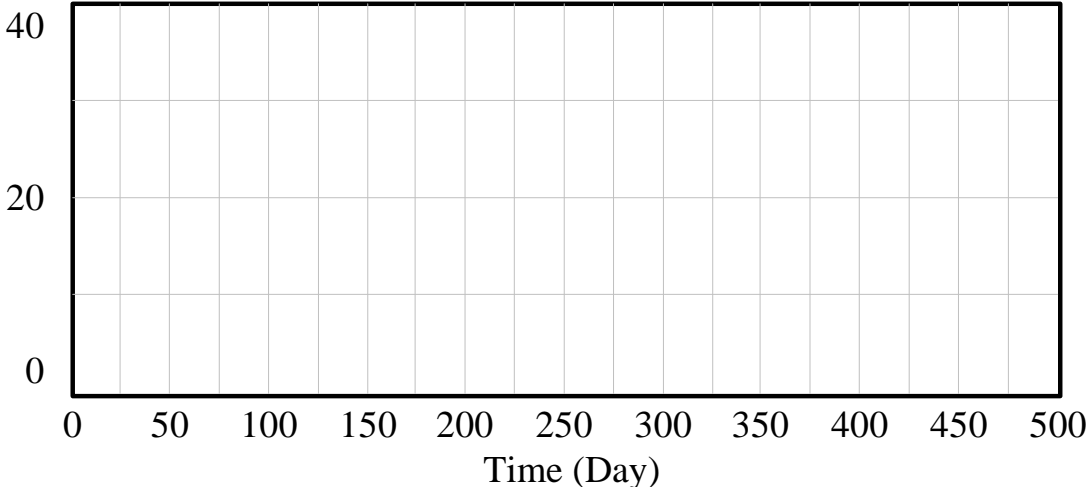
Experiment 1: Different Earnings Inputs

Graph for Volatility Measure



Volatility Measure : test2 ————— 1/(Day*Day)
Volatility Measure : test1 1/(Day*Day)

Graph for Price



Price : test2 ————— \$/share
Price : test1 \$/share

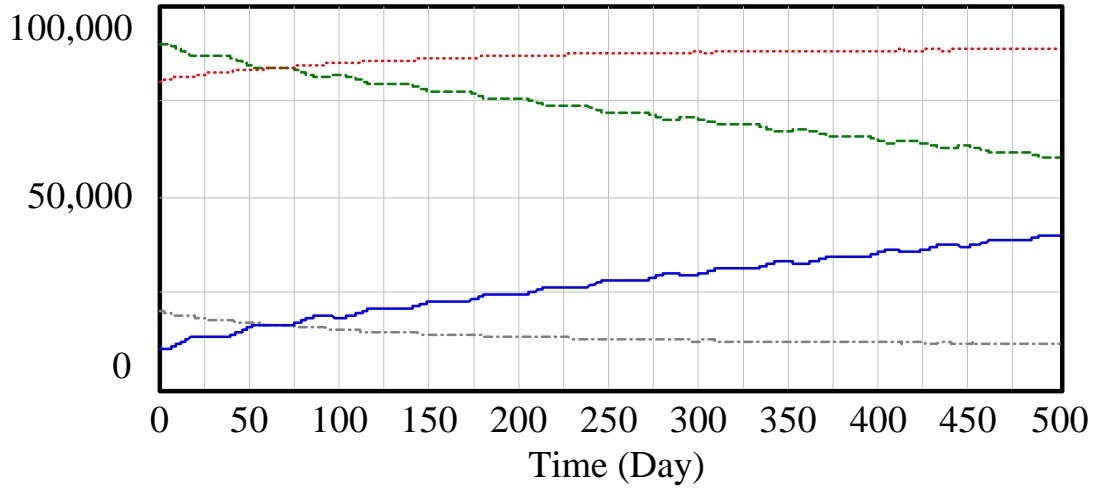
Test 1: Earnings = $1/63 + \text{RAMP}(0.0001, 2, 25)$ (\$/Share/Day)

Test 2: Earnings = $1/63$ (\$/Share/Day)

Note, when earnings are growing constantly for 23 days, behavior is more oscillatory with higher amplitude of oscillations compared to the run where earnings are constant.

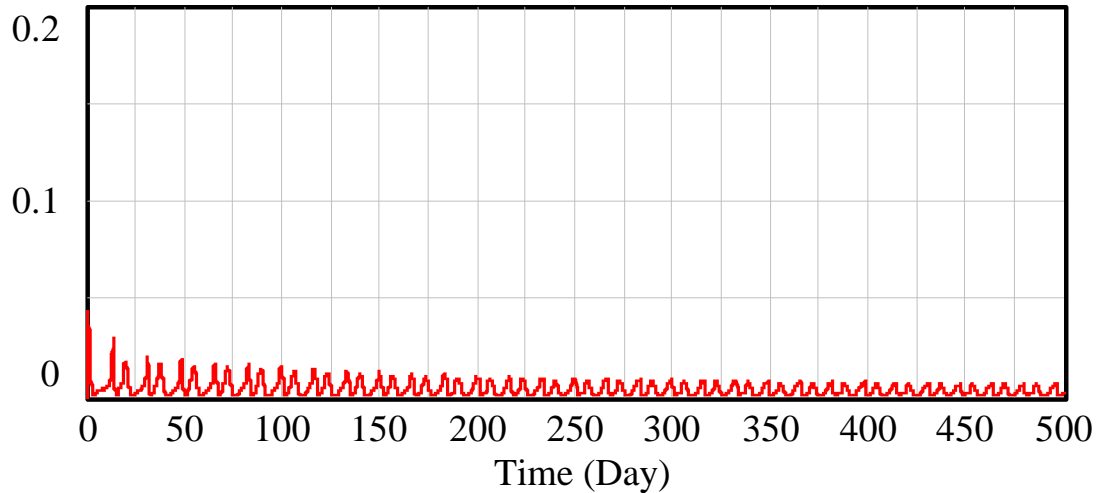
Experiment 2: Change in Fundamental Specification Mix

Graph for Shares



Shares[Fundamental] : test1 — shares
Shares[Fundamental] : test2 shares
Shares[Momentum] : test1 - - - shares
Shares[Momentum] : test2 - . - shares

Graph for Volatility Measure



Volatility Measure : test1 — $1/(\text{Day}*\text{Day})$
Volatility Measure : test2 $1/(\text{Day}*\text{Day})$

Test 1: Fundamental Specification Mix = 0.1

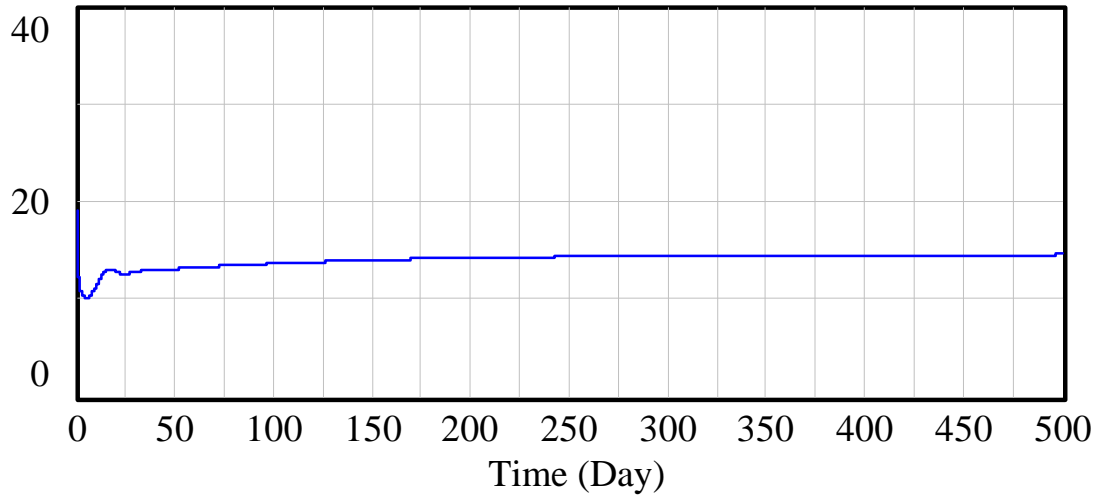
Test 2: Fundamental Specification Mix = 0.8

It is interesting and will be mentioned in Section E, that when most of the traders are fundamental investors (80%), they do not immediately drive out momentum investors. The number of momentum investors slowly decreases. However, when most of traders are irrational – momentum traders (90%), the majority is driven out by fundamental traders, but it takes time before momentum traders are driven out. However, the fraction of momentum traders does not reach 0 even if the simulation is run for a longer period of time. Momentum or irrational traders survive. As can be seen from test 1, the volatility is higher when initially there are more momentum traders than fundamental traders.

Experiment 3: Aggressiveness of Momentum Traders

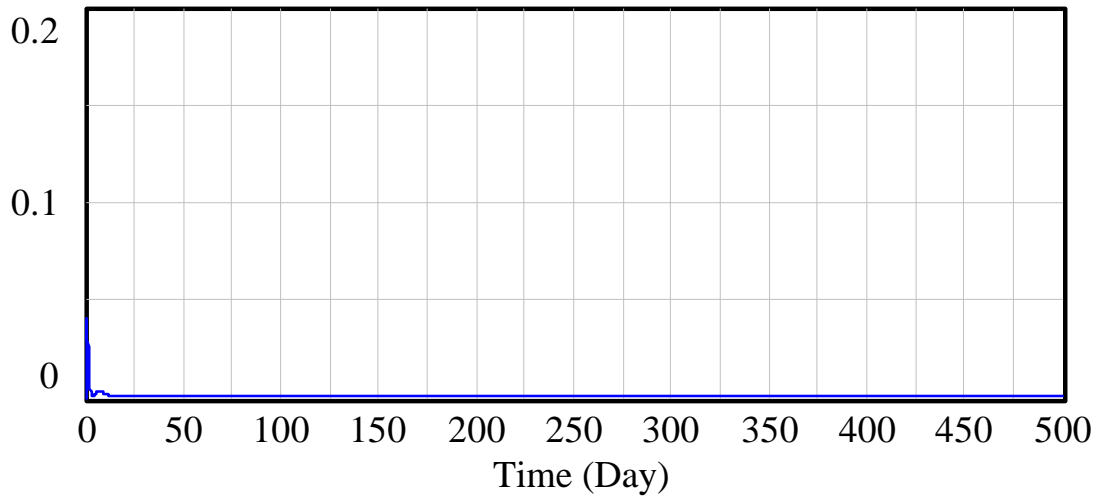
In this experiment, aggressiveness of momentum traders was changed by manipulating “Normal Trend in Price” value.

Graph for Price



Price : test2 ————— \$/share
Price : test1 \$/share

Graph for Volatility Measure



Volatility Measure : test2 ————— $1/(\text{Day}*\text{Day})$
Volatility Measure : test1 $1/(\text{Day}*\text{Day})$

Test 1: Normal Trend in Price = 0.015 (1/Day)

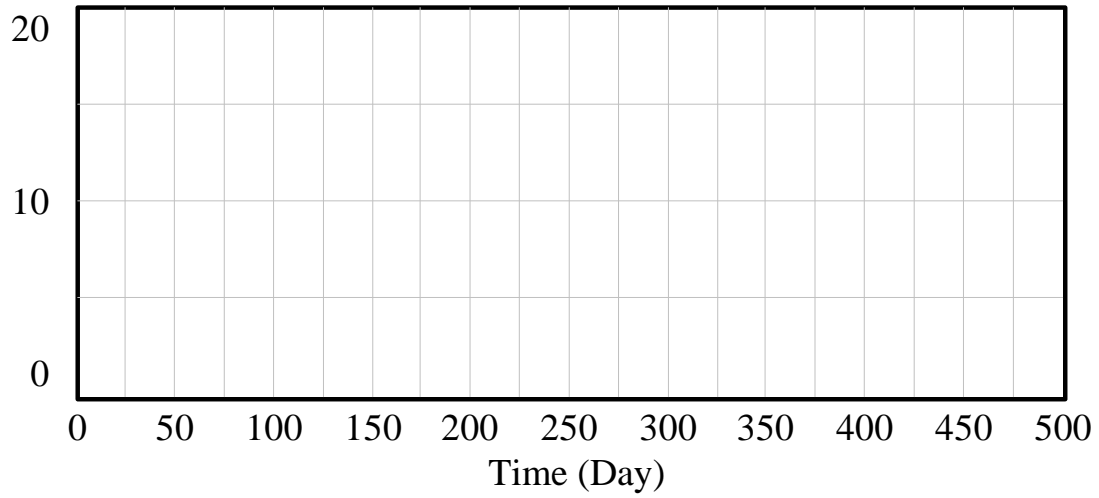
Test 2: Normal Trend in Price = 0.027 (1/Day)

According to Test 2, momentum investors do not try to buy all shares of the asset as soon as the price trend increases a little bit or sell everything in the opposite case. They wait until the trend is bigger. This behavior actually leads to the equilibrium behavior as depicted by the Price graph above. The price stabilizes and the volatility becomes 0 as expected.

Experiment 4: Perception Time By Momentum Investors

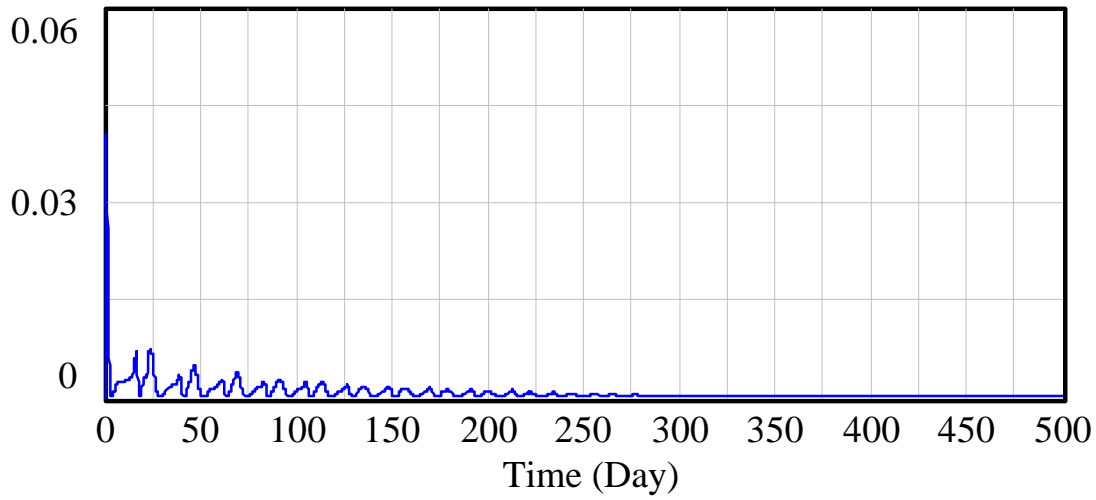
In this experiment, "Time to Perceive Price" by irrational investors is varied.

Graph for Price



Price : test2 ————— \$/share
Price : test1 \$/share

Graph for Volatility Measure



Volatility Measure : test2 ————— $1/(\text{Day}*\text{Day})$
 Volatility Measure : test1 $1/(\text{Day}*\text{Day})$

Test 1: Time to Perceive Price = 1 (Day)

Test 2: Time to Perceive Price = 2 (Day)

In test 2, oscillations decrease in amplitude and frequency and die out in approximately 400 days. Volatility goes to 0. Due to the increase in the delay, momentum investors are less likely to be in time to execute a buy (sell) when a price is increasing (decreasing). Therefore, they are less likely to exacerbate increases or decreases in price that lead to an increase in volatility.

E. Discussion

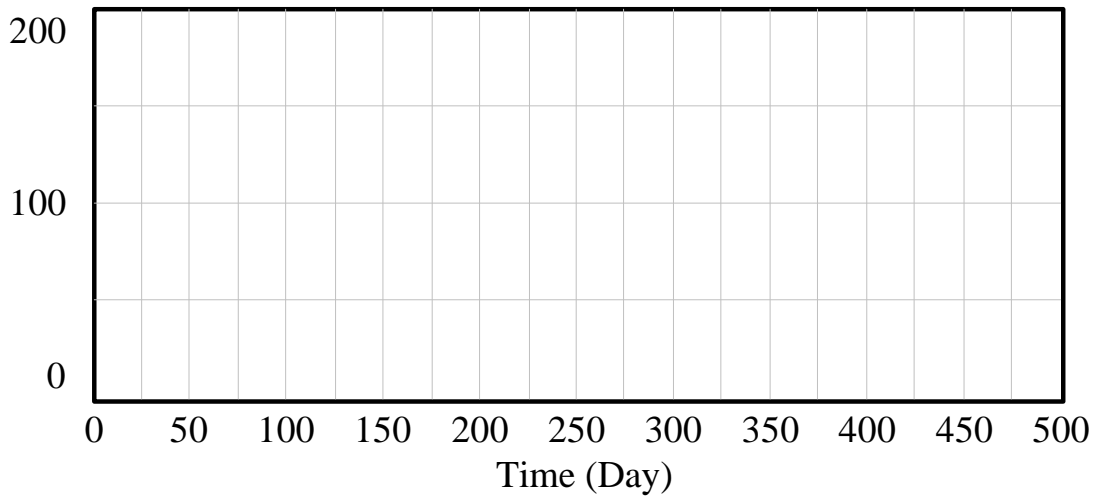
According to the results of the model, the equilibrium price is not reached instantaneously. Trading of fundamental and momentum investors lead to oscillations in prices, thus, an excess of volatility. This excess of volatility cannot be explained by the EMH. The excess of volatility is due to bounded rational behavior of the traders.

The model shows that trend chasers are not quickly forced out of the market by the fundamental investors. Indeed, it is possible to reach price equilibrium with a small fraction of momentum traders left. To illustrate these two points, the graphs for total buy

and sell rates and perceived demand/supply balance are provided below. As it can be seen, when momentum investors are at the peak to buy(sell), the fundamental investors are at the peak to sell(buy). The balance/supply balance oscillates until it reaches 1. As long as there is an imbalance and there are buy and sell orders from two types of investors, momentum investors are not driven out.

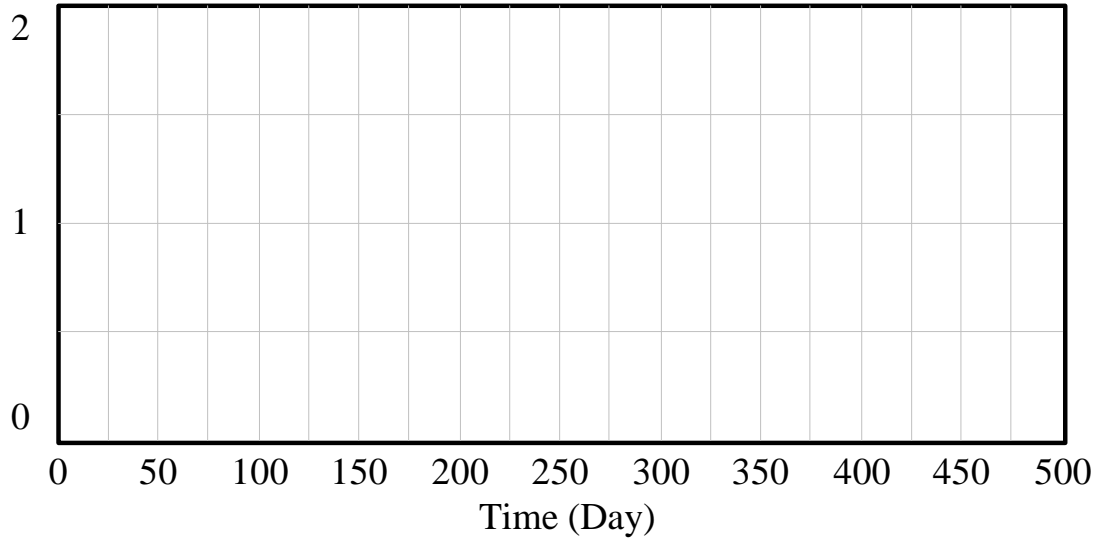
Potentially it will be interesting to include a third type of an investor: arbitrageurs who maximize utility as a function of the last period consumption. As it was shown by Shleifer (2000), the model with fundamental traders, momentum investors and arbitrageurs lead to more excess volatility than a model with only fundamental and momentum investors.

Graph for Total Buy Rate



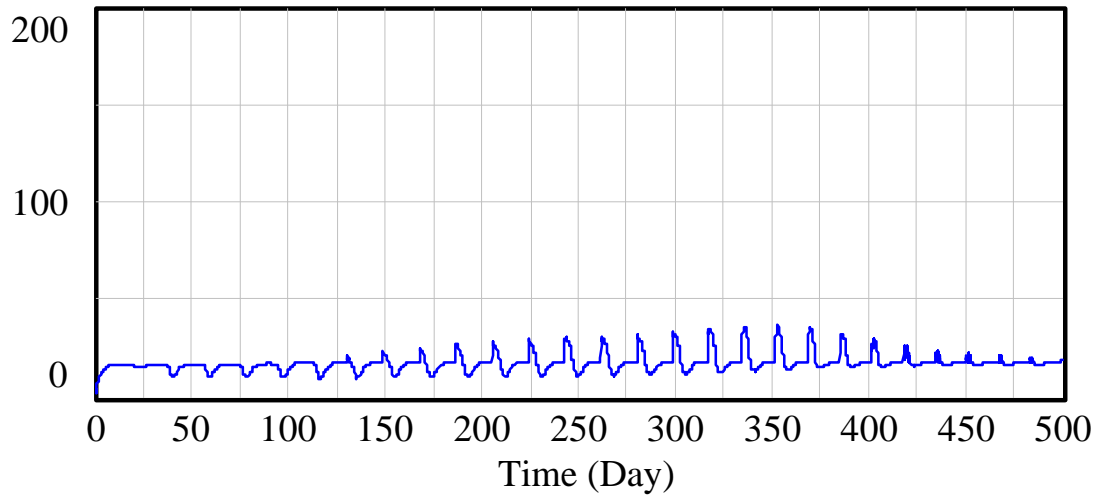
Total Buy Rate[Fundamental] : test1 ————— shares/Day
 Total Buy Rate[Momentum] : test1 shares/Day

Graph for Perceived Demand Supply Balance



Perceived Demand Supply Balance : test1 ———— fraction

Graph for Total Sell Rate



Total Sell Rate[Fundamental] : test1 ———— shares/Day
Total Sell Rate[Momentum] : test1 shares/Day

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Appendix: Model Equations

For model equations, please, contact Mila Getmansky at mgetman@mit.edu