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Understanding and Controlling a Law of Advancing Capitalism

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ABSTRACT

This paper refines a general model of capitalist reproduction. A compact state-space form of this model defines a hypothetical Law of capital accumulation. The state variables are the relative wage, employment ratio, gross unit rent, man-made capital-output ratio, natural capital-output ratio and indicated natural capital-output ratio. This paper identifies unobservable components of this Law through an application of the extended Kalman filtering to the US macroeconomic data. The retrospective statistical analysis (1958-99), univariate sensitivity analysis (1999-2057) and forecasting (1999-2507) support the analytical treatment of this Law. The latter ceases to impose itself as a blind force upon the human-beings and becomes more controllable stochastically. It is shown that the capital accumulation in the USA is a non-equilibrium quasi-periodic process described metaphorically as a long wave.

Keywords: Capital accumulation, learning, stochastic control, Kalman filtering, forecasting

Acknowledgement

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THE PREMISES OF THE MODEL

A Goodwin´s growth cycle model (Goodwin) reflects social contradictions in the process of income distribution as a cycle-generating factor. R. Solow has plotted a scatter graph of the employment rate and the relative wage for the US non-farm business economy for 1947-1986. His conclusion from the crudest sort of comparison with data (Solow: 39-40): “If the [Goodwin] model were an exact description for that place and that period, the successive data points would parade clockwise around in a closed orbit. ...there *is* a suggestion of predominantly clockwise motion, but in three episodes...The displacements are quite large and they suggest that the Goodwin model cannot be the only mechanism governing the relation between the wage share and the employment rate...It would make more sense for me if the Goodwin mechanism were to apply on a time scale considerably longer than the ordinary business cycle.”

This idea of the Nobel Prize winner has prompted an elaboration of Goodwin-like models of fluctuating economic growth to allow for induced technical change and the effects of composition of capital upon real wage, thus augmenting the key relationship – the real Phillips curve. The extended models reflect economies of scale, workers competition for jobs, the impact of economic activities upon natural environmental conditions and their influence on the growth rates of labor productivity and capital intensity. Policies, based on a perception of resource scarcity and pollution levels, have been also included in these models (Ryzhenkov).

The developed models explains causality, timing, economic-wide repercussions, and recurrence of a long wave (a non-equilibrium quasi-cyclical) process of capital accumulation. A stable limit cycle does not necessarily a characteristic of the long wave. The latter can be controlled through an appropriate selection of control parameters and policies. Exogenous shocks (driving and measurement noise, in particular) alter the dynamics. So a stochastic control of the long wave is appropriate.

The Model Assumptions

A closed capitalist economy is restricted by natural resources. Produced capital is an embodiments of knowledge and, similarly, natural capital is a stock of information. Some conversion factors are needed for aggregating information content of different constituents. Fixed assets, labor and natural assets are essentially complementary to each other and are also substitutes to some degree depending on relative price changes.

The other most important premises are such:

(1) two social classes (capitalists and workers); the State enforces the property rights, yet costs of such an enforcement are not treated explicitly;

(2) three factors of production -- labor force, man-made capital, natural capital --- are homogenous and non-specific;

(3) only one good is produced for consumption, investment and circulation purposes, its price is identically one;

(4) production (supply) equals effective demand;

(5) productive capacities can be partially idle;

(6) all wages consumed, the resource rent and a part of profits saved and invested;

(7) steady growth in the labor force that is necessarily not fully employed;
(8) a growth rate of a unit real wage rises in the neighborhood of full employment;
(9) a change in capital intensity and technical progress are not separable due to a flow of invention and innovation over time;
(10) a qualification of the labor force corresponds to technological requirements.

The product-money identity and the supply-demand equivalence stated in the third and fourth assumptions do not contradict the two-fold character of labor embodied in commodities. This model mirrors the twofold nature of labor power, the unity and contradiction of its value and use-value. The creative functions of labor market as an instrument for transmitting impulses to economic change are the focal point.

The model does not describe the formation of real income of the unemployed persons. It is assumed that a part of wages and salaries covers indirectly the needs of the unemployed. The latter do not play an active role in the model economy. Social security contributions and benefits are not shown unambiguously.

The model assumes supremacy of production over final demand. This assumption abstracts from the relative independence of final demand. It is more acceptable for the long-run as for the short-run: although in the shorter run aggregate demand influences output, in the very long run output dominates over demand. Capital adapts the output to the scale of production.

The model abstracts from over-production of commodities inherent in over-production of capital during certain phases of industrial cycles. The assumption (6) simplifies definitions of the investment, saving and profit rates. It may be a key to explanation of the fact that the rate of profit on capital of order of 12 or 15 per cent per annum is compatible with a rate of economic growth of two or three and half per cent per annum.

The assumption (5) reflects the existence of excessive productive capacities. It is important for interpreting an equation for a rate of change of labor productivity (below). The assumption (7) means that the labor force grows exponentially over time. This assumption may be substituted by an assumption of an asymptotic growth or another hypothesis. The assumption (6) corresponds to the immediate aim of capitalist production. Capital produces surplus product and profit as a monetary form of surplus-value.

THE BASAL MODEL EQUATIONS

The model is formulated in continuous time. Time derivatives are denoted by a dot, while growth rates will be indicated by a hat. The model consists of the following equations:

$$P = K/s \qquad (1)$$
$$a = P/L \qquad (2)$$
$$u = w/a \qquad (3)$$
$$\hat{a} = m_1 + m_2(\widehat{K/L}) + m_3 y(\hat{v}) + m_5 \widehat{F/L}, \qquad (4)$$

where $y(\hat{v}) = \mathrm{SIGN}(\hat{v})\mathrm{ABS}(\hat{v})^\wedge j$, $m_1 \geq 0$, $1 \geq m_2 \geq 0$, $m_3 \geq 0$, $m_5 \geq 0$, $0 < j$

$$(\widehat{K/L}) = n_1 + n_2 u + n_3(v - v_c) + n_5(Z/P), \qquad (5)$$

$$n_2 \geq 0,\ n_3 \geq 0,\ n_5 \geq 0,\quad 1 > v_c > 0,$$

$v = L/N$ (6)

$N = N_0 e^{nt}, \ n = const \geq 0, N_0 > 0$ (7)

$\hat{w} = -g + rv + b(\hat{K/L}) + q\hat{F/L}, \ g \geq 0, r > 0$ (8)

$P = C + \dot{K} + Y = wL + (1-k)M + \dot{K} + Y$ (9)

$\dot{F} = Y - Z$ (10)

$Z = eP, \quad 0 < e = const < 1$ (11)

$\dot{y} = (o_1(c - f) + o_2\hat{f})y, \ \ y = Y/P \geq 0$ (12)

$\hat{X} = i$ (13)

$f = F/P$ (14)

$c = X/P$ (15)

$\dot{K} = kM = k[(1 - w/a)P - Y] = k[(1-u)P - Y], \ 0 < k \leq 1.$ (16)

Equation (1) postulates a technical relation between the capital stock (K) and net output (P). The variable s is called capital-output ratio. Equation (2) relates labor productivity (a), net output (P) and labor input or employment (L). Equation (3) describes the shares of labor in net output (u).

Equation (4) is an extended technical progress function. It includes: the rate of change of produced capital intensity, K/L, the direct scale effect, $m_3 y(\hat{v})$, and the rate of change of natural capital intensity, F/L. ABS(x) is absolute value of x that is non-negative, x^j is x raised to the j-th power, SIGN(x) is a sign of x.

Equation (6) outlines the rate of employment (v) as a result of the buying and selling of labor-power. Labor force grows exponentially in (7). In the equation (8), the rate of change of the wage rate (w) depends on the employment rate (v), as in the usual Phillips relation, and on the rates of change of capital intensity (K/L) and (F/L), additionally. The capital intensity (K/L) is a proxy for qualification.

In the equation (9), final private and public consumption is $C = P[u + (1-k)(1-u-y)]$. The net formation of produced fixed capital is $\dot{K} = kM$, where K is man-made fixed assets. The gross accumulation of natural assets Y equals the gross resource rent in monetary (or information value) terms. Equations (9) and (16) show that profit ($M = (1-u-y)P$) and incremental man-made capital ($\dot{K}$) are not equal in monetary (or information value) terms if the investment share $k < 1$. Considering the latter as a variable and reflecting the workers saving is left for a future research.

In the equation (10), $\dot{F}$ is a net accumulation (loss) of the natural capital (F). Z is the net environmental damage in the equation (11), i.e., depletion and degradation of non-produced natural assets (land, soil, landscape, ecosystems) due to economic uses above the regeneration rate.[i] A key suggestion is that resource use or pollution has a fixed relationship to output (the linearity of this relationship constitutes a particular case).

The rate of change of capital intensity (K/L) in the equation (5) is a function of the relative wage (u), difference between real employment ratio and some base ('natural') magnitude ($v - v_c$), depletion/degradation of natural capital in relation to net output (Z/P). The rate of growth of capital intensity depends on the environmental damage per unit of output (an application of the principle 'a pollution prevention pays'), in particular. A high wage share and high employment ratio foster mechanization (automation).

The indicated natural capital, X, may remain constant, decrease or increases exponentially in the equation (13). The equation (12) defines an investment policy that is aimed to develop the natural capital in accordance with the indicated natural capital (y is the investment ratio for the natural capital). A combination of proportional and derivative control over the investment in natural capital is used hereby.

The stock of environmental assets is not treated explicitly in this model. The natural capital-output ratios -- real, f, and indicated, c, in the equations (14) and (15) -- belong to the state variables of the model.

The flow variables P, C, M, Y, and Z are measured in dollars per year, the stock variables K and F are measured in dollars. Respectively, these variables could be measured in bits per year and bits as well. Methods of an evaluation of their informational content need a special elaboration that goes beyond the scope of this paper.

The next peculiarity of the model is that it has only implicit delays. Due to them, the model gets rid of the instantaneous adjustment to an equilibrium with full employment of labor force used by the earlier neo-classical theories of economic growth. An explicit investment delay is still set aside.

Two profit rates and a net (resource) rent rate are defined for this model. The first is the average rate of return to man-made capital $(1- u - y)/s$. The second is a general one, it measures a ratio of the economic surplus to the total value of produced and natural capital $(1 - u - e)/(s + f)$. The rate of net rent is the ratio of net unit rent to natural capital – output ratio, $(y - e)/f$. The general rate of profit is a weighted average of the rate of return to man-made capital and the rate of net rent: $(1 - u - e)/(s + f) = [s/(s + f)](1- u - y)/s +[f/(s + f)](y - e)/ f$.

The average rate of profit can rise as a result of a rise in the capital share $(1 - u - y)$, a decline in the capital-output ratio (s), or decline in the relative price of capital goods (p/p_K). These three factors are not independent. The ratio p/p_K is identically one in this one-product model.

The Model and Law in a Compact Form

To get a compact model we need the following transformations.

$$\hat{s} = \widehat{(K/P)} = \widehat{(K/L)} - \widehat{(P/L)}$$
$$= n_1 + n_2 u + n_3(v - v_c) + n_5(Z/P) - (m_1 + m_2 \widehat{K/L} + m_3 y\,(\hat{v}) + m_5 \widehat{F/L})$$
$$= n_1 + n_2 u + n_3(v - v_c) + n_5 e - (m_1 + m_2(n_1 + n_2 u + n_3(v - v_c) + n_5 e) + m_3 y\,(\hat{v}) + m_5(\hat{F} - \hat{L}))$$
$$= -m_1 + (1- m_2)(n_1 + n_2 u + n_3(v - v_c) + n_5 e) - m_3 y\,(\hat{v}) - m_5(\hat{F} - \hat{K} + n_1 + n_2 u + n_3(v - v_c) + n_5 e)$$
$$= -m_1 + (1- m_2)(n_1 + n_2 u + n_3(v - v_c) + n_5 e) - m_3 y\,(\hat{v}) - m_5(\hat{f} - \hat{s} + n_1 + n_2 u + n_3(v - v_c) + n_5 e)$$
$$= -m_1 + (1- m_2 - m_5)(n_1 + n_2 u + n_3(v - v_c) + n_5 e) - m_3 y\,(\hat{v}) - m_5(\hat{f} - \hat{s});$$
$$\hat{v} = \widehat{(L/N)} = \hat{K} - (n_1 + n_2 u + n_3(v - v_c) + n_5 e) - n$$

$$= k\,\frac{1-u-y}{s} - (n_1 + n_2 u + n_3(v - v_c) + n_5 e) - n;$$

$$\hat{u} = \hat{w} - \hat{a}$$

$$= -g + rv + b(n_1 + n_2 u + n_3(v - v_c) + n_5 e) + qF\hat{/}L - (m_1 + m_2(n_1 + n_2 u + n_3(v - v_c) + n_5 e) + m_3 y(\hat{v}) + m_5(\hat{F} - \hat{L}))$$

$$= -g + rv - m_1 + (b - m_2 - m_5)(n_1 + n_2 u + n_3(v - v_c) + n_5 e) - m_3 y(\hat{v}) - m_5(\hat{f} - \hat{s});$$

$$\hat{f} = \hat{F} - \hat{P} = \frac{y-e}{f} - \hat{a} - \hat{L}$$

$$= \frac{(y-e)}{f} - (m_1 + m_2(n_1 + n_2 u + n_3(v - v_c) + n_5 e) + m_3 y(\hat{v}) + m_5(\hat{F} - \hat{L})) - \hat{L}$$

$$= (1 - m_5)\frac{(y-e)}{f} - m_1 - m_2(n_1 + n_2 u + n_3(v - v_c) + n_5 e) - m_3 y(\hat{v}) - (1 - m_5)(\hat{v} + n);$$

$$\hat{c} = \hat{X} - \hat{P} = i - \hat{K} + \hat{s} = i - k\frac{1-u-y}{s} + \hat{s};$$

$$\hat{y} = (o_1(c - f) + o_2 \hat{f}).$$

In a compact form, the model consists of the six non-linear ordinary differential equations (17) -- (22) that define the *hypothetical Law of capital accumulation*:

$$\dot{s} = -\frac{1}{(1-m_5)}(m_1 + (m_2 + m_5 - 1)(n_1 + n_2 u + n_3(v - v_c) + n_5 e) + m_3 y(\hat{v}) + m_5 \hat{f})s, \quad (17)$$

$$\dot{v} = (k\frac{1-u-y}{s} - (n_1 + n_2 u + n_3(v - v_c) + n_5 e) - n)v, \quad (18)$$

$$\dot{u} = (-g + rv - m_1 + (b + q - m_2 - m_5)(n_1 + n_2 u + n_3(v - v_c) + n_5 e) - m_3 y(\hat{v}) + (q - m_5)(\hat{f} - \hat{s}))u, \quad (19)$$

$$\dot{f} = ((1 - m_5)\frac{(y-e)}{f} - m_1 - m_2(n_1 + n_2 u + n_3(v - v_c) + n_5 e) - m_3 y(\hat{v}) - (1 - m_5)(\hat{v} + n))f \quad (20)$$

$$\dot{c} = (d - k\frac{1-u-y}{s} + \hat{s})c, \quad (21)$$

$$\dot{y} = (o_1(c - f) + o_2 \hat{f})y, \quad (22)$$

where $s > 0$, $1 \geq v > 0$, $1 \geq u > 0$, $f > 0$, $c > 0$, $1 > y$. The requirement for denominators to be positive is skipped. If $\dot{K} > 0, \dot{F} > 0$ for an every instant of time, the system (17) -- (22) defines a strongly sustainable development.

A stationary state is defined as

$$E_a = (s_a, v_a, u_a, f_a, c_a, y_a), \quad (23)$$

where

$$s_a = s_0,$$

$$v_a = (g + (1 - b - q)(d - n))/r,$$

$$u_a = (d - n - n_1 - n_3(v_a - v_c) - en_5)/n_2,$$

$$f_a = (1 - u_a - e)/d - s_a/k,$$
$$c_a = f_a,$$
$$y_a = e + df_a,$$
$$i = d;$$

the positive s_0 is determined exogenously and y_a is the stationary gross unit rent.

If $m_1 > 0$ and $m_2 + m_5 < 1$, the growth rate of labor productivity, wage rate and capital intensity is $d - n = m_1/(1 - m_2 - m_5) = w$ at this stationary state. The man-made capital stock, natural capital and net output increase at the rate $\hat{K}_a = \hat{F}_a = \hat{X}_a = \hat{P}_a = d$ that is equal to the net rent rate and less than (for $k < 1$) or equal to the average profit rate (for $k = 1$). The stationary average profit rate equals $(1 - u_a - y_a)/s_a = D = d/k$. The incremental and average capital-output ratios ($\bar{s}$ and s) are identical at the stationary state. The total capital-output ratio is $s_a + f_a = s_a + (1 - u_a - e)/d - s_a/k = (k - 1)s_a/k + (1 - u_a - e)/d$.

Typically, the higher the rate of technological progress (w), the low is the total capital-output ratio, the higher are the employment ratio, relative wage, profit and rent rates. Thus win - win solution is possible. This conclusion does not deny the fact that regeneration or recycling of the unemployed is the inescapable condition for capital accumulation. Notice that $u_a = (d - n - n_1 - en_5)/n_2$ for $v_a = v_c$. If $v_a \neq v_c$, there is a certain trade-off between the stationary employment and the stationary relative wage.

The fixed point E_a is not necessarily stable even in the sense of Liapunov. A development path deviates from this stationary state. The next sections tell where it goes.

A system identification by an extended Kalman filtering

The Kalman filter is a particular powerful tool for estimating unobservable part of a model (parameters and meta-parameters like variances) in one operation. Although the Kalman filter itself does not estimate the unknown parameters of the model, it provides a one-step-ahead prediction error with its covariance matrix. The prediction error decomposition of the likelihood function utilizes this information. Maximization routines can be used to determine the unknown parameters (Cuthbertson et. al.: 210-225).

The VENSIM professional soft-ware has served for performing such an extended Kalman filtering (EKF). EKF identifies the parameters of the above system of the six differential equations (17) – (22) and of an additional equation for the level of labor productivity (a). The number of measured levels is five since the indicated natural capital (c) and the gross unit rent (y) are treated as unobservable.

The filter control file specifies a covariance matrix for a driving noise and initial covariance of the state vector (see the file *kalman.prm* in the Appendix of this paper). It is known that the likelihood computation is exact only in the case of linear systems with Gaussian noise for driving and measurement errors (Peterson). This paper reports only about quasi-optimal estimates obtained so far for the presented non-linear model. A global optimal solution (the desired maximum likelihood estimates of genuine optimal filtering) has not been found yet. The sensitivity analysis below is about the best point the optimizer reached as presented in the File 2 in the Appendix.

A forecast of the US economic development, 1999-2507

The model with the identified quasi-optimal parameters is run for a long term forecast. The latter describes a complicated non-equilibrium process with a long transition (about 175 years) to a quasi-cyclical attractor with a period about 41-43 years.

EKF has created a VENSIM data file with estimated values of the model variables and constants. The following initial values parameters from this file are used in a POWERSIM simulation for the forecast starting year 1999: $i \approx 0.0239$, $e \approx 0.0263$, $k \approx 0.2729$, $m_1 \approx 0.0168$, $m_2 \approx 0.1222$, $m_3 \approx 0.0138$, $m_5 \approx 0.199$, $n_1 \approx -0.2413$, $n_2 \approx 0.3555$, $n_3 \approx 0.5$, $n_5 \approx 0$, $n \approx 0.0188$, $r \approx 0.0606$, $b \approx 0.5040$, $g \approx 0.0525$, $o_1 \approx -0.0460$, $o_2 \approx -8.1384$, $a_0 \approx 0.576$, $s_0 \approx 1.908$, $c_0 \approx 6.815$, $f_0 \approx 0.0855$, $y_0 \approx 0.0256$, $v_0 \approx 0.9578$, $v_c \approx 0.9254$, $u_0 \approx 0.7012$, $j \approx 0.2161$, $q \approx -0.005$. The coefficients are the same as in the File 2 but the magnitudes of the state variables are EKF estimates for the year 1999 (not 1958!). The forecasting uses no real data after 1999 in this paper.

Let us look at the forecast now. The growth rate of the indicated natural capital $i \approx 0.0239 < d \approx 0.0436$ (these parameters are defined in the equation (23)). It means that there is no stationary state with a periodic attractor nearby. Still the values v_a and u_a approximate the average, quasi-equilibrium values of the respective variables (v_{qe} and u_{qe}) in the very long run (over centuries) after the transition period is over (Figures 2 and 3). The auxiliary constant d also is a good approximation for the rates of capital accumulation and economic growth after the transition (Figures 1 and 6). During the transition period, the net unit rent $(y – e)$ will be positive and relatively stable after 2050 (Figure 5).

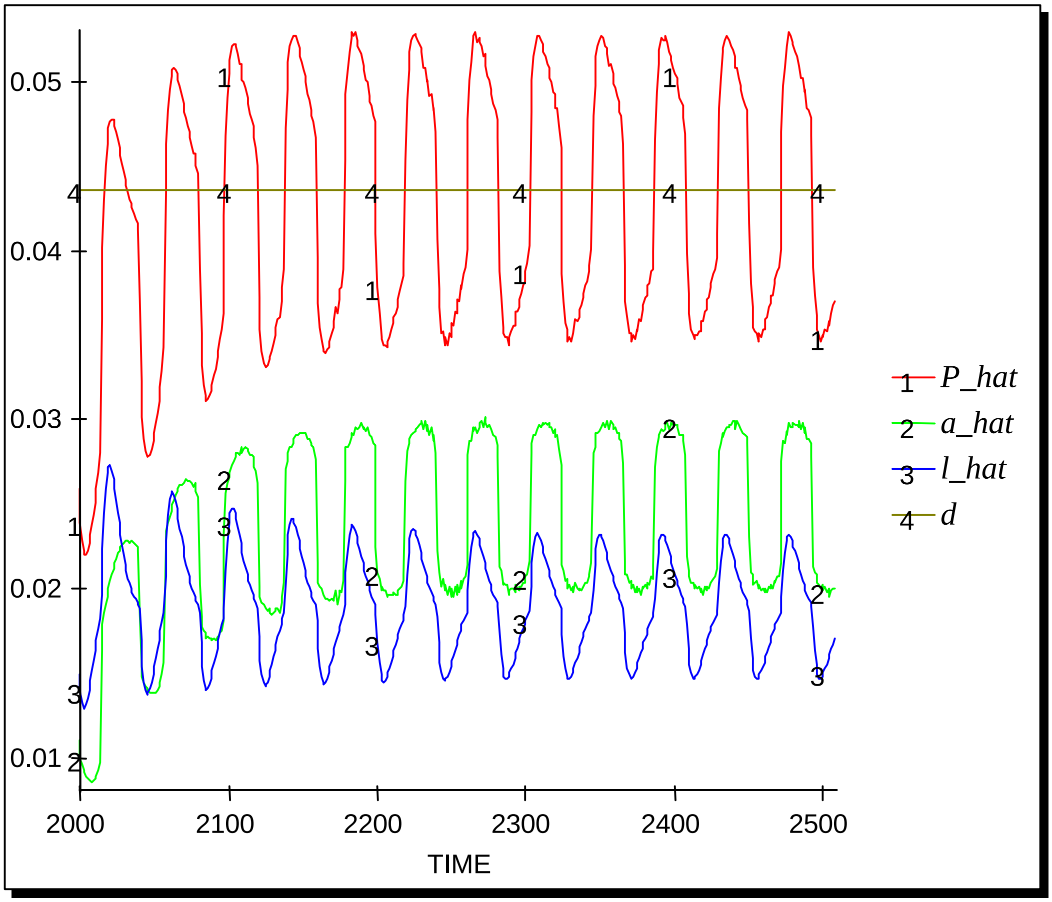

Figure 1 The rates of growth of net output ($\hat{P}$ =*P_hat*), labor productivity ($\hat{a}$ =*a_hat*) and employment ($\hat{l}$ =*l_hat*) versus the benchmark (*d*), 1999-2507

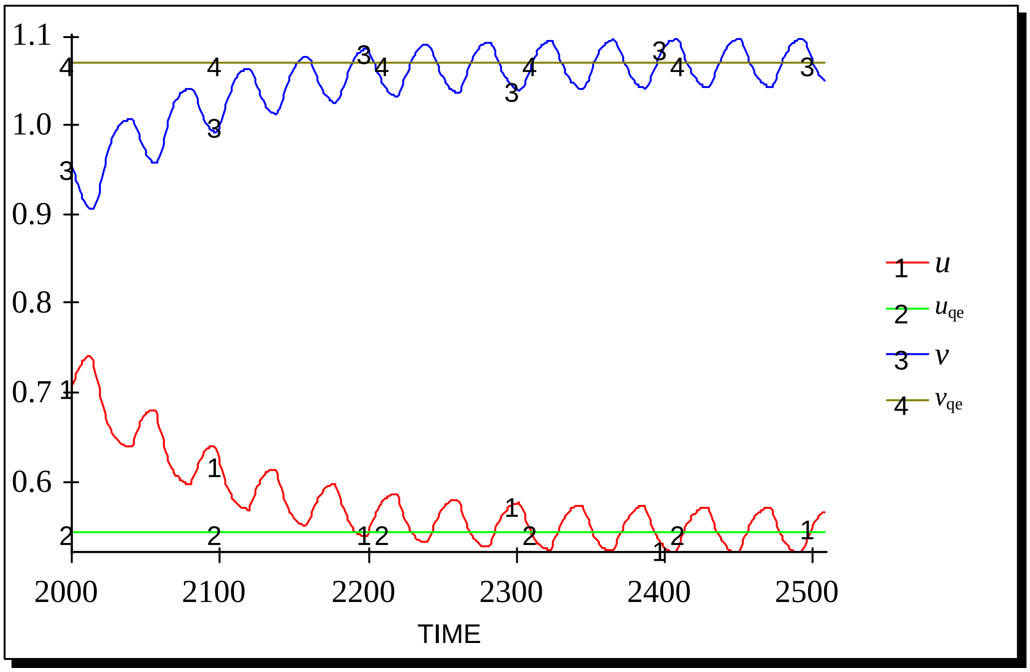

Figure 2 The relative wage (u) and employment ratio (v), 1999-2507

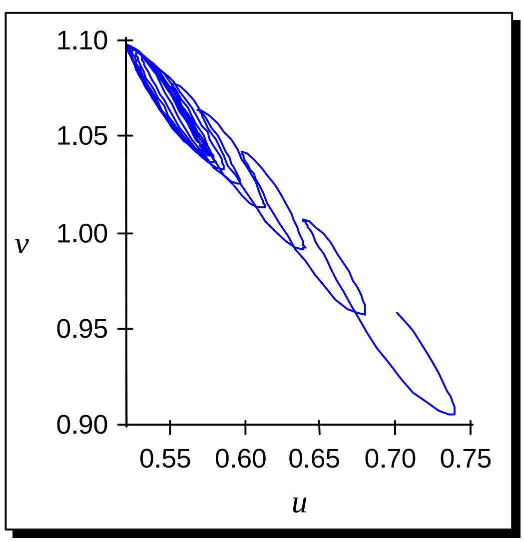

Figure 3 The employment ratio (v) versus the labor share (u) (for $r \approx 0.0605$), 1999-2507. Upward, clockwise

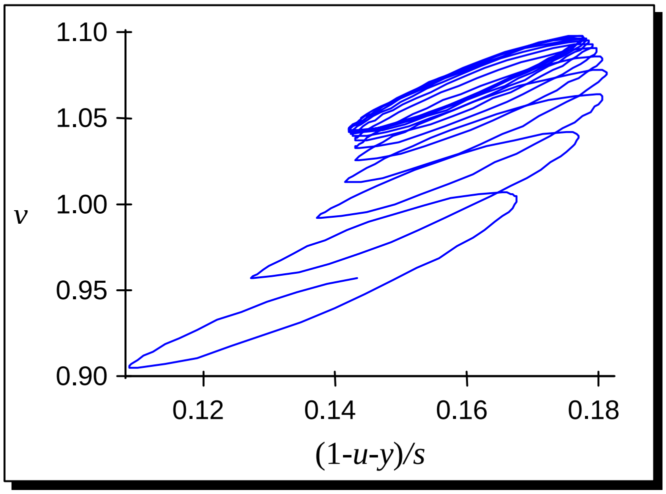

Figure 4 The employment ratio (*v*) and average profit rate ((1 – *u* - *y*)/*s*), 1999-2507. Upward, counter-clockwise

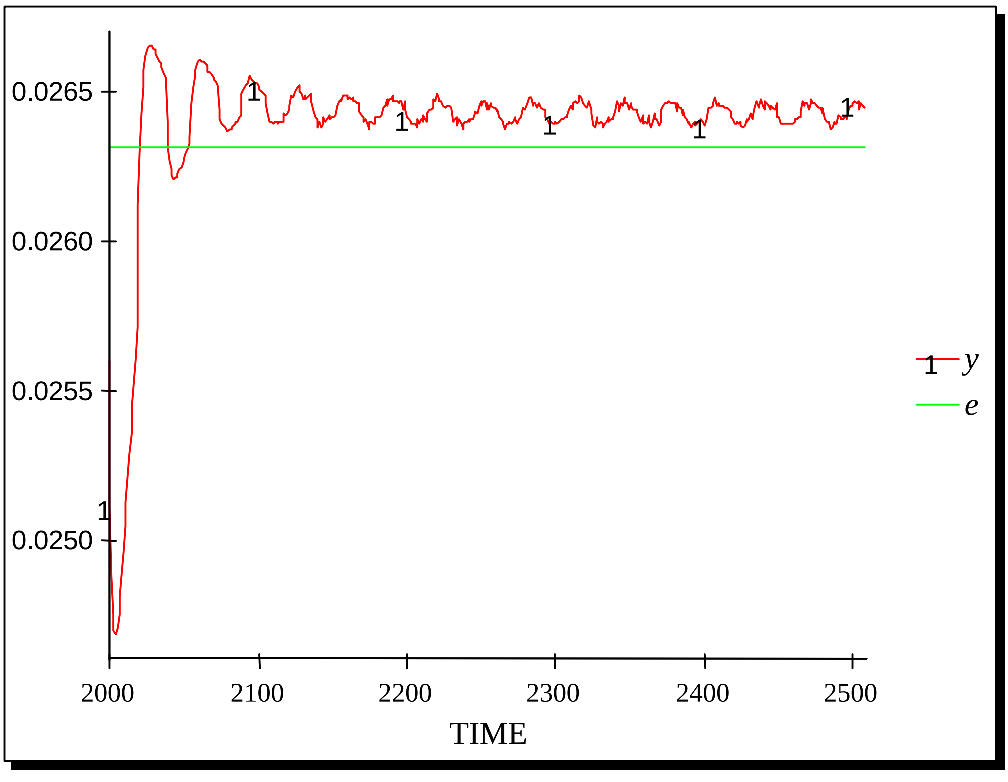

Figure 5 The unit gross rent (*y*) and unit depletion (degradation) of natural capital (*e*), 1999-2507

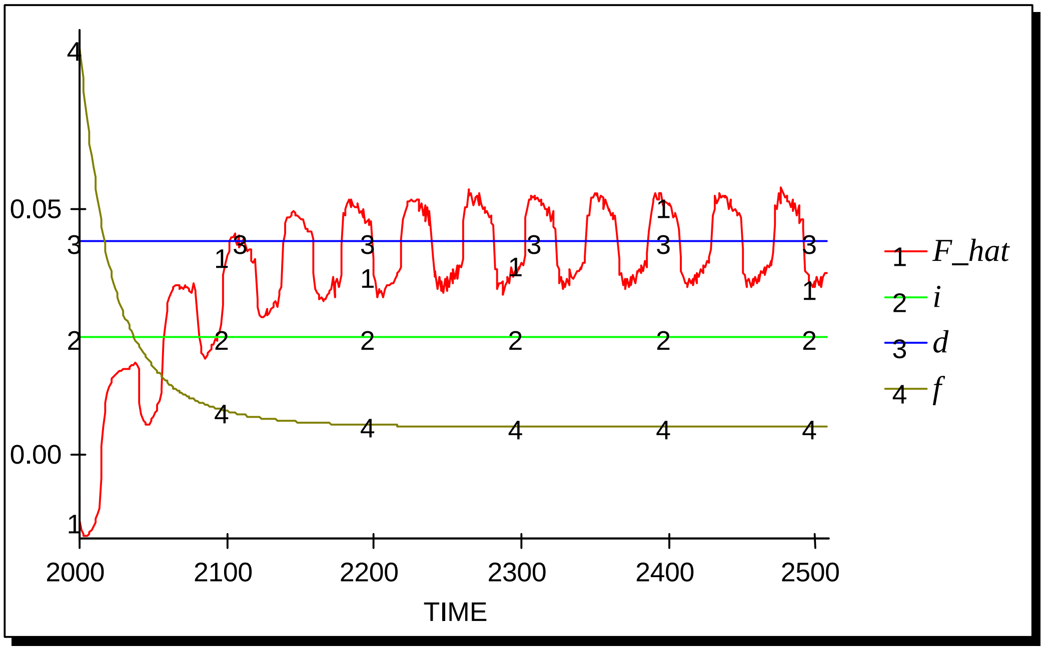

Figure 6 The natural capital – output ratio (*f*), growth rate of natural capital ($\hat{F}$ = *F_hat*) and benchmarks (*i*) and (*d*), 1999-2507

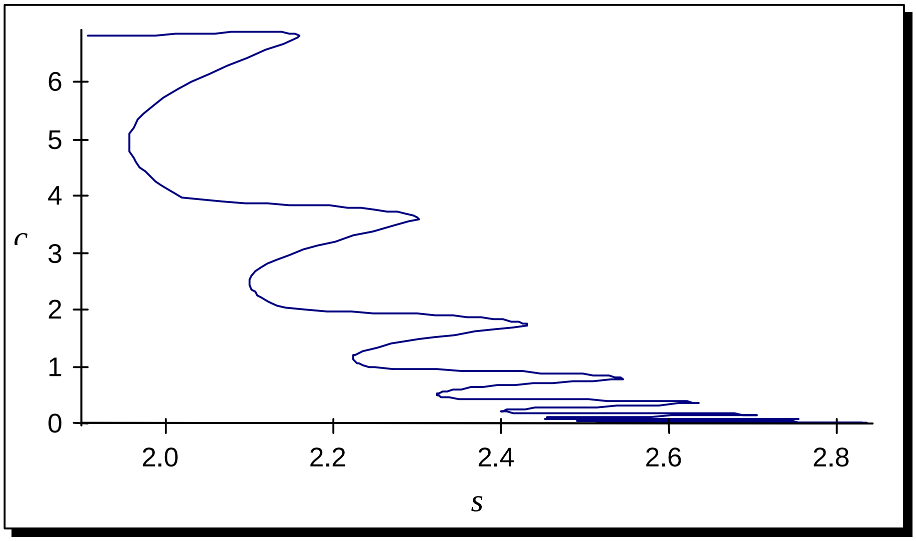

Figure 7 The produced capital-output ratio (*s*) and indicated natural capital-output ratio (*c*), 1999-2507. Downward

The transition itself is also a wave-like. After a decline in the period 1999-2014, the natural capital increases over the other years; it grows even relatively to the labor force and employment after 2056. The rates of growth of net national product (NNP) and natural capital become approximately equal between 2150 and 2200; this match remains thereafter (Figures 1, 5 and 6). Due to a declining labor share in NNP, the rates of profit $(1 - u - y)/s$ and $(1 - u - e)/(s + f)$ tend to increase over the whole transition period even with generally rising produced capital – output ratio (s) and total capital – output ratio ($s + f$). (See Figures 2, 4 and 7.)

The univariate sensitivity analysis

This forecast projects the internal tendencies of capital accumulations into the third millennium. It clearly requires a qualification. The employment ratio tends to exceed the upper limit ($v = 1$) after 2037 while the relative wage (u) declines dramatically. If $r \approx 0.0675$ (instead of being equal to 0.0605), the employment ratio will not exceed that upper limit. It will fluctuate around 0.958 after the transition while the relative wage will fluctuate around $u_a \approx 0.698$ compared with 0.544 before the adaptation of r. The both profit rates $(1 – u - e)/(s + f)$ and $(1- u - y)/s$ will tend to be at a level $d/k \approx 0.16$ again. The rate of net rent will tend to be equal to the same benchmark as before ($d \approx 0.0436$). This alteration reduces the period of fluctuations from 41-43 to 32-33 years approximately.

A sensitivity analysis sheds some light on the forecasting confidence bounds. For simplicity only one control parameter has been randomized: the capital investment ratio (k) is randomly uniformly distributed in the interval (0.2183, 0.3275) with the variation about 0.001. Two hundreds of Monte Carlo simulations with the initial noise seed of 1234 have been calculated using the Latin hyper-cube option. A shorter period of simulations (until the year 2057) facilitates a closer view of the long wave in the first half of the XXI century.

A data set sd-6v-9-5-1-KF-58-98-34 represents the sensitivity simulation run based on the EKF estimates (a data set sd-6v-9-5-1-KF-58-98-27). A data set C:\USA_economy\f-58-91-a-48-98-for-real-NNP\7-5-1 contains the real observations from Table 1 in the Appendix. They have been used for obtaining the EKF estimates.

The simulations display the confidence bounds for periods stating in the year 1999. These bounds are computed at each point in time by ordering and sampling all the simulation runs. For example, for a confidence bound at 50, a quarter of the runs has a value bigger than the top of the confidence bound and another quarter has a value lower than the bottom. The plot mean value at each point in time is displayed as a red line.

There is almost no difference between the estimates, generated by EKF, and observations for period 1958-1998 for a human eye. Table 2 reports on the forecast errors using EKF one step ahead.

The current downswing in the long term investment cycle

The first noticeable aspect of the real current recession is first of all its connection to the downswing in the long term investment cycle. The graphs show that in the first decade the US economy will experience decelerating growth rates of labor productivity and employment, declining rates of return to produced capital. The reader may also see the growing relative wage and increasing produced capital-output ratio that negatively affect the profitability (Figures 10-14). It is likely that the share of investment in natural capital (*y*) will remain stable, while the ratios of real and indicated natural capital to NNP (*f* and *c*) will continue their decline (Figures 8, 9, 15). The higher relative wage (*u*) will produce inflationary pressures.

The long term business upturn will not probably happen until 2008 or even 2015. It will proceed thereafter up to the beginning of the next long term downturn in 2035-2040.

Real development will be different from the offered description because of learning, external influences and counter-cyclical policies that are not taken into account. Still the model parameters can be adjusted by EKF and the forecast can be updated each period, based on new information.

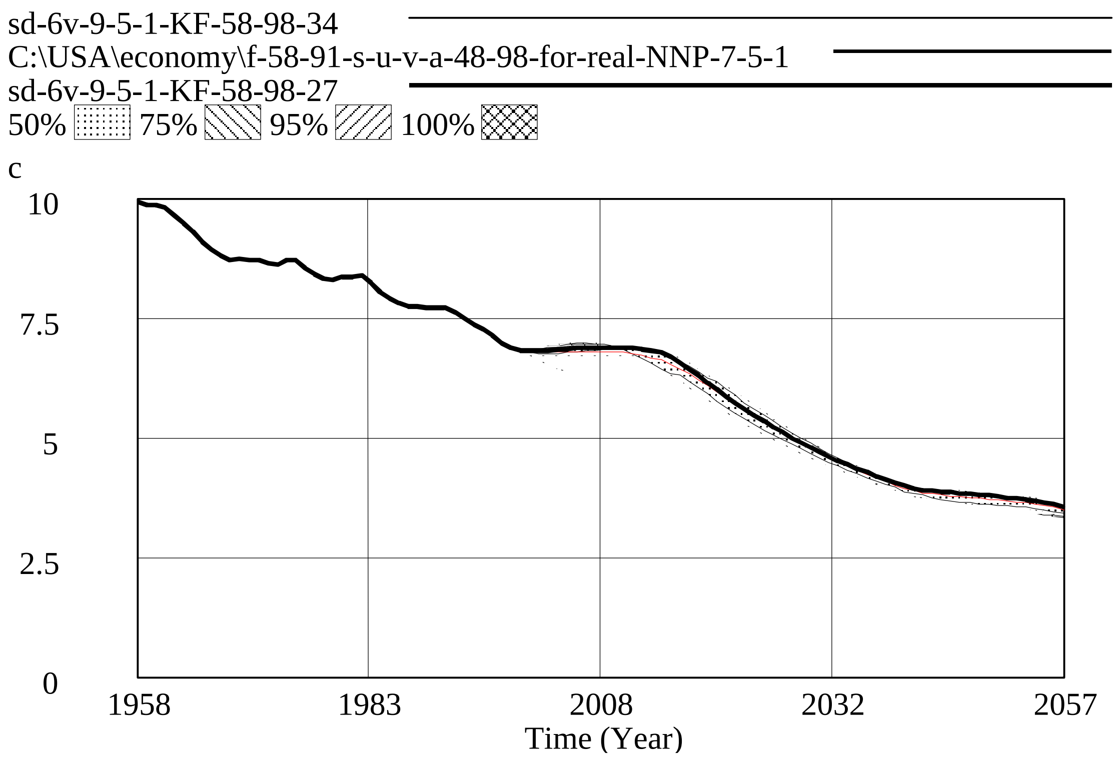

Figure 8 The indicated natural capital – output ratio (*c*)

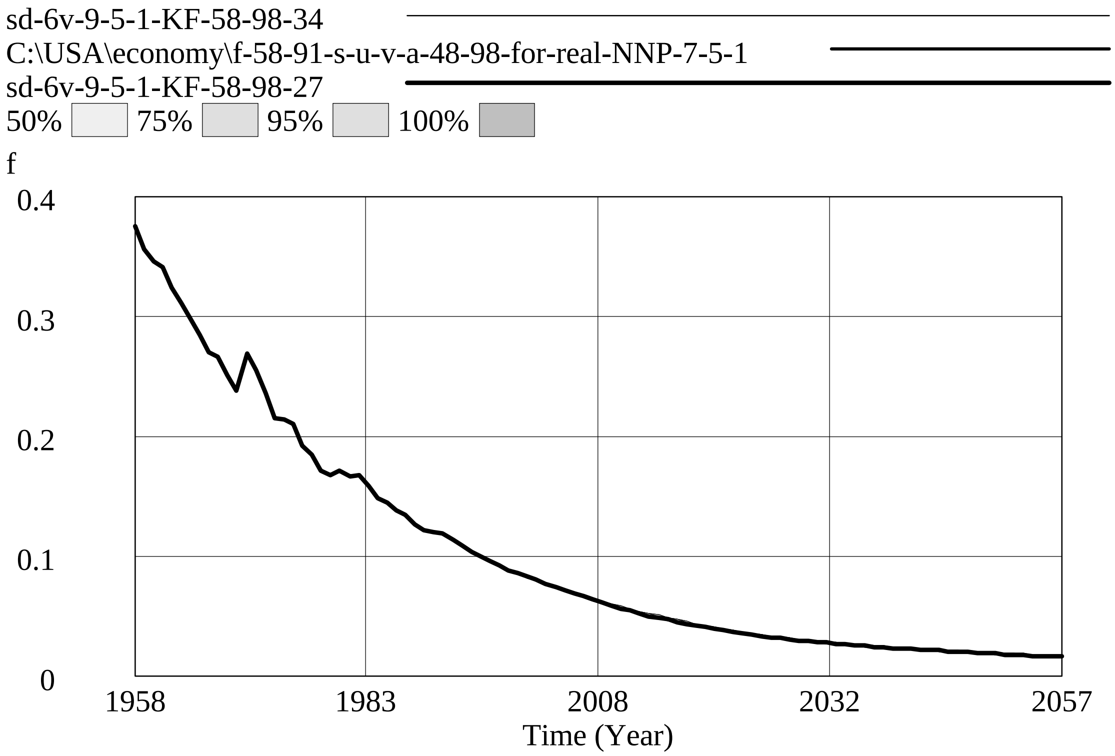

Figure 9 The expected real natural capital – output ratio (*f*)

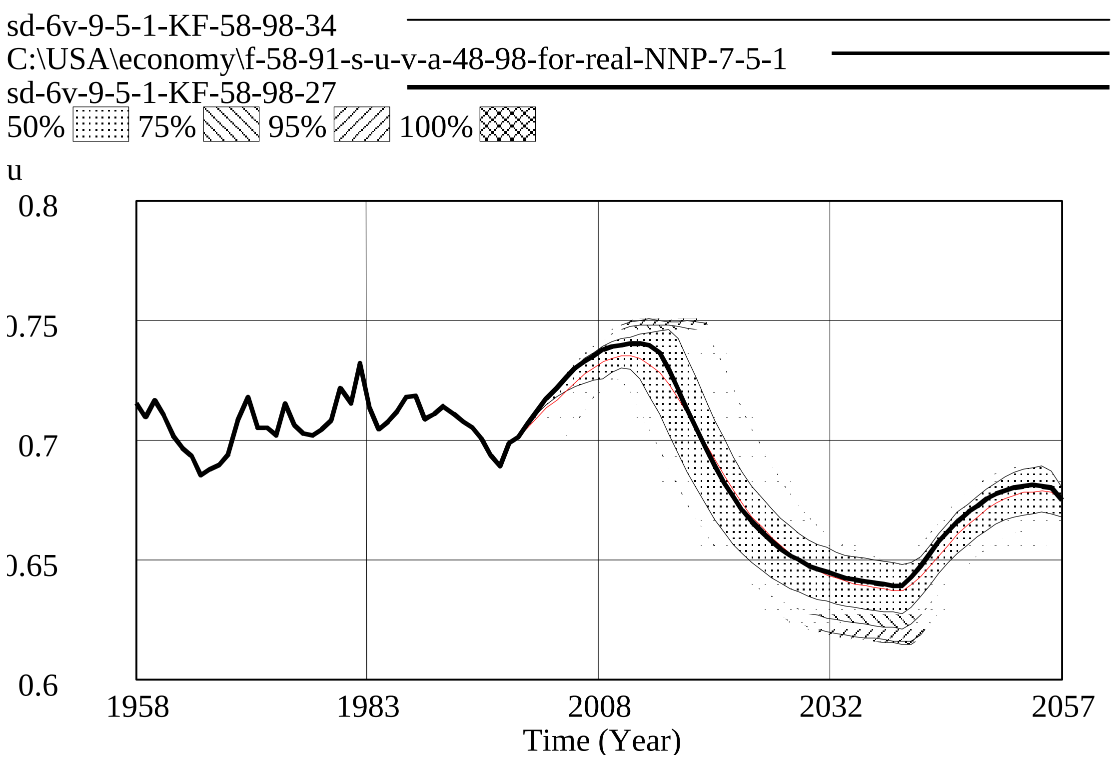

Figure 10 The relative wage (*u*)

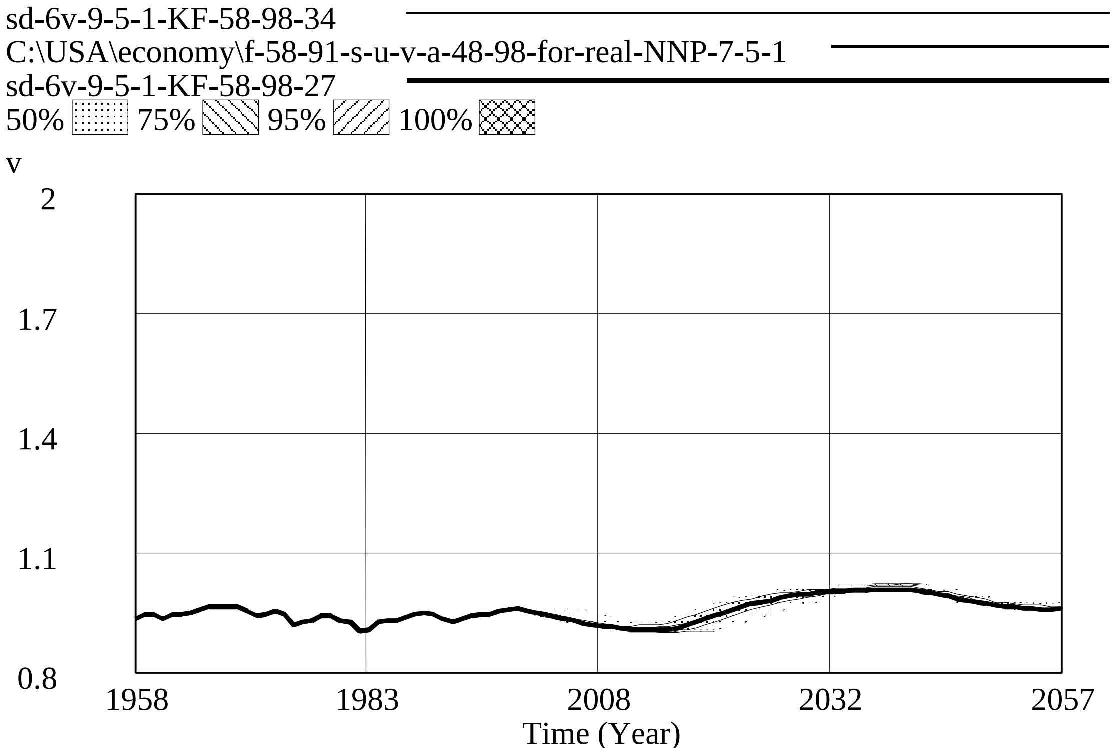

Figure 11 The employment ratio (v)

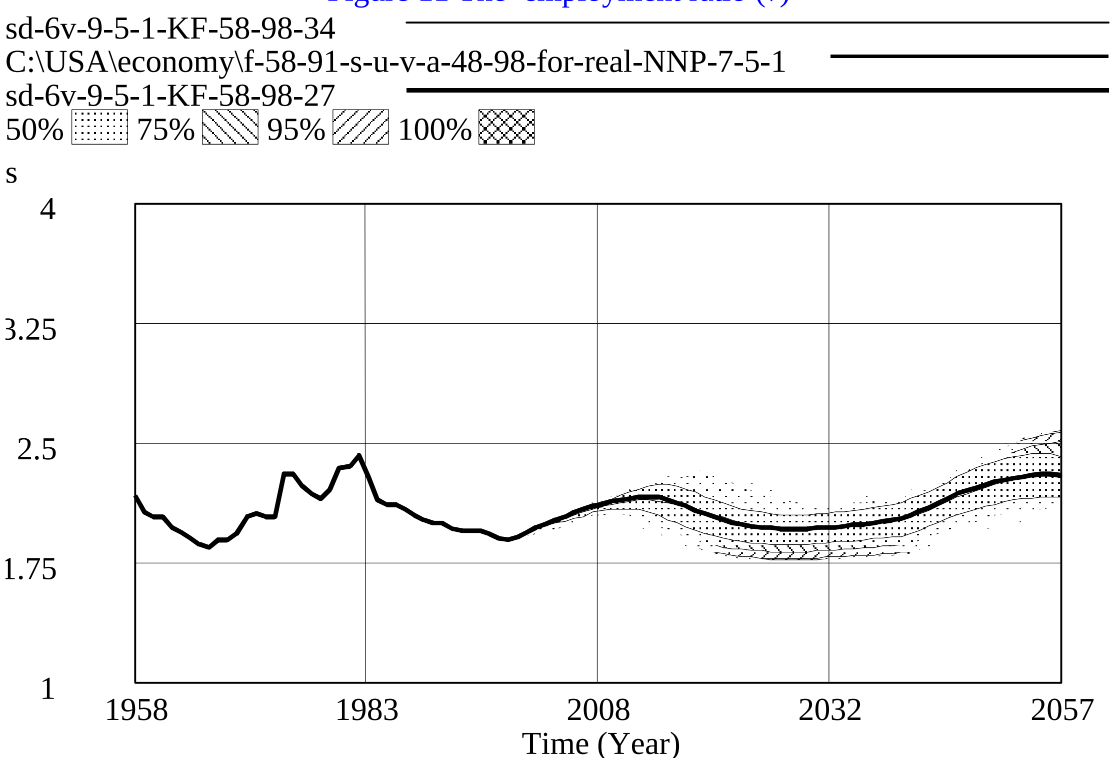

Figure 12 The produced capital -- output ratio (s)

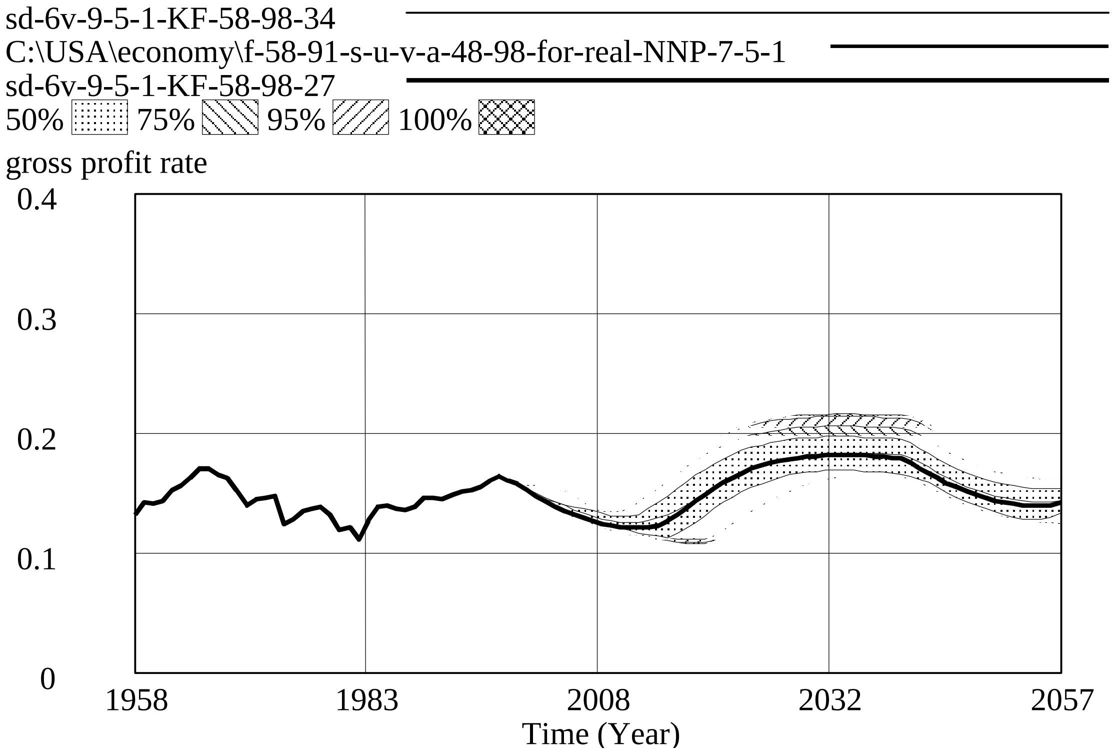

Figure 13 The gross rate of return to produced capital (1 – u)/*s*

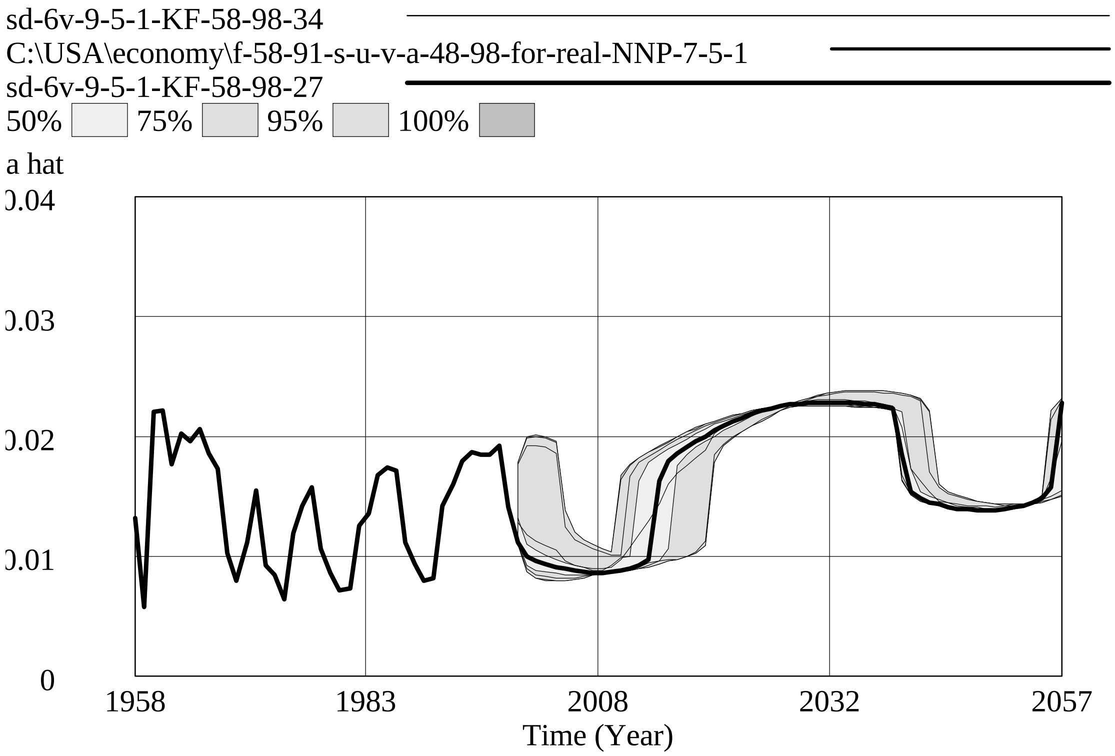

Figure 14 The growth rate of labor productivity ($\hat{a}$)

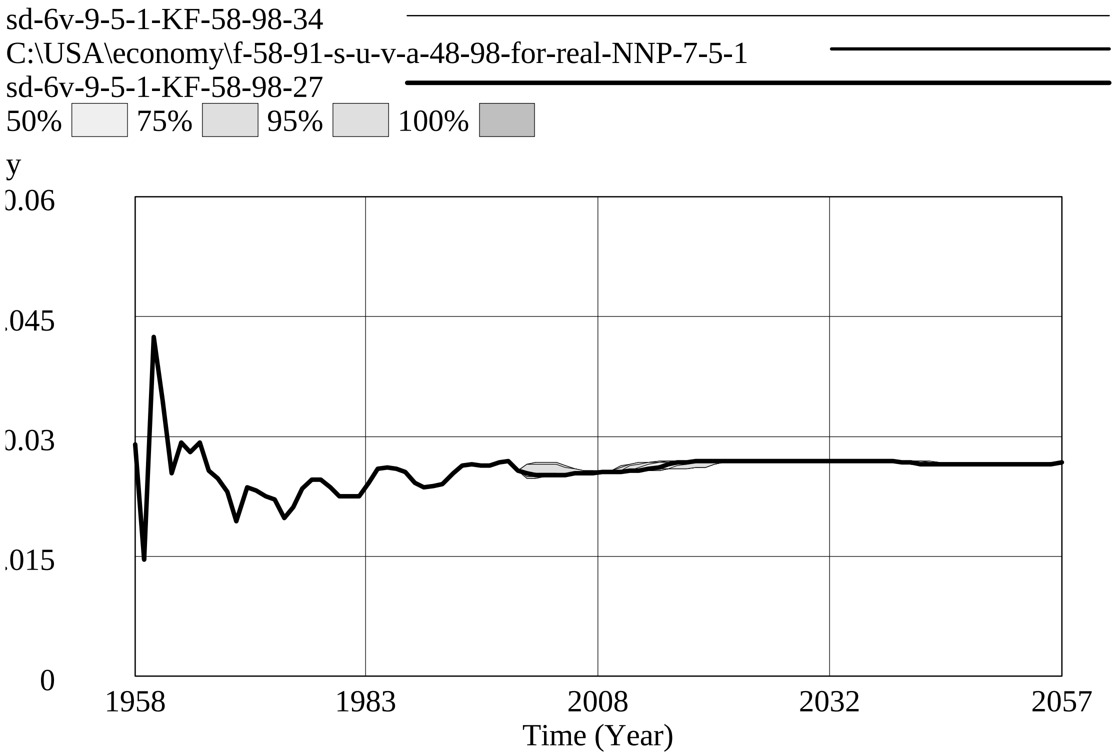

Figure 15 The gross unit rent (*y*)

Conclusion

The EKF realized in the VENSIM soft-ware has enabled to estimate the unobservable components of the compact model written in the state-space form. The identified model has been used for long-term projections together with the univariate sensitivity analysis. The found estimates are not as exact as required by optimal filtering. Still it may be interpreted an advantage rather then a drawback if we recall that the economic application of the optimal filtering assumes that learning agents use the given information optimally. It is clearly a very strong idealization for the world of bounded rationality. Without this idealization the identification of unobservable parameters and meta-parameters is even more complicated. It is a matter of combined theoretical and empirical research to find out how efficiently the information is really used by the economic agents. If it can be proved than a quasi-optimal usage of available information is more likely than optimal, then the common pragmatic usage of quasi-optimal estimates (in this paper, in particular) is justified theoretically.

A more sophisticated statistical analysis of the model consistency and validity together with a better model calibration is wishful. The reader have seen grounds for stipulation that the hypothetical Law of advancing capitalism presented in this paper reflects realistically important aspects of the dialectical interaction between factors that tend to lower the average rate of profit and those that counteract this tendency.

Conversion of profit into capital and sustained expansion for a number of years eventually results in a tight labor market, rising wages, and accelerated substitution of labor force by produced capital since capitalists try to economize on labor costs. As this process tends to raise the capital-labor ratio, it also tends to lower the average rate of profit. When this tendency outweighs the counteracting tendencies, the expansion is turned to a depression. These well-known relationships are expressed mathematically in a rather novel way paying attention to development of natural capital.

There is less agreement in the literature whether inner laws of accumulation of capital can similarly explain the turn from long periods of stagnation to those of expansion, or extra-economic factors are required to bring about such upward transitions. We have seen that the hypothetical Law can explain this turn as well.

The conscious element of the Law may play a key role in providing a better governance of the ecological-economic reproduction on the increasing scale in the third millennium. In any case, the real practice is the decisive final criterion in testing objectivity of this Law.

Appendix

Table 1. The author's estimates for real macroeconomic data based on the official statistics of the USA for 1958-1999

	f	*u*	*s*	*v*	*a*
1958	0.37537	0.71458	2.16474	0.93162	0.03149
1959	0.35589	0.70956	2.05753	0.94542	0.03312
1960	0.34513	0.71619	2.02956	0.94438	0.03333
1961	0.34072	0.71012	2.03005	0.93321	0.03413
1962	0.32338	0.70172	1.96848	0.94452	0.03579
1963	0.31121	0.69675	1.93987	0.94357	0.03679
1964	0.29774	0.69298	1.89615	0.94833	0.03813
1965	0.28434	0.68468	1.86284	0.95494	0.03962
1966	0.26946	0.68737	1.84462	0.96215	0.04117
1967	0.26558	0.68933	1.88948	0.96158	0.04122

1968	0.25116	0.69404	1.888	0.96446	0.04231
1969	0.2379	0.70784	1.92997	0.96493	0.04238
1970	0.26829	0.71733	2.03042	0.95015	0.04184
1971	0.25499	0.70482	2.04807	0.94048	0.04288
1972	0.23468	0.70488	2.03457	0.94396	0.04371
1973	0.21493	0.70169	2.03485	0.95124	0.04485
1974	0.21282	0.71508	2.29921	0.94375	0.04351
1975	0.20938	0.7058	2.30697	0.91533	0.04352
1976	0.19149	0.70207	2.22419	0.92306	0.04461
1977	0.18417	0.70181	2.17677	0.92962	0.04509
1978	0.17141	0.70391	2.15084	0.93948	0.0456
1979	0.16713	0.70826	2.20955	0.94155	0.04575
1980	0.17061	0.7217	2.3441	0.92829	0.04513
1981	0.16563	0.71519	2.35172	0.92384	0.04554
1982	0.16738	0.73149	2.41718	0.90281	0.04462
1983	0.1583	0.71291	2.27873	0.90411	0.04595
1984	0.14766	0.70398	2.14105	0.92487	0.04743
1985	0.14381	0.70729	2.10606	0.928	0.04797
1986	0.13807	0.71158	2.10566	0.93004	0.04827
1987	0.13395	0.71733	2.08067	0.93814	0.04856
1988	0.12582	0.71845	2.04407	0.94496	0.04952

1989	0.12148	0.70831	2.01227	0.94733	0.05011
1990	0.12008	0.71042	1.99851	0.9439	0.05037
1991	0.11898	0.71355	1.99378	0.93162	0.05028
1992		0.71058	1.95891	0.92497	0.05132
1993		0.70712	1.94959	0.9309	0.05196
1994		0.70476	1.94537	0.93914	0.0526
1995		0.70051	1.94455	0.94402	0.05325
1996		0.69304	1.92755	0.94602	0.05426
1997		0.68867	1.89862	0.95064	0.05524
1998		0.69822	1.88958	0.95494	0.05656
1999				0.95782	0.05769

This paper assumes that the annual value of the labor force of a proprietor is equal to the annual earnings of a hired employee. Calculating the relative labor income (u) requires two steps:

1) estimating the ratio of the total labor force to employees M = (CLF + AFP)/(CLF + AFP - SE),

where CLF is the civil labor force, AFP is Armed Force Population, SE is a number of self-employed in all industries; this account does not covers the part of the defense related personnel outside the AFP because of incompleteness of the official statistics available for the author;

2) estimating the labor share u = (W+S+NWC)*M/NNP,

where W, S, NWC are accruals for wage and salary income and disbursements for other labor income, NNP is net national product.

Units of measurement: u, v and y [dimensionless], c, f and s [years], a [billions of chained 1996 dollars per 1000 civil persons employed]. The values of y, e for 1958 are calculated using the estimates of additions and depletion relative to NNP (billions of 1987 dollars). The valuations of the closing stocks of subsoil natural assets, given by the BEA first current rent method, in billions of 1987 dollars have been converted into billions of 1996 dollars through multiplication by the NNP price index for 1987-1996. This index is calculated as (NNP 1987, billions of 1996 dollars)/(NNP 1987, billions of 1987 dollars) = 5460/4029 = 1.355. In calculating f and a, constant 1996 dollars are used for the nominators and denominators; calculations of u and s are done with the nominators and denominators valued in current prices. The employment ratio v is for the civil labor force.

The new BEA estimates of non-residential fixed assets (formerly "fixed reproducible tangible wealth") for 1998 and revised estimates for 1958-97 are used in calculating produced capital – output ratio (s) for different years. The net stock of durable goods owned by consumers is not included in calculations in this paper. The BEA estimates cover the net stock of equipment and software and of structures owned by business and government (Survey of Current Business, April 2000).

The value of the stocks of proved mineral reserves (a part of developed natural assets) in the aggregate has grown in current dollars (1958-91), while showing a little growth until 1970 and slow decline thereafter until 1991 (*Survey of Current Business*, April 1994, p. 58-60). So the ratio of this reserves to NNP has declined in constant and current dollars. This decline has been correlated with a growing reliance on import of mineral resources that is not taken into account explicitly in the above model of a closed economy.

The definition of the resource rent, offered in this paper, relates to the current rent method I and II of valuing mineral resources applied by the BEA (see *Survey of Current Business*, April 1994, p. 54-57). According to the first method, the value of closing stocks at the end of 1991 was 480.6 billions of current dollars or 519.7 billions of 1987 dollars. According to the second method, the value of closing stocks at the end of 1991 was 907.6 billions of current dollars or 1,018.7 billions of 1987 dollars.

Unlike the BEA, the above model is abstracting from revaluation effects due to changes of prices. The reader is not to overlook another principal difference: whereas the BEA derives the resource rent as a residual, the model interprets profit as a residual instead.

The current rent method I corresponds to the following transformations:
$Y_a = (1-u)P_a - M_a = (1 - u)P_a - \dot{K}_a/k$, $y_a = 1 - u_a - ds_a/k$, $f_a = (1 - u_a - e)/d - s_a/k$, and hence $s_a = k[-f_a + (1 - u_a - e)/d]$, and, finally, $y_a = 1 - u_a - ds_a/k = e + df_a$.

The BEA current rent method II needs a substantial refinement. It assumes implicitly that the profit investment ratio is equal to one ($k = 1$). The refined current rent method II without this assumption, would assume:
$F_a + K_a = [(1 - u_a - e)P_a - (1 - k)M_a]/d$, $F_a = - K_a + [(1 - u_a - e)P_a - (1 - k)\dot{K}_a/k]/d = - K_a + [(1 - u_a - e)P_a - (1 - k)dK_a/k]/d = (1 - u_a - e)P_a/d - K_a/k$, $\dot{F}_a = dF_a$, $\dot{K} = dK_a$, $\dot{F}_a = Y_a - Z_a$, or $Y_a = Z_a + dF_a = Z_a + (1 - u_a - e)P_a/d - K_a/k$;
after dividing the both sides by P_a, we get $y_a = e + df_a$. Thus both kinds of the current rent method are equivalent in our model after the refinement.

In fact, the BEA current rent method II overestimates F_a by $(1 - k)M_a/d = [(1 - k)/k]K_a$, since instead of $F_a = [(1 - u_a - e)P_a - (1 - k)M_a]/d - K_a$ it postulates $F_a = [(1 - u_a - e)P_a]/d - K_a$. This has not been noticed in the BEA final report that could not explain where from the important difference in the estimates comes. This unexplained difference is about 89 percent (current prices) and 96 percent (constant prices). The above model offers the explanation.

The POWERSIM soft-ware has been used to simulate figures 1-7, whereas VENSIM professional soft-ware served for performing an extended Kalman filtering and sensitivity analysis (Figures 8-15).

The current VENSIM implementation uses the following format for a specification of the noise:

kalman.prm

a/dr *a* variance/*a* ISC
f/*f* dr variance/*f* ISC
s/*s* dr variance/*s* ISC
u/*u* dr variance/*u* ISC
v/*v* dr variance/*v* ISC

In the first line, the noise influencing the level *a* has a variance of 3.71754e-007, and the initial variance of *a* is 1e-007. The other magnitudes are defined in an optimization output File 1 below. This file contains the best payoff so far, the reason the optimizer stopped, and the values of the search parameters needed to achieve that payoff. All variances from the file kalman.prm and variances of the measurement noise for the same five state variables have been included in the list of parameters to be estimated. The absence of missing data is not required by EKF.

File 1. The outputs of a penultimate optimization for 1958-1999

```
:COMSYS After 1093 simulations
:COMSYS Best payoff is 906.654
:COMSYS User terminated multiple search session
:OPTIMIZER=Powell
:SENSITIVITY=Off
:MULTIPLE_START=Random
:RANDOM_NUMER=Linear
:OUTPUT_LEVEL=2
:TRACE=2
:MAX_ITERATIONS=10000
:PASS_LIMIT=2
:FRACTIONAL_TOLERANCE=0.0003
:TOLERANCE_MULTIPLIER=21
:ABSOLUTE_TOLERANCE=1
:SCALE_ABSOLUTE=1
:VECTOR_POINTS=1.24418e-306
0 <= b = 0.359799  <= 1
0 <= c0 = 9.72738
0 <= f0 = 0.3756
0 <= g = 0.0524713  <= 1.5
0 <= m1 = 0.0168176  <= 0.02
0 <= m3 = 0.0148225  <= 0.1
0 <= m5 = 0.195932  <= 0.3
0 <= n = 0.01878  <= 0.022
0 <= n2 = 0.3555  <= 0.5
0 <= n3 = 0.5  <= 0.5
```

0 <= n5 = 0 <= 1
0 <= r = 0.0605975
0 <= y0 = 0.0292635
0.015 <= a0 = 0.0316 <= 0.05
0.05 <= j = 0.226619 <= 1
0.1 <= m2 = 0.100975 <= 0.75
0.2 <= k = 0.272922 <= 0.5
0.75 <= vc = 0.92536 <= 0.99
1e-007 <= a ISC = 1e-007
1e-007 <= f ISC = 1e-007
1e-007 <= MEAS f VARIANCE = 1e-007
1e-007 <= MEAS s VARIANCE = 1e-007
1e-007 <= MEAS u VARIANCE = 1e-007
1e-007 <= MEAS v VARIANCE = 1e-007
1e-007 <= s DR VARIANCE = 0.00371009 <= 0.01
1e-007 <= s ISC = 1e-007
1e-007 <= u DR VARIANCE = 5.99147e-005 <= 0.001
1e-007 <= u ISC = 1.08545e-007
1e-007 <= v ISC = 1e-007
1e-008 <= DR a VARIANCE = 3.71754e-007 <= 0.001
1e-008 <= f DR VARIANCE = 7.53926e-005 <= 0.001
1e-008 <= MEAS a VARIANCE = 1e-008 <= 0.001
1e-008 <= v DR VARIANCE = 4.9729e-005 <= 0.001
e = 0.0263151
i = 0.023724
n1 = -0.2413 <= 0.02
o1 = -0.0460085
o2 = -8.13836

Notice: c0, f0, y0 are for the year 1958.

File 2. The outputs of the last optimization for 1958-1999

:COMSYS After 809 simulations
:COMSYS Best payoff is 906.476
:COMSYS User terminated multiple search session
:OPTIMIZER=Powell
:SENSITIVITY=Off
:MULTIPLE_START=Random
:RANDOM_NUMER=Linear
:OUTPUT_LEVEL=2
:TRACE=2
:MAX_ITERATIONS=10000
:PASS_LIMIT=2
:FRACTIONAL_TOLERANCE=0.0003

:TOLERANCE_MULTIPLIER=21
:ABSOLUTE_TOLERANCE=1
:SCALE_ABSOLUTE=1
:VECTOR_POINTS=1.24418e-306
0 <= b = 0.504016 <= 1
0 <= c0 = 9.90707
0 <= f0 = 0.3756
0 <= g = 0.0524713 <= 1.5
0 <= m1 = 0.0168176 <= 0.02
0 <= m3 = 0.0137987 <= 0.1
0 <= m5 = 0.199002 <= 0.3
0 <= n = 0.01878 <= 0.022
0 <= n2 = 0.3555 <= 0.5
0 <= n3 = 0.5 <= 0.5
0 <= n5 = 0 <= 1
0 <= r = 0.0605975
0 <= y0 = 0.0289326
0.015 <= a0 = 0.0316 <= 0.05
0.05 <= j = 0.216074 <= 1
0.1 <= m2 = 0.122184 <= 0.75
0.2 <= k = 0.272922 <= 0.5
0.75 <= vc = 0.92536 <= 0.99
1e-007 <= a ISC = 1e-007
1e-007 <= f ISC = 1e-007
1e-007 <= MEAS f VARIANCE = 1e-007
1e-007 <= MEAS s VARIANCE = 1e-007
1e-007 <= MEAS u VARIANCE = 1e-007
1e-007 <= MEAS v VARIANCE = 1e-007
1e-007 <= s DR VARIANCE = 0.00364696 <= 0.01
1e-007 <= s ISC = 1e-007
1e-007 <= u DR VARIANCE = 5.89013e-005 <= 0.001
1e-007 <= u ISC = 1.01666e-007
1e-007 <= v ISC = 1e-007
1e-008 <= DR A VARIANCE = 3.65476e-007 <= 0.001
1e-008 <= f DR VARIANCE = 7.36972e-005 <= 0.001
1e-008 <= MEAS A VARIANCE = 1e-008 <= 0.001
1e-008 <= v DR VARIANCE = 4.98797e-005 <= 0.001
e = 0.0263151
i = 0.0238585
q = -0.00501026
n1 = -0.2413 <= 0.02
O1 = -0.0460085
O2 = -8.13836

Notice: c0, f0, y0 are for the year 1958.

Table 2 One step prediction errors for the state variables, 1958-1999

1958-1991

	a	*f*	*s*	*u*	*v*
1958	0.000107199	0.000235021	0.000259876	0.000423968	0.000376999
1959	-0.00124741	0.00866526	0.115749	0.00535792	-0.0152113
1960	-6.01076e-005	-0.00793988	0.0606837	0.000613213	-0.00301242
1961	-0.000226606	0.00224012	0.00740528	0.00468171	0.00554371
1962	-0.000961125	0.00769284	0.0487653	0.00186837	-0.0091061
1963	-0.000399157	-0.00169852	0.0297027	0.00268859	0.00252503
1964	-0.000645667	0.00166234	0.0370775	-0.000606835	-0.000351727
1965	-0.000777967	0.00142834	0.0298843	0.0048275	-0.00200826
1966	-0.000775434	0.00402987	0.0144112	-0.00619447	-0.00148535
1967	0.000671905	-0.00755474	-0.037866	-0.00265807	0.00245667
1968	-0.000373866	0.00298327	0.011552	-0.00447935	-0.00242347
1969	0.000318509	0.00321993	-0.012092	-0.00738823	-0.00412738
1970		-	-	-	

	0.000865564	0.041195	0.0579084	0.000118911	0.00466275
1971	-0.000601459	0.00407287	0.0129385	0.0179856	0.000950933
1972	-0.000173431	0.0084693	0.01615	-0.00165027	-0.0027104
1973	-0.000768311	0.0095883	0.0208557	0.00728631	-0.00876856
1974	0.00168328	-0.00728275	-0.237223	-0.00777453	0.00407571
1975	0.0002978	-0.00654104	0.0271852	0.0159156	0.0199124
1976	-0.000503816	0.00572766	0.0657854	-0.000999987	-0.000268161
1977	0.000171877	-0.00257476	0.0319307	-0.00469637	0.000178158
1978	0.000233307	0.00400624	0.0143347	-0.00663936	-0.00505054
1979	0.000297938	-0.00265357	-0.0445459	-0.00216138	-0.00273114
1980	0.000986271	-0.0104059	-0.111593	-0.00925624	0.0091911
1981	-7.73072e-005	-0.00250609	0.014462	0.0102577	-0.000572503
1982	0.00124465	-0.00954773	-0.0531332	-0.0142845	0.0202
1983	-0.000702497	0.000308692	0.12328	0.0132466	0.00078851
1984	-0.000805438	0.00253308	0.110147	0.00138944	-0.0111164

1985	0.000251859	-0.00269124	0.0184505	-0.00922948	0.00326604
1986	0.000540022	-0.000384584	-0.0117357	-0.009579	0.00231248
1987	0.000554491	-0.00161196	0.0178733	-0.010092	-0.00627083
1988	-0.000476521	0.00340068	0.0572989	0.00217408	-0.0113271
1989	-0.000179447	-0.000573896	0.0615978	0.0157575	-0.00989169
1990	0.000137988	-0.00381722	0.0409981	0.00345981	-0.000857174
1991	0.000509851	-0.00403332	0.0297095	0.00186872	0.00882274

1992 -1998

	a	*s*	*u*	*v*
1992	-0.000241328	0.0349802	0.000530303	0.00823772
1993	0.000233717	-0.00105929	-0.00167066	0.00022012
1994	0.000316147	-0.00528419	-0.00274199	-0.00290489
1995	0.000337016	-0.00367856	0.000251949	-0.00192529
1996	-1.18092e-006	0.0143441	0.00403756	0.000602603
1997	5.30407e-005	0.0233687	0.000434518	0.000391841
1998	-0.000233412	0.00411773	-0.0133591	0.00106645

1999

	a	*v*
1999	-	-

	0.00045351 3	0.00408012

The optimal filter provides several internal consistency measures (Peterson). In particular, the residuals from Table 2 may be normalized and tested for whiteness and unit variance. Still there is an additional uncertainty in such testing since the optimisation has not been accomplished precisely.

References

Cuthbertson, K., Hall, S. G., Taylor, M. P. 1992. Applied Econometric Techniques. Simon & Schuster International Group: New York.

Goodwin, R. 1972. A growth cycle. In Socialism, Capitalism and Economic Growth, Feinstein C. (ed.). Cambridge University Press: Cambridge; 165-170.

Peterson, D. W. 1980. Statistical tests for system dynamics. In Elements of the System Dynamics Method, Randers J. (ed.). The MIT Press: Cambridge (Math.): 224-245.

Ryzhenkov, A. 2000. Unfolding the Eco-wave: Why Renewal Is Pivotal. John Wiley & Sons, Inc., Chichester UK.

Saeed, K. 1994. Development Planning and Policy Design: A System Dynamics Approach. Ashgate/Avebury Books: Aldershot, England.

Solow, R. M. 1990. Goodwin's growth cycle: reminiscence and rumination. In Nonlinear and Multisectoral Macrodynamics: Essays in Honour of Richard Goodwin, Velupillai K. (ed.). The Macmillan Press: London; 31-41.

[i] The rate of regeneration is given by a function $Q(F, Y)$, satisfying $Q(0, Y) = 0$, $\partial Q/\partial Y > 0$ (at least for F above a certain minimal level of F) in a more detailed model of sustainable development. There is a perceived social need of directing technological progress to the development of material resources with a shorter regeneration time after the epoch of the increasing aggregate regeneration time of the resource package in use (Saeed: 124-130). These aspects are skipped in this paper.