

# Flexibility in Manufacturing Processes: a relational, dynamic and multidimensional approach<sup>1</sup>

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## Abstract

*We propose an approach that introduces a relational, dynamic and multidimensional conception of flexibility in manufacturing systems. In this approach, two inquiries must be introduced by the analyst: what is the field of variations on which flexibility is going to be observed? and, what is the field of tensions grouping the resistences to changes on the field of variations? In order to obtain an operational model of flexibility, the analyst must define the current and the expected states of the observed system and construct a model of the factors of tension's influences on its state. Thus, three flexibility dimensions are proposed: the degree of adjustment, the effort and the time necessary to achieve this adjustment. It is our contention that the model construction substends the analyst's conjecture about a system's inner logic calculating the expected states.*

**Key words:** *flexibility, multidimensional approach, field of variations, field of tensions, manufacturing.*

## 1. Introduction

Because an organization needs several types of flexibility, there are multiple definitions and evaluation schemes. The origins of this diversity may be linked to the variety of uncertainty factors, the possible time perspectives or the different possible dimensions to evaluate for flexibility (Gerwin,1993; Carlsson,1987). Consequently, there are several operational difficulties in achieving an unambiguous understanding of notions of flexibility (Pereira,95): intuitive definitions; misclassification, misdefinition and misevaluation; single-dimensional and transversal approaches, etc.

We propose an approach in which the system's flexibilities are specified by means of a frame of analysis. Our intention is to define clearly and simply what flexibility is, what a dimension of flexibility is and how it must be evaluated. Our aim is to characterize flexibility in a relational, multidimensional and dynamic way. We argue that this perspective conduces researchers and practitioners to an unambiguous definition of flexibility which enables them to better evaluation schemes.

The paper is organized as follows: in Section 2 our approach and the relevant concepts to be used are introduced. In Section 3, we introduce a single-line push-based manufacturing ordering system. Subsequently, in Section 4 we characterize flexibility in that ordering system to exemplify our approach. Finally, the conclusions are presented in Section 5.

## 2. The proposed approach

The flexibility is the capability of a system to adapt to changes that occur in its environment. This intuitive definition brings two questions: adaptation to what? and how? In order to answer these questions, we are going to consider the system as any logical and/or a physical device which we might wish to evaluate for flexibility. In according to Maturana and Varela (1984), the *adaptation degree* is properly a subject of an observer who evaluates the congruence between the system and its environment. In this evaluation he/she actually uses an implicit or explicit deviation function which tells him or her how much the system is adapted to its environment. Consequently, the adaptation capability will be attributed by an observer to a modelled system and then, the system flexibility will be a model-related property also.

For a given system's model, an observer will be compelled to investigate what is to change in the environment and the system, what is to be defined as the congruence between them, what is to be defined as the deviation function and how he can measure this deviation.

### 2.1 The field of variations

Let  $S$  be the *field of variations*, representing the set of states in which an observer accords to characterize the behaviour of a system and its environment through the trajectories that they take in. Let's suppose that he accepts to model  $R \subset S$  as the subset of realizable states of the system. Also, he defines  $E$  as the set of the states in which the environment *moves*.

Now, let  $s_t \in R$ ,  $e_t \in E$ ,  $s_t^* \in S$  be the observed current state of the system, the observed current state of the environment and the *expected* current state of the system, respectively. We will suppose that the observer assumes the existence, in the system, of a logic  $L$  such that:

$$L(e_t, s_t) = (s_t^*, |s_t^* - s_t|), \quad (1)$$

i.e., given  $s_t$  and  $e_t$ , the  $L$  logic allows to determine the expected state and the norm between  $s_t^*$  and  $s_t$ . Then, we can say that the system is in *partial equilibrium* when  $L(e_t, s_t) = (s_t^*, 0)$ , or not in partial equilibrium if  $|s_t^* - s_t| \neq 0$ .

#### Definition

*If  $|s_t^* - s_t| \neq 0$ , then flexibility is the property that tends to realize the partial equilibrium in the system.*

Thus, a flexible system has the capability to *adjust* its current state in response to the deviation  $|s_t^* - s_t|$  and the observer models the relationship between the system and its environment by the construction of a logic  $L$ . Note that, in this perspective, the system does not adjust to the environment, but to the  $L$ -defined expected states.

Let  $D = s_d^*, \dots, s_f^*$  be the succession of expected states as determined by  $L$  between two arbitrary periods  $d$  and  $f$ . In general,  $D \not\subset R$ . Additionally, let  $F = s_d, \dots, s_f$  be

the succession of states adopted by the system when it seeks to adjust to the  $D$  succession. Whatever an adjustment degree would be defined, it should consider a measure of similarity between  $D$  and  $F$ .

## 2.2 The field of tensions

Let the *factors of tension* be the set of one or more factors of resistance to change which imply an effort and time interval for the adjustment. Let  $\Omega_i^S$  be the set defining the variations of the factor of tension  $i = 1, \dots, n$  when the field of variations  $S$  is considered. We define the *field of tensions* as  $\Omega^S = \bigcup_{i=1}^n \Omega_i^S$ . Additionally, we define the *level of tension* by a function  $T$  such that  $T : S \rightarrow \Omega^S$ . Therefore, any system state transition on  $S$  implies a change of the level of tension. Because of resistences, this change occurs with an effort and a time interval. Thus, those time and effort must be considered in relation to a specific adjustment situation, what we call the *longitudinal approach*, and not independently of it, what we call the *transversal approach*.

## 2.3 The dynamical and relational flexibility

In a dynamic approach, the observer models the logic  $L$  to produce a succession  $D$  of expected states. Also, the system's *responses* to these demands are defined as a  $F$  succession, tracing the actual system moves. Whatever the transition of a  $s_i$  state to a  $s_{t+\Delta t}$  state be in  $F$ , it demands effort and time. Therefore, we say that the system dynamically adjusts to the *demanded* changes defined by the  $D$  succession.

The static approaches in the literature propose to relate the transition efforts to the  $R$  set (cf. Section 2.1). In such perspectives, we can't correctly associate dynamic efforts or time to  $R$ . In our approach, only a specific environmental process may explain time and effort of adjustment of the system. We claim that this is a congruent dynamical approach to conceive flexibility. Thus, we propose a change of perspective that relates  $F$  (and not  $R$ ) to the system's efforts and times of transition. This implies moreover that flexibility is a relative property: it depends on a well specified environment. The following section presents an example to illustrate how this approach may be used.

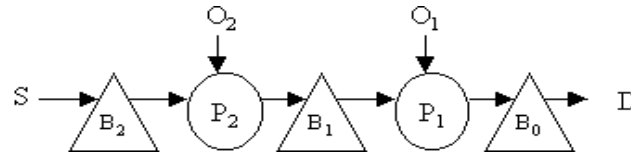
# 3. Example: a push-based manufacturing ordering system

## 3.1 Introduction

In this section, in order to illustrate the proposed approach, a model of one push-based manufacturing ordering system is introduced. The push ordering method is well known in the literature and different articles have been dedicated to model, evaluate and simulate it (Krajewski *et al.*,1987; Shingo,1983; Molet,1993; Takahashi *et al.*,1994; Pereira,1995). First of all, we are going to consider the ordering method as a management system working over a single-line manufacturing system's model. Thereby, we introduce the later and the involved variables. Then, the push model is defined.

### 3.2 The basic manufacturing system model

Let us suppose a single-line manufacturing system composed by a set of manufacturing stages and stockage sites in between. This system is represented in Figure 1:



**Figure 1. A single-line manufacturing system**

In Figure 1, the rough arrows indicate the product flow direction which is regulated, in quantity and delay, by production orders  $O_i$  ( $i = 1,2$ ) on each manufacturing stage  $P_i$ ; additionally, the production rates regulate the stockage sites  $B_i$ . A specific manufacturing management method establishes a regulation model to define these orders. In this way, two methods will be distinct if the production orders are calculated in a different manner (Crespo,1992). In order to introduce the model, we present below the variable notations:

- $i$  : index,  $i = 1,2,\dots$ ,
- $t$  : the time interval,
- $\tau$  : the manufacturing delay, the same on each stage,
- $D_t$  : the demand rate on stockage  $B_0$ , during  $t$ ,
- $\hat{D}_{t,t+i}^i$  : the  $t+i$  demand estimate, calculated at the end of  $t$ ,
- $\hat{D}_{t,acc}^i$  : the sum of the demand estimate (SDE), calculated at the end of  $t$ , for the stage  $P_i$ ,
- $\Delta\hat{D}_t^i$  : the marginal change of SDE, calculated at the end of  $t$ , for the stage  $P_i$ ,
- $O_t^i$  : the production order on the stage  $P_i$ ,
- $P_t^i$  : production rate on stage  $P_i$  during  $t$  placed on stock  $B_{i-1}$  at the beginning of  $t+1$ ,
- $B_t^i$  : stock level of  $B_i$  at the end of  $t$ ,
- $S^i$  : security level of  $B_i$ ,
- $EC_t^i$  : work-in-process level on stage  $P_i$ , calculated at the end of  $t$ ,
- $LT^i$  : accumulated manufacturing delay between stages  $P_i$  and  $P_1$ .

Therefore, a set of basic equations must be defined:

a) *The stock level on site  $B_{i-1}$ :*

$$B_t^{i-1} = \begin{cases} B_{t-1}^{i-1} + P_{t-1}^i - D_t & \text{if } i = 1 \\ B_{t-1}^{i-1} + P_{t-1}^i - P_{t-1}^{i-1} & \text{if } i > 1. \end{cases} \quad (2)$$

b) *Production rate<sup>2</sup> on stage  $i$ :*

$$P_t^i = O_{t-\tau}^i. \quad (3)$$

The variable  $\tau$  indicates the necessary delay before the production quantity arrives to the stockage site.

c) *The sum of the demand estimate (SDE):*

Noted  $\hat{D}_{t,acc}^i$ , this variable is defined by ( $LT^0 = 0$ ):

$$\hat{D}_{t,acc}^i = \sum_{j=1}^{\tau+1} \hat{D}_{t,t+LT^{i-1}+j}^i. \quad (4)$$

d) *The work-in-process level:*

Noted  $EC_t^i$ , this variable represents the production quantities ordered to the stage  $P_i$ , between  $t - \tau$  and  $t - 1$ , arriving to stockage site  $B_{i-1}$  after  $t - 1$ :

$$EC_t^i = \sum_{j=1}^{\tau} O_{t-\tau-1+j}^i. \quad (5)$$

### 3.3 The push method

Actually, the push method is a demand-estimate-based system: the production order for each manufacturing stage is calculated considering to a demand estimate function; whereas, in other methods like the pull systems (in its ideal-type conception; Pereira,1995) the production order on each stage is calculated only by the real demand rate. In this section, we will present the necessary equations to model the former. In Section 4, we will use these equations to determine the flexibility dimensions.

#### Proposition:

Let  $\Delta\hat{D}_t^1 = \hat{D}_{t,acc}^1 - \hat{D}_{t-1,acc}^1$  be the marginal change of SDE for the first stage, calculated at the end of  $t$ . If this manufacturing stage is managed under the push method, the production order is given by

$$O_t^1 = D_t + \Delta\hat{D}_t^1, \forall t. \quad (6)$$

*Dem:*

1) The production order equation at the end of  $t$  is given by (Pereira,1995)

$$O_t^1 = \hat{D}_{t,acc}^1 + S^0 - (B_t^0 + EC_t^1), \forall t. \quad (7)$$

Then, subtracting  $O_t^1 - O_{t-1}^1$  we have

$$O_t^1 - O_{t-1}^1 = \Delta \hat{D}_t^1 - \Delta B_t^0 - \Delta EC_t^1. \quad (8)$$

Additionally, it is easy to show

$$B_t^0 - B_{t-1}^0 = P_{t-1}^1 - D_t, \quad (9)$$

$$EC_t^1 - EC_{t-1}^1 = O_{t-1}^1 - O_{t-\tau-1}^1. \quad (10)$$

- 2) Therefore, the equations (9) and (10) finally imply  $O_t^1 = D_t + \Delta \hat{D}_t^1 + O_{t-\tau}^1 - P_t^1$ , which demonstrates the proposition. ■

In a push method, the equations for production orders in the upstream stages have a similar structure to the first stage, but they include a delay on the real and estimated demand; the next proposition establishes it.

**Proposition:**

Let  $D_{t-(i-1)\tau}$  be the demand rate on the first stage at the end of  $t - (i - 1)\tau$  and  $\Delta \hat{D}_{t-(i-j)\tau}^j$  be the marginal change of SDE, calculated for the stage  $j$  ( $1 \leq j \leq i$ ). If the stage  $i$  is managed under the push method, the production order is given by

$$O_t^i = D_{t-(i-1)\tau} + \sum_{j=1}^i \Delta \hat{D}_{t-(i-j)\tau}^j. \quad (11)$$

*Dem:*

Let us consider a recurrent procedure:

- 1)  $i = 1$ : The equation 6 establish the truth value for the first stage.

- 2)  $i = m$  ( $m > 1$ ): The recurrence hypothesis says:

$$O_t^m = D_{t-(m-1)\tau} + \sum_{j=1}^m \Delta \hat{D}_{t-(m-j)\tau}^j.$$

- 3)  $i = m + 1$ : In general, the production order for the stage  $i$  is defined by (Pereira,1995)

$$O_t^i = P_t^{i-1} + \Delta \hat{D}_t^i, \quad (12)$$

Then, one has  $O_t^{m+1} = P_t^m + \Delta \hat{D}_t^{m+1}$ . Furthermore,  $O_{t-\tau}^m = P_t^m$ . Thus, we obtain

$$\begin{aligned} O_t^{m+1} &= O_{t-\tau}^m + \Delta \hat{D}_t^{m+1} \\ O_t^{m+1} &= D_{t-m\tau} + \sum_{j=1}^m \Delta \hat{D}_{t-(m+1-j)\tau}^j + \Delta \hat{D}_t^{m+1} \\ O_t^{m+1} &= D_{t-m\tau} + \sum_{j=1}^{m+1} \Delta \hat{D}_{t-(m+1-j)\tau}^j. \end{aligned}$$

which demonstrates the proposition. ■

#### 4. Flexibility in push methods

The results obtained in the precedent section will serve us to specify the constructs representing the flexibility dimensions. Thus, in the following sections, we develop our framework. Firstly, the field of variations and one adjustment measure are defined. Secondly, the field of tensions, the effort and the time dimensions are defined and analyzed.

##### 4.1 The field of variations: an adjustment measure

Actually, in a manufacturing system, we may define several fields of variations, each one related to one kind of expectations: the coupling of, production and demand rates, real and desired stock or real and desired work-in-process (Forrester,1969). Additionnally, several objectives may be defined for the ordering system and it may be evaluated in relation to the success or failing to reach them (Lenard,1995). Thus, the first choice to be made by the analyst is what is to count as the field of variations? In this particular instance, we will select the common space of changes of demand and production rates. Thereby, let  $S = \mathfrak{R}$  be this space. We are going to consider the demand rate process as the  $D$  succession of expected states and the production rate process as the  $F$  succession of the system responses (cf. Section 2.1). Then, to define an adjustment measure we must find a similarity function for the demand and production signals.

In Figure 2, three manufacturing stages are managed in a push ordering system. We may observe that there is no a great similarity between the production curves and the demand process, represented by a rough line<sup>3</sup>.

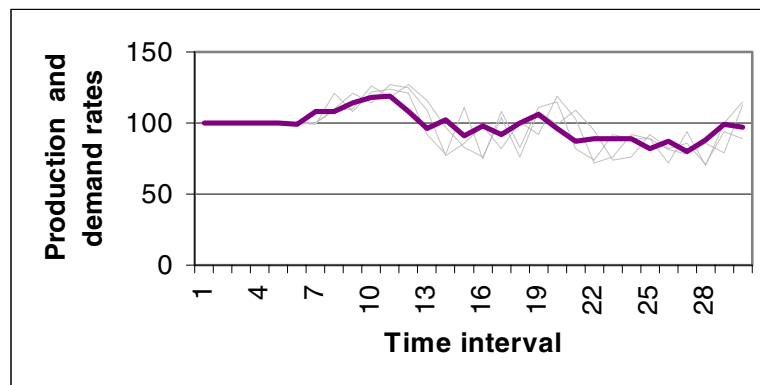
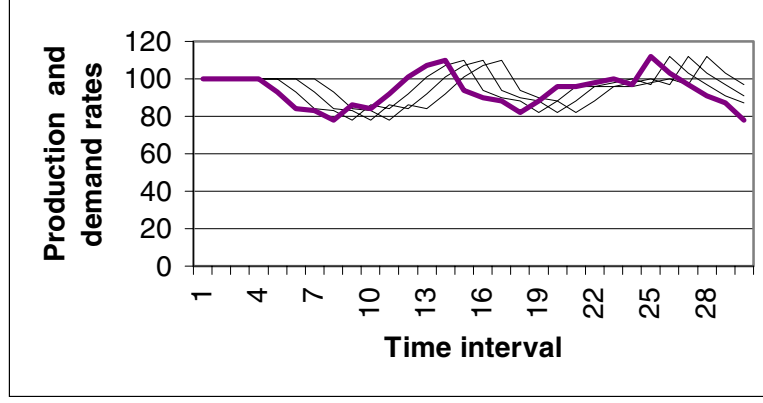


Figure 2. Production rates in a push ordering system (three stages)

In contrast, in Figure 3, the same three stages are managed in a pull ordering system. In this case, we may appreciate an astonishing similarity between curves, excepting the sliding effect caused by the production delay. According to these examples, an appropriate measure for the adjustment may be a similarity or dissimilarity demand-production indicator. However, it is important to take into account the production delay. In fact, we know that  $P_t^i = O_{t-\tau}^i$ .



**Figure 3. Production rates in a pull ordering system (three stages)**

Then, the difference (cf. Equation (11))  $P_t^i - D_{t-i\tau} = \sum_{j=1}^i \Delta \hat{D}_{t-(i-j+1)\tau}^j, \forall t$  may give us an idea of the gap between the expected state  $D_{t-i\tau}$  and the system's response  $P_t^i$ . In general, the production rate is defined by  $P_t^i = D_{t-i\tau} + \theta_t^i$ , where  $\theta_t^i$  depends on the ordering system. In other words, the distance variable between the production and demand rates corresponds to  $\theta_t^i = P_t^i - D_{t-i\tau}$ . Then, we define the adjustment degree of the stage  $i$ , in relation to the demand rate, by the following expression (the condition  $E[\Delta \hat{D}_t^i] = 0 \Rightarrow E[\theta_t^i] = 0, \forall i, t$ , must be satisfied):

$$\vartheta^i = \frac{V(\theta_t^i)}{V(D_t)}, i \geq 1. \quad (13)$$

Now, we have  $t - (i+1-j)\tau = t - \tau - (i+1-1-j)\tau$ , then, in order to obtain a developed expression for the adjustment degree, we can establish:

$$\theta_{t-\tau}^{i-1} = \sum_{j=1}^{i-1} \Delta \hat{D}_{t-(i+1-j)\tau}^j, i \geq 1, \quad (14)$$

which will lead to

$$\theta_t^i = \theta_{t-\tau}^{i-1} + \Delta \hat{D}_{t-\tau}^i, i \geq 2.$$

Note that a pull method is characterized by the absence of the demand estimate term, that is,  $\theta_t^i = 0, \forall i, t$  (Pereira,1995). Also, we may consider an hybrid ordering system in which the first stage is managed in a push method and the upstream stages in pull. In Table 1, the  $\theta_t^i$ 's for these three ordering systems are presented. It should be noted that the push method induces an upstream propagation of the demand estimate signal.

Stage	Push	Hybride	Pull
$i = 1$	$\Delta \hat{D}_{t-\tau}^1$	$\Delta \hat{D}_{t-\tau}^1$	0
$i > 1$	$\theta_{t-\tau}^{i-1} + \Delta \hat{D}_{t-\tau}^i$	$\theta_{t-\tau}^{i-1}$	0

**Table 1. Distance to demand rate in different ordering systems.**

Indeed, Table 1 suggests that, it suffices to determine the variances for the push method and the other methods are resolved. Thus, the variance on the stage  $i > 1$ , for the push ordering system, corresponds to  $V(\theta_t^i) = V(\theta_{t-\tau}^{i-1}) + V(\Delta \hat{D}_{t-\tau}^i) + 2 \text{cov}(\theta_{t-\tau}^{i-1}, \Delta \hat{D}_{t-\tau}^i)$ . Using the equation (14), let us define the



following terms:

$$G = \frac{V(\Delta\hat{D}_{t-\tau}^1)}{V(D_t)},$$

$$H_i = \frac{V(\Delta\hat{D}_{t-\tau}^i) + 2\text{cov}(\sum_{j=1}^{i-1} \Delta\hat{D}_{t-(i+1-j)\tau}^j, \Delta\hat{D}_{t-\tau}^i)}{V(D_t)}.$$

In according to these expressions, the adjustments degrees are specified in the Table 2.

Stage	Push	Hybride	Pull
$i = 1$	$G$	$G$	0
$i > 1$	$\vartheta^{i-1} + H_i$	$\vartheta^{i-1}$	0

**Table 2. Adjustment degree in different ordering systems.**

When the manufacturing system is managed in a push method, the upstream stages potentially raise the dissimilarity between production and demand rates. As a result, the stages are not synchronized, whereas, in the pull or the hybrid methods, they are. These are the evident behaviours showed in Figures 2 and 3. Another conclusion may be obtained. It should be pointed out that the production-demand distance measure depends on delay  $\tau$ . In consequence, the higher the  $\tau$  delay, the slower the ordering system will adjust production rates.

#### 4.2 The field of tensions: the effort and time consequences

Muramatsu *et al.* (1985) have proposed an amplification measure,  $V(P_t^i)/V(D_t)$ , which should characterize flexibility in a manufacturing stage. The desirable management systems satisfy  $1 \geq \text{Amp}^1 \geq \dots \geq \text{Amp}^n$ , where  $\text{Amp}^i$  ( $i = 1, \dots, n$ ). Indeed, the amplification evaluates the relative average raise or decrease of production rate. In fact, in a single time interval, several lot-related setup operations may occur. In that case, a lot-sizing problem may be resolved in which one of constraints imposes the demand satisfaction (Spence y Porteus, 1987). Thereby, the amplification ratio indicates the opportunity cost incurred by the ordering system when it fails to determine the optimal production rate. Then, the *adjustment effort* increases with the non unitary amplification.

It is clear that any variable contributing to the production costs may be considered as a factor of tension<sup>4</sup>: the facility availability for processing, the nominal and relative time setups, the direct setup cost, the fixed cost, the unit cost of production, the production lot-size, the opportunity cost of capital (DeGroot, 1994).

Now, we know that  $P_t^i = O_{t-\tau}^i$  and (cf. Equation (12)):

$$P_t^i = \begin{cases} D_{t-\tau} + \Delta\hat{D}_{t-\tau}^1 & \text{if } i = 1 \\ P_{t-\tau}^{i-1} + \Delta\hat{D}_{t-\tau}^i & \text{if } i > 1. \end{cases}$$

(15)

Thus, an expression for the variance of production rate may be found.

#### Proposition

*Let us consider a push ordering system and a stationary stochastic demand process satisfying the equation (15), then the variance of the production rate  $P_t^i$  is given by:*

$$V(P_t^i) = \begin{cases} V(D_t) + V(\Delta\hat{D}_t^1) + 2\text{cov}(D_t, \Delta\hat{D}_t^1) & \text{if } i = 1 \\ V(P_t^{i-1}) + V(\Delta\hat{D}_t^i) + 2\text{cov}(P_t^{i-1}, \Delta\hat{D}_t^i) & \text{if } i > 1. \end{cases}$$

(16)

Dem:

1) When  $i = 1$ , one has  $V(P_t^1) = V(D_{t-\tau}) + V(\Delta\hat{D}_{t-\tau}^1) + 2\text{cov}(D_{t-\tau}, \Delta\hat{D}_{t-\tau}^1)$ . Nevertheless, the stationarity hypothesis implies

$$\begin{aligned} V(\Delta\hat{D}_{t-\tau}^1) &= V(\Delta\hat{D}_t^1), \\ \text{cov}(D_{t-\tau}, \Delta\hat{D}_{t-\tau}^1) &= \text{cov}(D_t, \Delta\hat{D}_t^1). \end{aligned}$$

2) In the same manner, one concludes on the variance for  $P_t^i$  when  $i > 1$ . ■

It can be seen that the amplification ratios for the manufacturing stages may be deduced. Moreover, with this result, it is easy to find the variances expressions for the pull and hybrid cases. Next, to do this, let us define the following variables:

$$\begin{aligned} A &= \frac{V(\Delta\hat{D}_t^1) + 2\text{cov}(D_t, \Delta\hat{D}_t^1)}{V(D_t)}, \\ B_i &= \frac{V(\Delta\hat{D}_t^i) + 2\text{cov}(P_t^{i-1}, \Delta\hat{D}_t^i)}{V(D_t)}. \end{aligned}$$

The ordering systems behaviour (see Table 3) is very similar to the patterns found for the adjustment degree. It should be noted, however, that the amplification ratio is not null for the pull method. Again, the push method introduces an additional term in the upstream direction, whereas the hybrid ordering system introduces an initial term  $A$  which propagates upstream in the manufacturing line.

Stage	Push	Hybride	Pull
$i = 1$	$1 + A$	$1 + A$	1
$i > 1$	$Amp^{i-1} + B_i$	$Amp^{i-1}$	1

**Table 3. Amplification ratio in different ordering systems.**

Because of the potentially higher amplification of the push ordering system, we establish that, under the specified conditions, this method induces a larger adjustment effort than the other two (Takahashi *et al.*, 1994).

## 5. Discussion

We have proposed an approach that introduces a relational, dynamic and multidimensional conception of flexibility in manufacturing systems. In this approach, we define two fields of analysis: the field of variations and the field of tensions. In our approach, any analysis, evaluation or definition of a specific system flexibility, must begin by introducing two important inquiries: what's the field of variations on which flexibility is going to be observed? and, what is the field of tensions grouping the factors imposing resistances to changes on the field of variations? In order to obtain an operational model of flexibility, the analyst (observer) must define the vectors of the current and the expected states of the observed system. A model of the factors of tension relationships and their influences on the state of the system must be achieved to determine the adjustment degree, the effort and the time of this adjustment. It is our contention that the model construction sustends the analyst's conjecture in a system's logic which *calculates* the expected states of the system.

In the example of manufacturing ordering system, we have shown the necessity of a correct definition of the field of variations. Subsequently, we have proposed that, in the manufacturing ordering example, our adjustment degree measure differs from those presented in other articles (Kimura and Terada, 1981; Muramatsu *et al.*, 1985; Takahashi *et al.*, 1987; Takahashi *et al.*, 1994) because the amplification ratio is better apprehended as an effort indicator and not as a production-demand adjustment measure. Additionally, we have shown that the time factor is directly considered in the adjustment and

amplification ratios. A further analysis of time aspect may be found in Pereira (1995). The inner logic, in our case the defined push, pull or hybrid methods, strongly determines the flexibility evaluations. Thus, in Sections 4.1 and 4.2 we show that adjustment and amplification ratios depend on the demand estimate functions: the push and hybrid methods are very sensible to these functions, meanwhile the pull method is sensible only to the real demand rate.

We conclude that, in the manufacturing ordering system example, the proposed approach establishes a well structured framework to define and evaluate flexibility.

<sup>1</sup> This research is supported by a DIUT project, at the Universidad de Talca.

<sup>2</sup> Here, we suppose that there is no production shortages.

<sup>3</sup> The demand curve corresponds to an AR(1) random process, autocorrelated ( $\lambda = 0.8$ ), and the production delay is a constant number ( $\tau = 1$ ).

<sup>4</sup> An analysis model from these factors goes beyond the scope of this article. A flexibility approach to this problem has been undertaken in Pereira (1997).

## Acknowledgments

I wish to acknowledge Martin Schaffernicht, at the Universidad de Talca, for his critical comments on several aspects of this article.

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