

# A Heuristic Mathematical Method for Improving the Behavior of Forrester's Market Growth Model

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## Abstract

It is difficult for system dynamics models with high order and nonlinear property to straightforwardly obtain high leverage policies by mathematical method. In current literature, almost all of these mathematical methods were originated from the linear system mathematical skill of control engineering. They must simplify and linearize the system dynamics model first, and then use sophisticated mathematical skill to obtain a "guiding equation" for the simplified and linearized model; the equation then directs the design of high leverage policy for the original system dynamics model. However, the new mathematical method developed by this study neither needs to simplify and linearize the system dynamics model, nor uses sophisticated mathematical skill, but utilizes the insight from system archetypes and fundamental calculus to obtain a high leverage policy. The new method has been successfully applied to Forrester's "market growth model". The underlying idea is as follows. In the condition of the demand growth due to the dominance of the positive feedback loop, we fix "the adequacy of capacity to satisfy demand" to calculate how much capacity is able to prevent it from decreasing. If "the adequacy of capacity to satisfy demand" does not decrease, the negative feedback loop which inhibits the growth of the positive feedback loop will not be activated, and the growth can thus sustain. From the calculated capacity function, we can then derive high leverage investment policy. According to the new method, this study further developed a quantitative planning framework for dynamic growth. The framework can clearly calculate how to allocate critical resources to sustain corporation's growth.

## Introduction

There are three types of formal approach for obtaining high leverage solution in system dynamics: mathematical methods, optimal algorithms, and guideline methods. The mathematical methods are almost originated from the control theory (Talavage, 1980; Mohapatra and Sharma; 1985; Ozveren and Sterman, 1989). The user without profound mathematical background can not apply the mathematical methods. The study believes if we want to develop easier heuristic mathematical method, the system archetypes (Senge, 1990) will provide useful insight. Forrester's market growth model (Forrester, 1968) will be used as a basic example in the study. The

system archetype of “Growth and Underinvestment” will help develop an easier heuristic mathematical method to obtain high leverage solution for the model.

The paper will first introduce how we get the idea of this method from the insight of system archetypes. Then the solution procedure will be presented, followed with a comparison between the simulation results of the solution and the improvement policy suggested in Forrester’s original model. This method is also applied qualitatively to the case of People Express M.F.S. (Sterman, 1988). A general planning procedure for dynamic growth of firms is proposed at the end of the paper.

### Forrester s market growth model

The market growth model is shown in Figure 1. The names of variables and equations are identical to those of Forrester’s original model. Interested readers can refer to Forrester’s original paper (1968). The only difference is the elimination of the investment policy acting on PCO, because we will develop a high leverage investment policy acting on PCO. Besides, in order to meet the need of seeking solution, we will replace the Delay3 function of PCOO with three orders level variables.

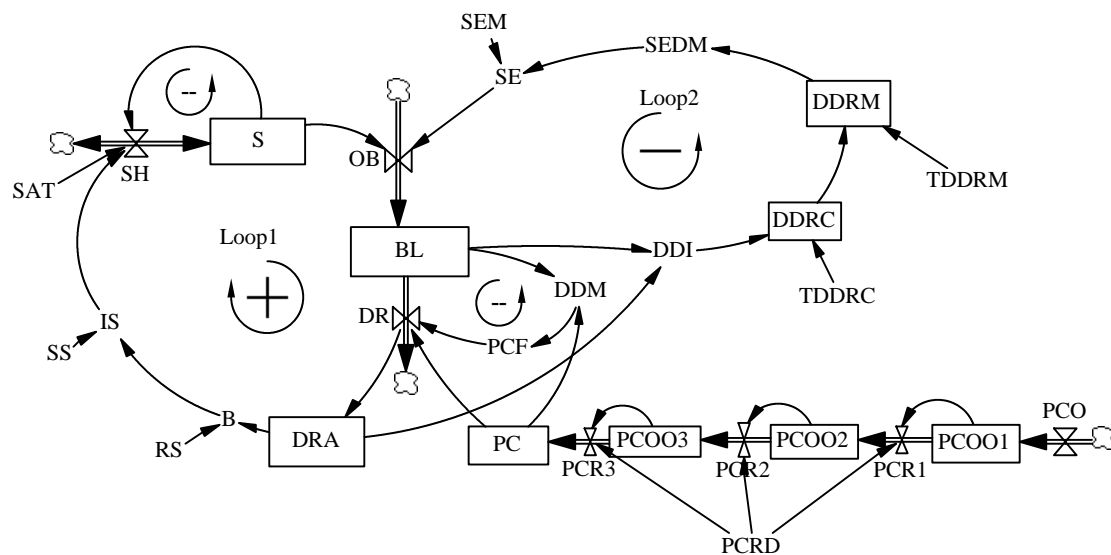


Figure 1: The flow diagram of the modified market growth model

### The solution concept for Forrester s market growth model

The system archetype “Growth and Underinvestment” is generalized from Forrester’s market growth model. The insight of this system archetype is that in order to have a sustained growth, the expansion of capacity should be fast enough to support an exponential growth of the positive feedback loop. However, owing to the delay for capacity expansion, capacity should be expanded in advance of growth. According to this insight, we first let the positive feedback loop grow without restriction from the connected negative feedback loop. Then we try to decide the required capacity for maintaining such growth trajectory. The key how to decide required capacity is to keep the adequacy of capacity to satisfy demand unchanged. The time delay in the

expansion of capacity is then considered, we can thus derive high-leverage investment policies for capacity expansion.

**The solution procedure of the heuristic mathematical method**

**Step 1:** Find the index of the adequacy capacity to derive the required capacity to fix that adequacy.

The adequacy of capacity for this model is DDI (=BL/DRA). Firstly, we will solve DDI in the given condition which the growth of the positive feedback loop will not be limited. This will be decided later by SE, a variable affects the positive feedback loop directly.

Given  $SE=SEM*SEDM$

$SEM=400$

$SEDM=f(DDRM)$

$DDRM=DDRC-TDDRM*dDDRM/dt$

$DDRC=DDI-TDDRC*dDDRC/dt$

Because DDI is fixed, the negative feedback loop (Loop 2) will maintain in a steady state. Therefore,  $DDRM=DDRC=DDI$ . So,  $SEDM=f(DDRM)=f(DDI).....(1)$

And given  $DDI=BL/DRA.....(2)$

$DRA=DR-DRAT*dDRA/dt$

Suppose DRAT is very short, it can be ignored. Therefore,  $DRA=DR$

Given  $DR=PC*PCF.....(3)$

$PCF=g(DDM).....(4)$

$DDM=BL/PC.....(5)$

Substituting Eq.(4) into Eq.(3), and substituting Eq.(3) into Eq.(2),

we obtain  $DDI=BL/(PC*g(DDM))=DDM/g(DDM).....(6)$

Substituting Eq.(6) into Eq.(1), We have  $SEDM=f(DDI)=f(DDM/g(DDM))$

$SEDM=f(DDM/g(DDM))$  will reach its maximum value; as a result, it will reduce the influence of the negative feedback (Loop 2) on the positive feedback loop.

The graphic function of  $SEDM$  is a monotonically decreasing function, so the smaller  $DDM/g(DDM)$  is, the bigger  $SEDM$  will be.

**Obtain min DDM/g(DDM):**

For graphic function  $PCF=g(DDM)$ ,

when  $DDM$  is within  $[0,1]$ , the graphic function will be a straight line.

when  $DDM$  is within  $[1,5]$ , the graphic function will be a curve. Substitute every point on the curve into  $DDM/g(DDM)$  and compare its value, we will realize that when  $DDM$  is within  $[0,1]$ ,  $DDM/g(DDM)=2$  is the minimum value.

And since  $DDM=BL/PC$ , and  $DDM \in [0,1]$ , we are able to assume there are unlimited relations between  $BL$  and  $PC$ . However, the highest production efficiency will occur when  $BL=PC$ .

Under the known growth situation, we obtain the needed capacity equal to the growth, that is,  $BL=PC$ .

**Step 2:** Consider time delay during the process of capacity expansion to derive required capacity for investment policy.

Given  $BL=PC$ , we will solve for  $PCO$

$$BL=PC \Rightarrow dBL/dt=dPC/dt$$

Given  $dBL/dt=OB-DR$ , let  $OB-DR=y$

Therefore,  $dPC/dt=PCR3=y$

$$PCR3=PCOO3/(PCR/3)$$

$$\Rightarrow PCOO3=(PCR/3)y$$

$$dPCOO3/dt=PCR2-PCR3=PCOO2/(PCR/3)-PCOO3/(PCR/3)$$

$$\Rightarrow (PCR/3)y' = PCOO2/(PCR/3) - y$$

$$\Rightarrow PCOO2 = (PCR/3)y + (PCR/3)^2 y'$$

$$dPCOO2/dt=PCR1-PCR2=PCOO1/(PCR/3)-PCOO2/(PCR/3)$$

$$\Rightarrow (PCR/3)y'' + (PCR/3)^2 y'' = PCOO1/(PCR/3) - (y + (PCR/3)y')$$

$$\Rightarrow PCOO1 = (PCR/3)y + 2*(PCR/3)^2 y' + (PCR/3)^3 y''$$

$$dPCOO1/dt=PCO-PCR1=PCO-PCOO1/(PCR/3)$$

$$\Rightarrow (PCR/3)y'' + 2*(PCR/3)^2 y'' + (PCR/3)^3 y''' = PCO - ((PCR/3)y + 2*(PCR/3)^2 y' + (PCR/3)^3 y'')/(PCR/3)$$

$$\Rightarrow PCO = y + 3*(PCR/3)y' + 3*(PCR/3)^2 y'' + (PCR/3)^3 y'''$$

High Leverage Investment Policy

$$PCO = (OB-DR) + 3*(PCR/3)*d(OB-DR)/dt + 3*(PCR/3)^2*d^2(OB-DR)/dt^2 + (PCR/3)^3*d^3(OB-DR)/dt^3$$

The initial value of  $PCOO1$ ,  $PCOO2$ ,  $PCOO3$  are as follows:

$$PCOO1 = (PCR/3)y + 2*(PCR/3)^2 y' + (PCR/3)^3 y''$$

$$PCOO2 = (PCR/3)y + (PCR/3)^2 y'$$

$$PCOO3 = (PCR/3)y$$

### Simulation results and comparison with Forrester s results

The above investment policy derived from the heuristic mathematical method will then substitute the investment policy affecting  $PCO$  in the original model, and change the initial values of  $PCOO1$ ,  $PCOO2$  and  $PCOO3$ . Figure 2 shows the result of simulation: both orders booked and backlogs have smooth exponential growth without fluctuation. The delivery delay keeps at two month. Figure 3 shows a comparison of the results of this method with those of the base run and new policy of the original model. The comparison of orders booked which representing the index for profitability shows that the results of this method is apparently better than that of the original model. The comparison of sales effectiveness which representing the index for potential growth in the future again shows that the result of this method is clearly better than that of the original model.

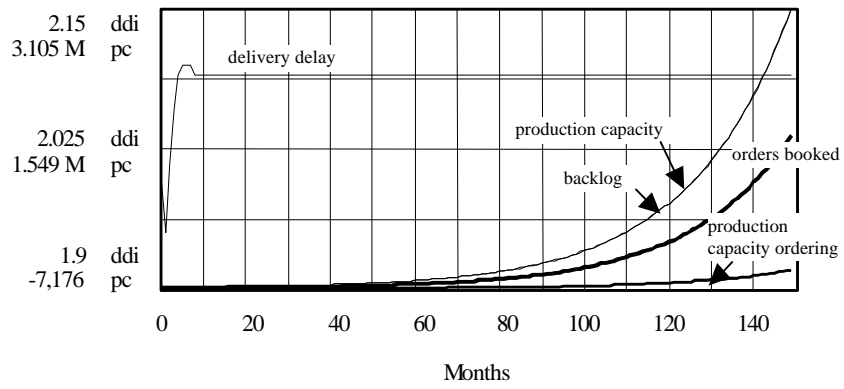


Figure 2: Simulation results of the policy derived from this method

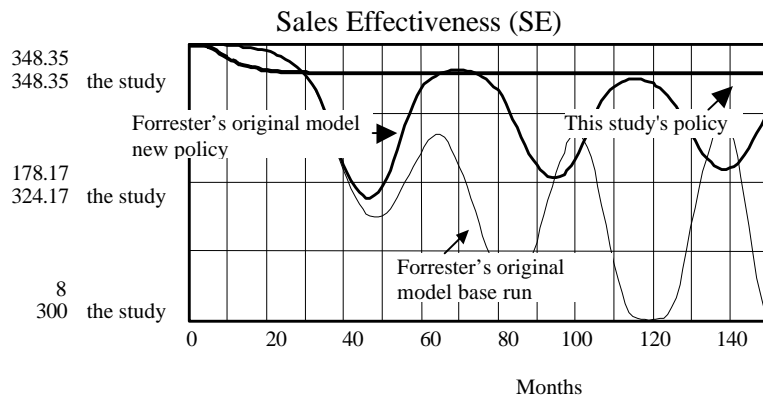
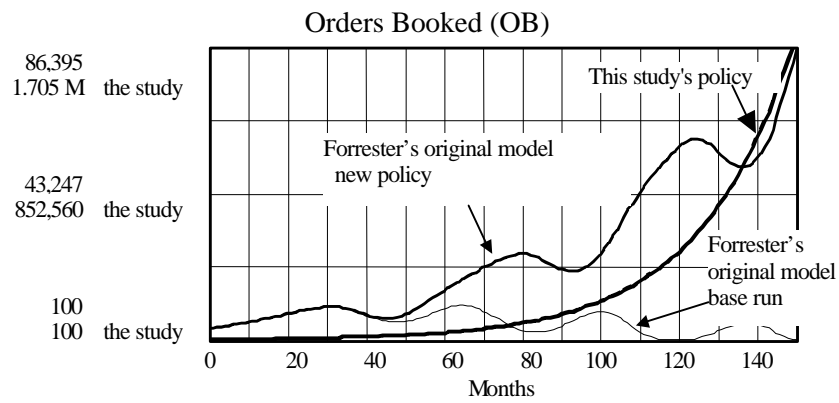


Figure3: Comparison with Forrester's results

### **A qualitative application of this method to People Express M.F.S.**

This method can also be applied qualitatively to cases with characteristics of “Growth and Underinvestment” even when quantitative equations are not available. Here we will take People Express M.F.S. as an example to illustrate a qualitative application of this method. As to the actual solution of high leverage, equations of the original People Express M.F.S. model are required.

Figure 4 shows the essential causal loop diagram constituted by variables of People Express M.F.S. Here we only consider two major decision-making points: buying aircraft and hiring, neglect the other decision points. Meanwhile, we assume that growth strategy is adopted in the model, that is to say, the behavior of buying aircraft will be processed according to certain percentage of revenue. Thus, a positive feedback loop for growth will be formed. Under this circumstance, if there is no hiring policy to meet the need of rapid growth, workload of workers will be too heavy and service capacity will not be enough. The average workweek will cause worse service quality that will reduce numbers of customers and hinder continuous growth. Problems of “Growth and Underinvestment” will occur. Since the structure of the People Express model and Forrester’s market growth model have characteristics of “Growth and Underinvestment”, so we may look for solution by our method.

We will employ this method to derive a high-leverage policy for hiring policy. There are two steps. First, we have to find out the index for the adequacy of capacity and its required capacity to fix the index. Here the index for the adequacy of capacity is average workweek. When there is a change in average workweek, it will trigger negative feedback loop of the service quality. It will also trigger two of what we coined “landmine” uncontrollable positive feedback loops (Young and Chen, 1996; Young, et al., 1997). One landmine is the turnover positive feedback loop; another landmine is the word of mouth positive feedback loop. If we can not figure out a way to fix them but let them increase, once a landmine is triggered, it will be very difficult to control the system.

Average workweek is determined by work load and service capacity. As long as we fixed workweek, we will be able to know service capacity through workload. On the other hand, workload is also determined by the positive feedback loop for growth. It is possible for us to obtain service capacity needed to maintain desired speed of growth of every time period through workload.

Taking into account the time delay of the learning curve for rookies, we also can use the required service capacity to calculate the number of workers needed at every time period and derive the high leverage hiring policy.

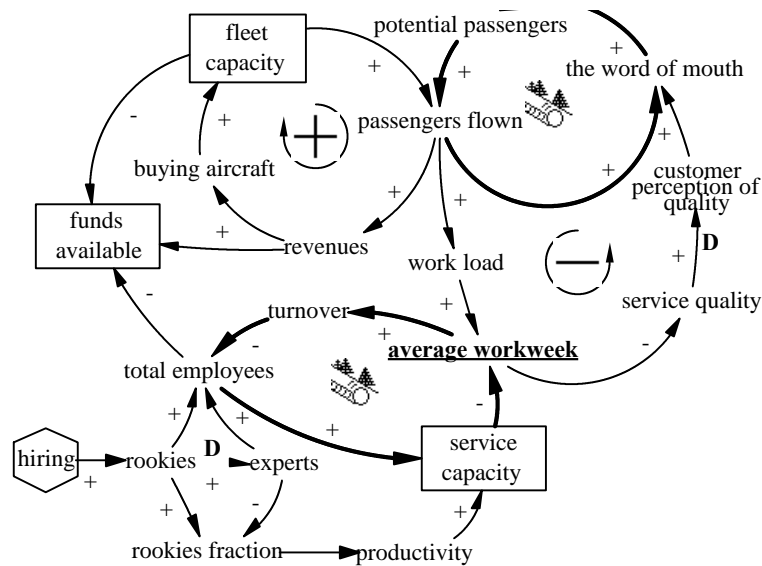


Figure 4: The essential causal loop diagram of People Express M.F.S.

### A proposal of a general planning procedure for dynamic growth

Growth is a major target for enterprises; however, it is quite often to encounter problems of dynamic complexity such as “Limits to Growth”, “Growth and Underinvestment” and “the attraction principle”. Current planning processes in management are quite static. Based on our heuristic mathematical method, this study proposes a general planning process for dynamic growth. There are three major steps of heuristic mathematical method.

#### Step 1: Identify three kinds of resources

Morecroft (1997) combines the resource-based view with system dynamics, which leads us to think more about the key level variables in a system. We borrow Morecroft’s idea as a base to identify three kinds of resources concerning the dynamic growth of a company: financial resources, service quantity resources and supportive quality resources. Financial resources are the available funds, which maintain in the company for investing in the other two resources. The financial resources will thus be converted to service quantity resources or supportive quality resources. Service quantity resources are the quantity directly used to serve the customers. Since the amount of customers will have direct influence on profitability, so it has the growth generating function for the company. For example, the service quantity resources in the market growth model are the quantity of salesmen, the service quantity resources in People Express M.F.S. are fleet capacity. Supportive quality resources are resources which will have major influences on the service quality to the customers, which will also support the service quantity resources to generate growth and profit. For example, in the market growth model, supportive quality resources are delivery delay, and in People Express M.F.S. they are service quality.

### Step 2: Create a positive feedback loop for growth

Create a policy so that when the financial resources increase, service quantity resources increase accordingly. When service quantity resources increase, the relative attractiveness remains strong, and more financial resources can be generated. When policy is well designed, financial resources and service quantity resources will both increase accordingly, they will thus create a positive feedback loop for growth. In the market growth model, a well designed policy can match sales revenue with the quantity of salesmen and create a positive feedback loop for growth.

### Step 3: Sustain the growth feedback loop

- (1) Determine a growth rate. Growth rate will directly influence the growing speed of the positive feedback loop. In the market growth model, RS is the service quantity variable.
- (2) Under above growth rate, calculate how much service quantity resources should be invested in each time period to increase the service quantity resources.
- (3) Estimate how much financial resources will be generated from the positive feedback loop for growth in each time period.
- (4) Estimate the evidence of growth resulted from the positive feedback loop in each time period. In People Express M.F.S., the evidence of growth is workload. In the market growth model, it will be backlog.
- (5) Find the proper index for the adequacy of capacity and calculate the supportive quality resources in each time period.
- (6) Take time delay in the process of expansion for the supportive quality resources into consideration to calculate how much financial resources have to be invested in each time period to obtain needed supportive quality resources.
- (7) Calculate balance of financial resources of each time period in (2), (3), (6) for the purpose of deciding the amount of loans.
- (8) Consider the costs of loans, that is interest rate, to find out the suitable growth rate under various interest rates.

Any growth will encounter its limits after a certain periods of time. Here we distinguish two kinds of limits, real limits and bottleneck limits. Real limits are the limits no matter how hard we work, we still have difficulties to overcome, such as saturation in the market and the actual limit of technology. Thus, the best solution will be innovating another growth loop and avoid those real limits. Bottleneck limits are formed when the expansion speed of the growth supportive factors cannot catch up with the growth itself. Hence, if the time and amount of expansion are adequate for supporting the growth, the bottleneck limits will be resolved. The planning procedure for dynamic growth suggested in this study is designed to be used for breakthrough in bottleneck limits. From the resources viewpoints mentioned above, bottleneck limits are formed when the expansion of the supportive quality resources can not catch up with the growth speed of the service quantity resources.

Actually, each service quantity resources can have more than one supportive quality resources. The matches among their amount and time delays during expansion are not the same. We need to consider various possible supportive quality resources. Otherwise, we will encounter bottleneck limits one after another in the process of growth.

Structural evolution will occur during growth. Along with the growth, supportive quality resources will form positive feedback loop and thus generating financial resources. Figure 5 shows that if we increase our investment to supportive quality resources according to the growth loop, then fitness will affect relative attractiveness and generate financial resources, and a new positive feedback loop for growth will be formed. Since the supportive quality resources are now in a positive feedback loop, its growth needs to incorporate with its own supportive quality resources. For example, in the case of People Express M.F.S., if the expansion of capacity will enhance service quality and create a new positive feedback loop for growth, its own supportive quality resources of service capacity will be management ability or the internalization of organizational culture. If its own supportive quality resources cannot be expanded, bottleneck limits will not be overcome either.

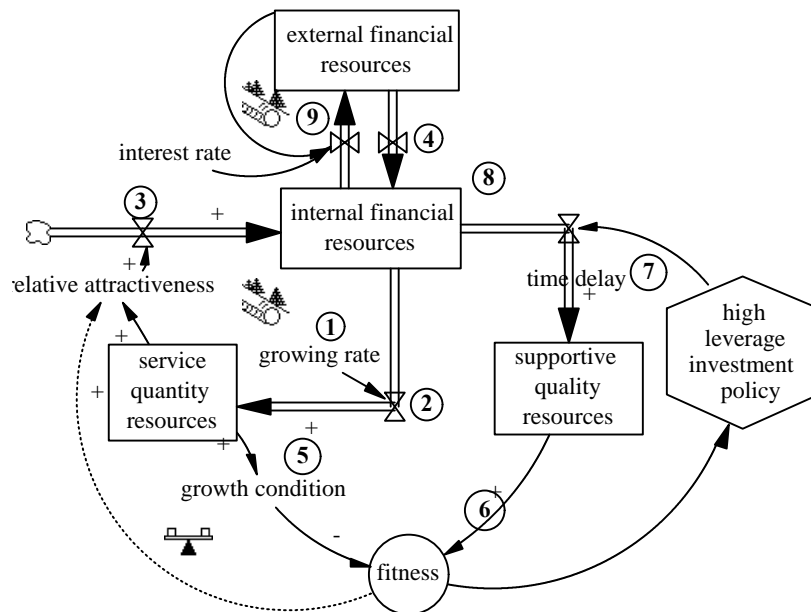


Figure 5 Flow diagram of the planning procedure for dynamic growth

### Conclusion and further research

Heuristic mathematical method developed in this research is based on the insight of the system archetype “Growth and Underinvestment”. The application of this method to Forrester’s market growth model which has the same structure has shown its effectiveness. The simulation result indicates that the policy derived through this method is much better than the policy suggested in the original model. And the policy is also very robust with respect to its support to the growth of the positive feedback loop at various growth rates. As to the case of People Express M.F.S., since its dominant structure is also “Growth and Underinvestment”, the concept of our method

is applicable too. However, useful high leverage policy can only be acquired through the equations of the People Express model. We believe that this method is able to be applied to solve various problems with “Growth and Underinvestment” dynamic complexity as the dominant structure to derive its high leverage policy.

If in the future this method can pass various tests, using the insight of system archetypes to conduct heuristic mathematical method for deriving high leverage policy may become a way worthwhile for more explorations. While solving differential equation in the mathematical field, techniques vary with various type of equations. This leads us to think that maybe we should give up developing a general approach which is applicable to various structures to derive high leverage solution, and models with different dominant structure should be solved through different mathematical method.

As far as the application in management is concerned, we combine the heuristic mathematical method and the resource-based view to propose a planning procedure for dynamic growth. For further studies, we believe that besides practical applications, the method itself needs further explorations. For example, when there are more than one kind of supportive quality resources, we need to consider the influences of interactions among supportive quality resources on the distribution of financial resources. Will the influence on relative attractiveness be added or multiplied? We think those issues are very interesting and deserve in-depth further studies.

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