Causality and Conjugate Variables in System Dynamics Modeling: Enhancements or Impediments

by
Maciej Zgorzelski, Professor of Mechanical Engineering
Timothy M. Cameron, Associate Professor of Mechanical Engineering
Kettering University (formerly General Motors Institute), Flint, MI 48504

Abstract

Causality has been condemned by many system thinkers. It is generally perceived as a synonym of reductionism, thinking in terms of simple cause-effect chains. System dynamics teaches that everything operates in the form of interacting feedback loops, and in these it is inherently impossible to distinguish between cause and effect. The balancing behavior of negative feedback, or the instability of positive feedback, is a structural property of a system, irrespective of which variable is considered the input to start the (computational) analysis. Classical system dynamics models, however, from Forrester and the early Dynamo, impose causality. One may argue that causality is incorporated because of computational requirements, nevertheless changing the input variable(s) requires creating a new model. Consequently, we can only determine reactions of a system to specific inputs (causal). We are unable to draw behavioral conclusions about a system based only upon its structure.

In the study of engineering systems these issues have been approached before. It is widely known that an electrical circuit diagram (without current direction arrows!) is an acausal model, which will exhibit oscillatory behavior (if it contains L and C elements), regardless of input choice. The technique of bond graphs used for physical systems is also inherently acausal, and it also approaches dynamic system modeling utilizing the concept of conjugate variables (such as voltage and current – the effort variable type and the flow type). This is a powerful concept in modeling multi-disciplinary engineering systems. The extensions to social and economic systems have never been fully explored. The present authors study the bond graph technique for modeling the dynamics of social systems, focusing on the causality and conjugate flows issues. This may lead to potential extensions of the current system dynamics modeling techniques.

Introduction

Causality, from the philosophical point of view, is generally perceived as a synonym of reductionism, of thinking about complexity of the world of systems in oversimplifying terms of straightforward cause-effect chains. The undisputed father of the General System Theory, Ludwig von Bertalanffy, wrote (back in 1955):

> *... In the world view called mechanistic, which was born of classical physics of the nineteenth century, the aimless play of the atoms, governed by the inexorable laws of causality, produced all phenomena in the world, inanimate, living and mental. No room was left for any directiveness, order or telos. ... The only goal of science appeared to be analytical, i.e., the splitting up of reality into ever smaller units and the isolation of individual causal trains. ... Causality was essentially one way: one sun attracts one planet in Newtonian mechanics, one gene in the fertilized ovum produces such and such inherited character, one sort of bacterium produces this or that disease....We may state as characteristic of modern science that this scheme of isolable units acting in one-way causality has proved to be insufficient. ... We must think in terms of systems of elements in mutual interaction.*[1]

To quote a more contemporary source - Peter Senge says in his famous *Fifth Discipline:*[2]

> *... Reality is made up of circles but we see straight lines. Herein lie the beginnings of our limitation as systems thinkers....In systems thinking feedback is a broader concept. It means any reciprocal flow of influence. In systems thinking it is an axiom that every influence is both cause and effect. Nothing is ever influenced in just one direction.*

System dynamics teaches us thus that everything in complex systems operates in the form of interacting feedback loops, and that in these it is inherently impossible to distinguish between cause and effect. The balancing behavior of a negative feedback loop, or the instability of the positive feedback, are both structural properties of systems, irrespective of which one of the variables describing the system we will consider the input to start the (computational) analysis.

Causality in Traditional System Dynamics Modeling

Classical system dynamics modeling techniques, however, since the time of Jay Forrester and the early Dynamo, impose causality. One may argue that this is the type of causality necessitated by the computational needs, and that this does not mean that system dynamicists adopt the simple cause-effect philosophy. We have to notice, however, that in classical system dynamics modeling, in order to change what we intend to consider as the input variable(s) of a problem we have to create a new, different system model. In consequence, we can only determine reactions of a model to specific inputs (i.e all our modeling results are causal). In effect we still remain within the realm of causal reasoning, the change we have made amounts only to replacing simple causal chains with feedback loops, wherever we consider this appropriate. We are unable thus to draw behavioral conclusions about systems based solely on their structure. We will return to this issue shortly.

Causality in Physics and in Physical Systems Analysis

In the study of physical and engineering systems some of the philosophical problems surrounding the causality issue have also been noticed. F. Cellier, H. Elmquist and M. Otter, known authorities in the field of engineering systems simulation techniques, write in a recent paper:

> *... Many engineers believe that physics is essentially causal in nature. Someone takes a conscious decision to affect the world in a particular way, thereby causing the world to react to his or her actions. Sir Isaac Newton followed the same line of reasoning when he formulated his famous law about* actio *being equal to* reactio. *... Yet, the distinction between* actio *and* reactio *is a deeply human and moral concept, not a physical one. There is no physical experiment in the world that can distinguish between* actio *and* reactio.*... The relationship between voltage across and current through an electrical resistor can be described by Ohm's law: $u=R*i$ – yet, whether it is the current flowing through the resistor that causes the voltage drop, or whether it is the difference between the electrical potentials on the two wires that causes current to flow is, from a physical perspective, a meaningless question.... State-space models are written in assignment*

> *statement form. There is always exactly one variable to the left of the equal sign, and the model implies that the expression to the right of the equal sign is evaluated, and the result of this evaluation is assigned as a new value to the left. Consequently, the modeler needs two different models to describe an electrical resistor: a* voltage-drop-causer *model and a* current-flow-causer *model. As mentioned earlier, from a physical perspective, this makes no sense whatsoever.*[3]

The authors of the above fragment touch upon some very critical questions here. These are the issues of the role of *conjugate variables* (i.e. current and voltage in the above example) in the analysis of physical dynamic systems, (which is very closely tied to the causality question), and the issue of *constitutive relations* (such as the resistance R of the resistor mentioned above) of the elements of a dynamic system, defining largely its behavior.

Bond Graphs for Physical Systems Modeling

Roughly at about the same time and in the same place that Jay Forrester had been working on the development of system dynamics modeling methodology for social and economic systems, another MIT professor, Henry Paynter started the development of the bond graph technique for modeling engineering systems[4]. This technique, further developed by many worldwide, is inherently acausal, and it approaches dynamic system modeling utilizing the concept of conjugate variables. This turned out to be a very powerful idea, in particular for modeling mixed-domain physical systems. The conjugate variables such as *voltage* and *current* in the electrical systems domain, *force* and *velocity* in mechanics, or *pressure* and *volume flow* in hydraulics– in bond graph terminology, are known as the *effort* variable and the *flow* variable. The product of effort and flow equals the *power* flowing through a *port* (cabling, hydraulic hose, force application point in mechanics) to (or from) the system *element.*

It is obviously impossible to summarize the wealth of concepts developed and research done on bond graph modeling of dynamic systems in this short paper. Extensive bibliography of this field can be found on the Internet[5]. Here, we present just a few basic concepts needed for further discussion. In this exposition we follow Karnopp and Rosenberg.[6]

The bond graph consists of *bonds* which carry *power* (represented by a half-arrow, or harpoon, indicating the direction of power flow which we assume to be positive, these arrows signify thus the power sign convention, not the causality). Power manifests itself in the conjugate effort and flow variables. There are three basic dynamic elements: a generalized resistor, a generalized capacitor and a generalized inertia, shown respectively in Fig. 1, 2 and 3.

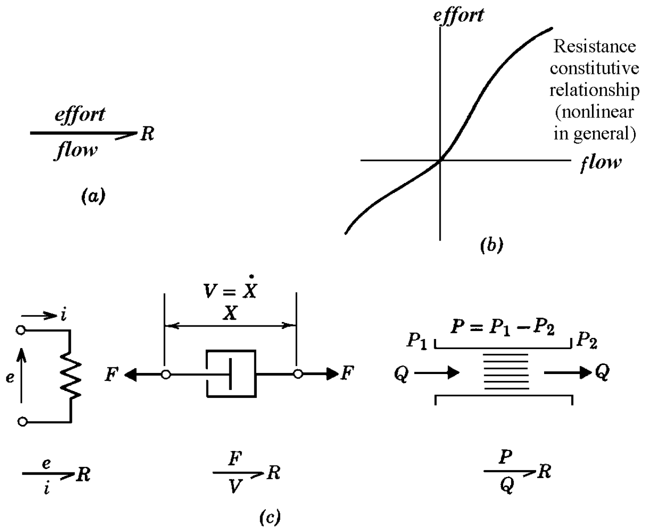

Figure 1.

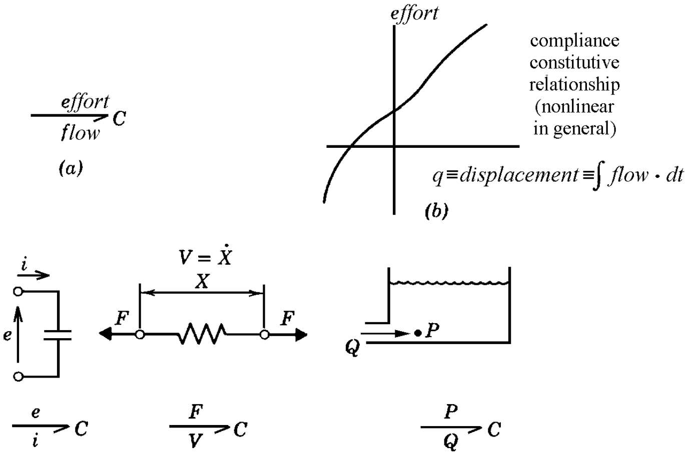

Figure 2.

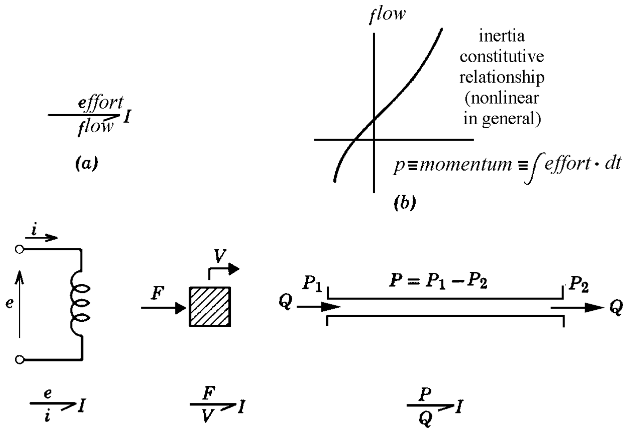

Figure 3.

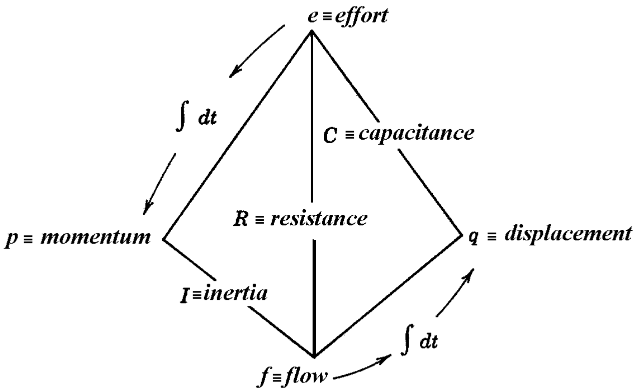

Figure 4.

Figure 4 shows the so-called tetrahedron of state, illustrating the relationships between four possible state variables. Besides effort and flow we have here their respective time integrals: a *generalized momentum* and a *generalized displacement*.

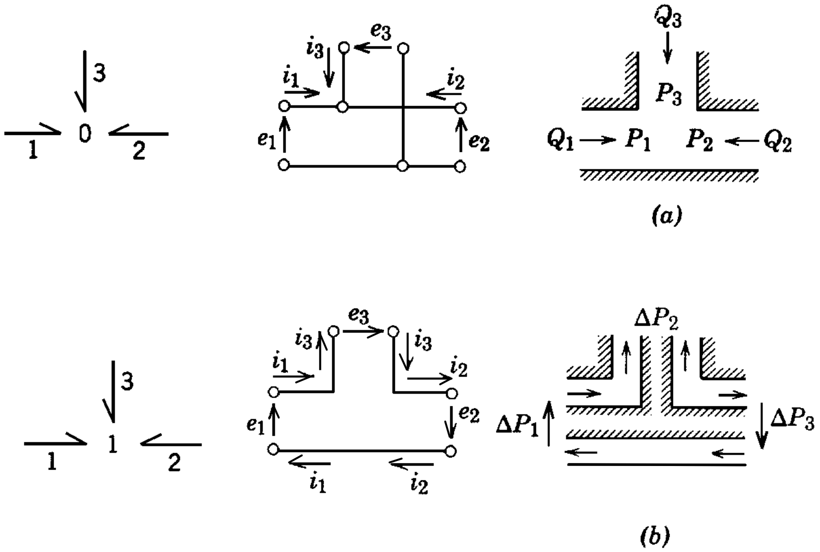

Figure 5.

The bond graph elements become connected using two types of junctions, known as 0 (zero) and 1 (one) junctions, illustrated in Fig. 5, which enforce the generalized Kirchhoff laws. There are several other types of elements: sources and sinks, transformers and gyrators. Bond graphs can also accomodate pure signal flows (e.g. a voltage signal for information transfer purposes) – we do need to get into any of these issues for the purposes of this paper.

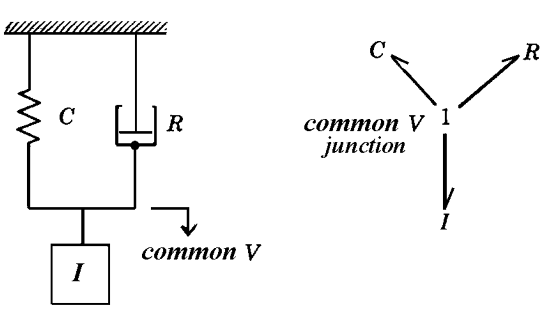

Figure 6.

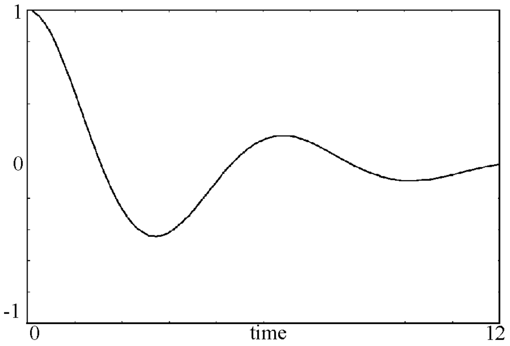

Figure 7.

Figure 6 illustrates the development of a complete bond graph model for a very simple case of a damped mechanical oscillator, and Fig.7 - a typical result - velocity of the common junction, as obtained using the bond graph (and control system) simulation software 20-Sim.[7]

For comparison we show in Fig. 8 the system dynamics model of the same damped oscillator problem, as developed in Vensim[8], and the same simulated result. The purpose of presenting both approaches here together lies not so much in comparing the results (which are, as one would expect, identical) but rather in comparing the approaches to model creation and underlying assumptions about causality, as well as about conjugate variables and about generalized Kirchhoff laws (0 and 1 junctions in bond graphs).

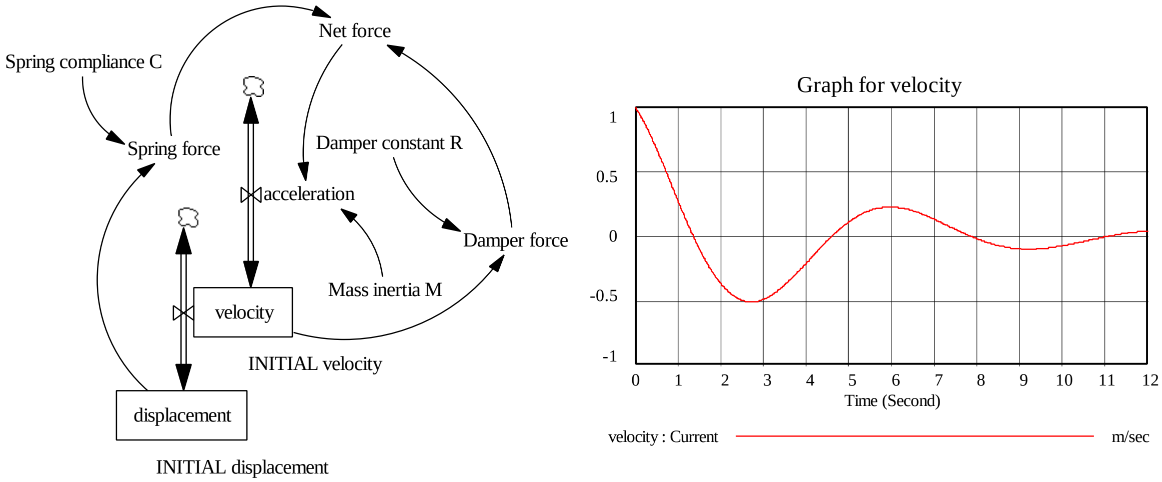

Fig. 8 System dynamics model of the damped mechanical oscillator. The same parameters were used as in the bond graph model shown in Fig. 6 and 7

It seems important to notice here that the bond graph model does not require the user to build feedback loops for the existing couplings between variables – the modeling technique takes care of this problem, as each of the bonds represents the conjugate flow (velocity in this case) and effort (mechanical force). The only things we have to specify are the three elements (inertia, compliance and resistance) and their constitutve relations.

Here these are the linear laws: *F=m*a* for inertia, *F=-q/C* for spring (where *C* is the spring's compliance value – inverse of the so-called spring constant) and *F=-R*v*, where *R* is the damping coefficient of viscous friction damper. (The relations used here are linear, but the bond graph technique is by no means limited to linear cases.) Besides the three elements and their constitutive laws, all we have to define are: the "1" junction, assigning a common velocity link to all three bonds, and – obviously – the initial conditions. *Derivation of equations of state, and assignment of causality – this one strictly for computational purposes – is done in modern bond graph systems by the computer system*. The modeling process, the resulting bond graph and the simulation results would have been identical for, say, an electrical oscillating circuit consisting of a capacitance, a resistance and an inductance.

Our intent is to show that many elements of the bond graph modeling technique could be incorporated into system dynamics modeling of social and economic systems, enhancing and – to a great extent – automating the modeling process, improving consistency of the models, as well as modelers' understanding of their inherent dynamics.

Approaches to Causality and Conjugate Variables

To illustrate the differences in approaches to causality and conjugate variables in classical system dynamics modeling and in bond graph models we will discuss the straightforward case of an inertia element – a simple mass *M* accelerated by force *F*, resulting in acceleration *a,* which may be inegrated to yield velocity *v* and displacement *q*. There will be *<u>two</u>* resulting different system dynamics models (Fig. 9) of this mechanical problem, depending upon its assumed causality: model (a) shown on the left assumes that force *F* is the input, velocity *v* the output, model (b) on the right assumes the opposite. Of course, in accordance with what we said earlier, physics is essentially acausal, and neither of these two models is more justified than the other.

What is modeled in classical system dynamics as a feedback mechanism is part of the inherent physical behavior of the system. The conjugate variables in the bond graph capture this. Consequently, in the bond graph technique, there is a useful distinction between the "reciprocal flow of influence" inherent in the dynamics of the system, and feedbacks that arise by human intervention, i.e. by design. Classical modeling techniques fail to recognize this distinction, and it is in their use of causal feedbacks to model the inherent dynamics – the reciprocal flow of influence between conjugate variables – that requires a different model for different boundary conditions (inputs) despite the constancy of the underlying system structure.

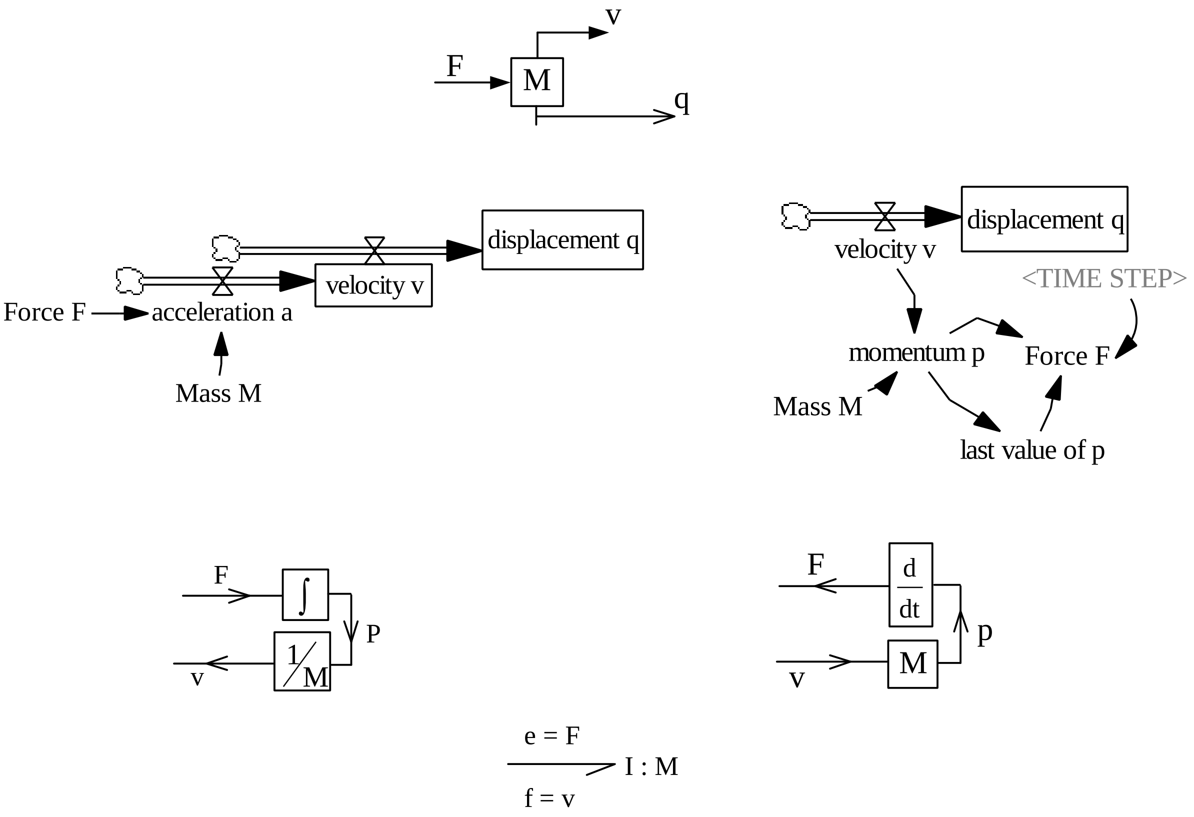

Figure 9. Two different models are needed in traditional system dynamics modeling methodology, depending on the causality of the problem. Block diagrams, used extensively in process automation, illustrated above are also causal. There is, however, only one bond graph model.

Application of Bond Graphs to Social and Economic Problems

It remains to show that bond graphs can in fact be applied to social and economic systems modeling. Some interesting results in that direction were obtained by Brewer.[9] He proposed using unit price p (\$/unit) of a specific product (or service) as the effort variable, and the flow of orders (units of product/unit time) as the bond graph flow f variable. Brewer's tetrahedron of state is shown in Fig. 10. Their product $p*f$ is the equivalent of power in engineering systems and obviously represents *cash flow.* The time integrals of f and p have interesting meanings: the integral of f is the accumulation of orders ("economic displacement", which, when negative, represents simply inventory of products), and the integral of p is something not recognized normally in economics, that Brewer names the *economic impulse.* The three basic bond graph elements – compliance C, inertia I and resistance R can be identified as representing, respectively: C – an inventory (also, as Brewer shows, a natural resource), I – is obviously the effect associated with the build-up of production (I represents the investment cost needed to increase production by one unit), and finally R – represents market effects. The other crucial element of the bond graph technique, the “0” and “1” junctions, represent the economic equivalent of Kirchhoff laws, Walras’ law.

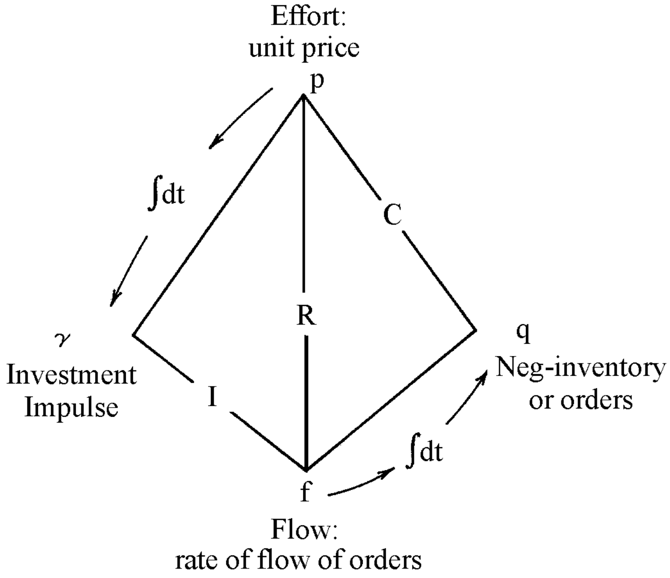

Figure 10.

The present authors believe that introducing some of these concepts into the practice of system dynamics modeling may greatly enhance the modeling efforts. Bond graphs use conjugate variables instead of feedbacks (obviously not eliminating feedbacks from where they really do belong: from economic control systems, i.e. company policies). This immediately leads the modeler to the conclusion that he/she *must* include financial considerations into the study of dynamics of economic systems: an inventory cannot be any more considered strictly as an accumulating level of products, monetary value *must* be assigned to it (the constitutive relationship of a compliance requires that). Similarly, build-up of production in an enterprise cannot be considered as a simple time delay: the conjugate variable of the cost of this build-up arises immediately. Finally, bond graphs are inherently acausal, so that changing the assumed input does not change the underlying model structure.

Besides the above outcomes, the inherent consistency of bond graph models (which assures the conservation of financial and material flows in a system), automated derivation of state equations by modern software, and automated (or, at least semi-automated handling of causality issues) make, in the present authors’ opinion an attempt to use bond graphs in economic and social system modeling – worthwhile undertaking.

---

[1] Ludwig von Bertalanffy: *General System Theory – Foundations, Development, Applications*; George Braziller, New York, 1995.

[2] Peter M. Senge: *The Fifth Discipline – The Art and Practice of the Learning Organization*; Doubleday, New York,1994

---

[3] Francois E. Cellier, Hilding Elmqvist, Martin Otter: *Modeling from Physical Principles.*

[4] Henry M. Paynter: *Analysis and Design of Engineering Systems*; MIT Press, Cambridge, MA, 1961.

[5] Bond graph bibliography is available at http://www.ece.arizona.edu/~cellier/bg.html

[6] Dean Karnopp, Ronald Rosenberg: *System Dynamics: A Unified Approach*; Wiley, New York, 1975.

[7] *20-Sim*, University of Twente, Controllab Products, P.O.Box 217, 7500 AE Enschede, The Netherlands

[8] *Vensim DSS ver. 3.0*, Ventana Systems Inc., 60 Jacob Gates Road, Harvard, MA 01451

[9] (a) John W. Brewer: *Structure and Cause and Effect Relations in Social System Simulations*, IEEE Trans. on Systems, Man and Cybernetics, June 1977, pp. 468 – 474.(b) John W. Brewer, Paul P. Craig, Mont Hubbard, Kenneth E.F. Watt: *The Bond-Graph Method for Technological Forecasting and Resource Policy Analysis;* Energy Vol.6, No.6, pp. 505-537, 1982. (c)John W. Brewer: *Progress in the Bond-Graph Representations of Economics and Population Dynamics*; J. of the Franklin Institute, Vol. 328, No. 5/6, pp. 675-696, 1991