Seamless Integration of System Dynamics into High School Mathematics: Algebra, Calculus, Modeling Courses

Diana M. Fisher, Franklin HS, 5405 SE Woodward, Portland, OR USA
Ron Zaraza, Wilson HS, 1151 SW Vermont, Portland, OR USA

Introducing system dynamics concepts is very natural in mathematics. The reform Calculus movement that has been in progress for ten years in the United States sets a useful backdrop for introducing systems. The reform movement has as its fundamental precepts the use of a four pronged approach to a conceptual understanding of Calculus and functions. To understand functions one must view them symbolically, graphically, numerically, and verbally. This is referred to as "the Rule of Four." Adding a fifth rule would provide a natural link to other disciplines and real applications. The fifth rule would be to view functions from a system dynamics perspective. This perspective is natural in Calculus. Functions can be viewed from their characteristic behavior-over-time/rate-of-change patterns. The "Rule of Five" can be implemented as early as Algebra I.

The introduction is most easily accomplished with the use of a motion detector, connected to an analog to digital converter interfaced to a computer. The computer is also connected to an overhead viewing system so the class can observe the activities. Students are asked to walk in front of the motion detector slowly and steadily or quickly and steadily. The graph produced is linear and a discussion centers around the characteristics of the motion that caused the graph to be linear. It is noted, via a questioning strategy (so the students make the determination) that the slope is dependent upon the speed. The connection between speed and slope is used as the foundation concepts for the study of all other functions from Algebra I through Calculus. Additional exercises are used to guide students to the obvious conclusion that, in order for a graph of motion to be non-linear there must be some acceleration/deceleration. Students know this, intuitively, but experiences crystallizing this concept are not usually provided in math classes. Students in Algebra I are then expected to interpret written explanations of movement into graphs, distance graphs into velocity graphs, and distance graphs into written explanations. In Algebra II students are also expected to continue this interpretation to include parabolic and oscillatory motion. Additionally they are expected to translate velocity graphs into corresponding distance graphs, velocity graphs into written explanations of motion, and velocity graphs into acceleration graphs. In pre-calculus classes the extensions include translation of acceleration graphs into corresponding velocity and/or distance graphs. As simple as these experiences may seem most students have not had concrete experiences in a math class, with the attendant vocabulary and reinforced connections that are so important to interpreting the equations and word problems that are found in the courses. The exercises truly crystallize for the average student, the connection between slope and speed that is fundamental to understanding Calculus. The exercises begin with motion producing straight line graphs and evolve to demonstrate first and second derivative concepts.

A vocabulary using a systems perspective can be developed using an intuitive set of exercises that most students find easy to understand. Some of the vocabulary is introduced in the motion detector activities. Using "characteristic behavior-over-time" in addition to "rate of change" to describe the standard linear, quadratic, exponential, and periodic functions is reinforced repeatedly. Using the motion detector as early experiences for students at each level allows repeated reference to the motions and their interpretations on the graph.

Lessons follow that use the method of finite differences, a numerical view, to reinforce the vocabulary introduced in the earlier motion exercises. Tables of values for linear, quadratic, and exponential functions are studied to show that linear functions have first differences that are always constant (first differences indicating velocity), quadratic functions have second differences that are always constant (second differences indicating acceleration), and exponential functions have first quotients that are always constant. Additional exercises are given where the function is not specified but the student must determine, via analysis on the tabular output of the function, it's characteristic behavior over time.

Finally, a modeling software, such as STELLA is introduced. With the vocabulary and rate-of-change concepts previously emphasized it is not difficult to expand problems to include a wider scope. The first set of lessons begin with problems that occur in most traditional math texts. Most standard Algebra, Pre-Calculus, and Calculus texts contain "word" problems that are suppose to provide students with applications of the functions they are studying. It is a simple task to choose those problems that involve time as the independent variable and create a handout where students design very simple STELLA models to solve those problems. Discussion with the class about the standard structure of the diagrams can refer to the earlier motion exercises. Once the standard diagrams are developed, students should be able to apply the correct diagram to the appropriate problem. This is not a very high level use of system dynamics, but is connects system dynamics to the traditional curriculum smoothly, providing a leverage point for expanding analysis of applications and functions via the system perspective in future exercises.

Students can, as a class exercise with the teacher, expand a simple problem. Again vocabulary is important. Assuming students have created simple STELLA models of the problems in the text, a problem of particular interest can be expanded/analyzed as an exercise with the entire class. Students have the opportunity to apply both growth and decay components to the same problem (something that is noticeably absent in most textbook problems before Calculus level). They can also combine functions within the same problem, applying for example, exponential growth and linear decay. Additionally students can be given exercises to expand the simple textbook problems into models on their own and explain their enhancements, thus providing a natural vehicle for including more written explanation in mathematics, as the US national math standards propose. It seems to be easier for students to explain STELLA models they have created, since the structure of the model is more closely connected to the application components than traditional symbolic representations of problems.

Once students have become accustomed to representing problems generally presented in their texts they become comfortable using lessons that introduce problems that would have been beyond the scope of the course, via traditional symbolic, numeric, or graphical expressions. For example, periodic functions are presented in most second year Algebra courses. The application problems usually accompanying this study often rely on study of Ferris Wheels, oscillating springs, and swings. While these are useful problems, there are other applications which may appeal to students who do not have a particular interest in these physics concepts. One such example is predator/prey interaction scenarios. Using a structured diagram approach, such as provided by the STELLA software, students are able to design a model and study it , answering the traditional questions about period, amplitude, including determining an appropriate symbolic representation for the model. Beyond this, however, students may extend their study to include potential problems that may arise in an ecosystem and test scenarios for controlling problems that may require legislation. Students could support certain legislation using experiments conducted on their model to provide rationale for their approach. Hence, now there is a connection between mathematics and the social sciences and/or law classes in the school, another objective of the US national standards for mathematics instruction.

STELLA models demonstrating the connection between exponential, convergent (Newton's Law of Cooling, as an example of convergent), and logistic structures illustrates beautifully the similarities and differences between these three growth patters. System dynamics and structured diagrams using STELLA illustrate elegantly the simplicity and connection between related function types. This view is not afforded by other methods.

The use of differential equations can be expanded formally in the development of models in a Calculus class. Generally differential equations is given very little time in most introductory Calculus classes. Unfortunately, this delays or (for many) eliminates the study of some of the most interesting applications in high school mathematics. Obviously, the study of models from the perspective of differential equation analysis was meant for system dynamics study using STELLA. As previously stated, providing experiences for students to study what may have been beyond their grasp via the traditional views is very powerful. Students can experiment with SIR infection models and Lotka-Volterra predator/prey models, among others. Designing and playing with models using STELLA provide useful insights into problem structures that provide deeper understanding when these topics are studied using more traditional methods later in a student's educational career.

In earlier classes (Algebra and Pre-Calculus) the vocabulary when studying functions can focus on the flow equations as a description of the behavior of the system over time. The system dynamics perspective, beginning as early as introductory Algebra classes, sets the foundation for those concepts that are at the core of Calculus. Starting with the motion detector and gradually studying and building STELLA models support what is currently being taught in mathematics. A course in modeling using a system dynamics approach is where the difference between what is currently being taught in high school mathematics and what can be taught is dramatically different. Students in system dynamics modeling classes have produced models and technical papers that are a quantum leap above the traditional work of high school students in math in the United States.

The ultimate development of a system dynamics view of problems is in a systems modeling course. Here students proceed through exercises, during the first half of the school year, that develop their ability to look at problems differently, to look at problems from varied disciplines, to develop simple models from scratch, and explain them to others. During the second half of the year students choose a partner with whom to work. They choose a problem they want to study. With the help of the instructor they find a reference/expert who understands the problem they want to model. Using the library, the Internet, various books, and various databases they try to collect data about the problem. They design a model, often finding they do not understand the problem well enough, or their expert cannot communicate effectively with them, or the data they found is inadequate or insufficient. Almost half of the students find they need to change their model topic in the first two weeks of data collection and early model construction. Once the groups have topics that appear to be appropriate the students work to design a model that represents the structure of the problem sufficiently and try to validate the results produced by the model. To validate their models students may use theoretical information, or their expert, or the data they have collected, or, if all else fails, a comparison of results that match reasonable expected behavior. Students then write a ten to twenty page technical paper explaining their model, how it works, what the graphs indicate, how they validated their results, and what they conclude. These papers never fail to impress all adults who have seen them, as they are far beyond the traditional expectations of students at high school level. For the past two years there has been a systems modeling competition, called SYM*BOWL, for high school students in Portland, Oregon. Students must explain their models to a panel of judges who are expert modelers.

At Franklin High School in Portland, Oregon the number of students taking the systems modeling class has increased from 11 to 60 in the last 5 years. At Wilson High School, also in Portland, the number has increased from a few students doing independent study to over 50 in the same amount of time. Three additional high schools will be adding a SD modeling course next year.