Determination Methods of Rate Variables Using Fuzzy Theories

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1 Introduction

In the real world, most rate variables are determined by human decision makings. However, in the system dynamics (SD) simulation method, a rate variable is often assigned to difference (or a ratio of the difference) between a level variable and its normal level (fig.1).

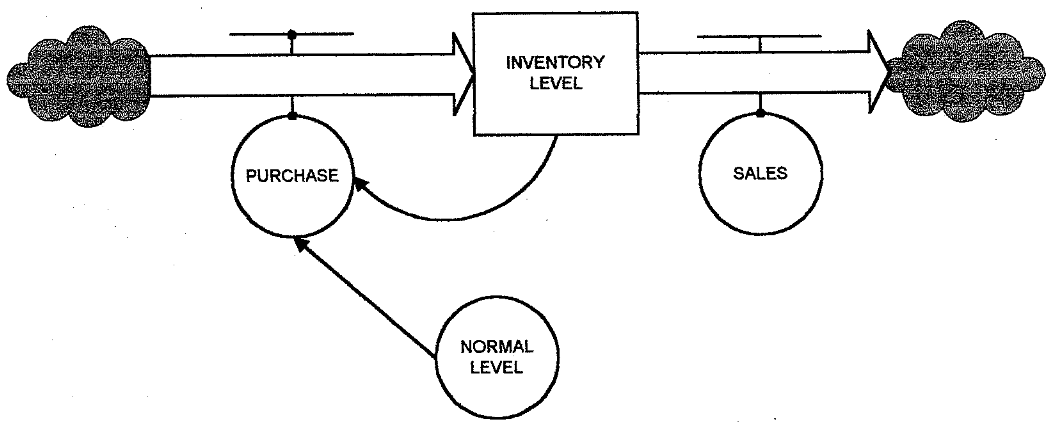

Fig. 1: Normal SD Model

Practically, most decision makers have some (fuzzy) rules and apply them depending on its situation. If an intermediate situation among the rules occurs, they interpolate the rules.

In this paper, to simulate such the human decision makings, we study to apply the fuzzy reasoning to determination method of rate variables. We discuss the design of fuzzy reasoning for SD in section 3. We show an application of this method using DYNAMO in section 4.

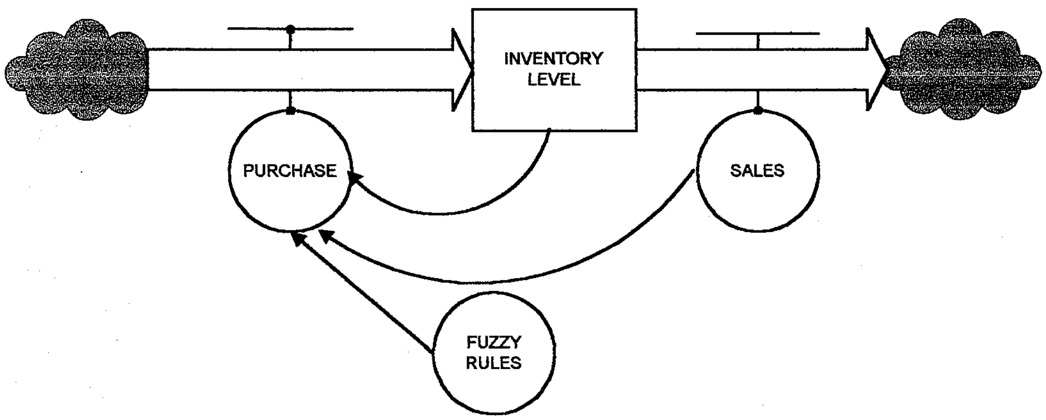

Fig. 2: Fuzzy SD Model

2 Fuzzy Reasoning

Taking an example to control an air conditioner, we will explain the most common fuzzy reasoning (fig.3).

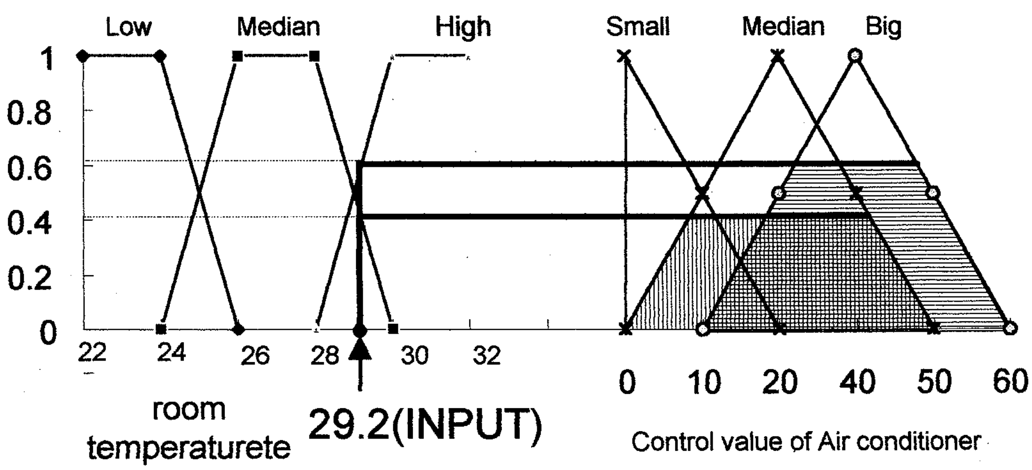

Fig. 3: Fuzzy Reasoning

Fuzzy reasoning have k control rules of form "R_i: IF x is A_i THEN y is $B_i(i = 1, \ldots, k)$" such as:

R_1: If the room temperature is high, then the output of the air conditioner is big.

R_2: If the room temperature is median, then the output of the air conditioner is median.

R_3: If the room temperature is low, then the output of the air conditioner is small.

$A_i(i = 1, \ldots, k)$ are fuzzy sets. For example, the membership function of "A_1(high)" is defined as

$$\mu_{A_1}(x) = \quad 1, \quad \text{if } x \geq 30, \tag{1}$$
$$\mu_{A_1}(x) = \tfrac{x-28}{2}, \quad \text{if } 28 \leq x < 30, \tag{2}$$
$$\mu_{A_1}(x) = \quad 0, \quad \text{if } x < 28. \tag{3}$$

"x" is a input such as the room temperature. $B_i (i = 1, \ldots, k)$ are also fuzzy sets. The output of the rule R_i is a fuzzy set C_i which is the B_i truncated at the a high value of $w_i = \mu_{A_i}(x)$, that is

$$\mu_{C_i}(y) = \min\{w_i, \mu_{B_i}(y)\}. \tag{4}$$

If the condition of the i-th rule is plural, that is $A_{i,1}, \ldots, A_{i,m}$, then w_i is

$$w_i = \min\{\mu_{A_{i,1}}(x_i), \ldots, \mu_{A_{i,m}}(x_m)\}. \tag{5}$$

When a input "x is 29.2" is given, the output of the rule R_i is calculated as

1. $w_i = \mu_{A_1}(29.2) = \frac{29.2-28}{2} = 0.6$

2. $$\mu_{C_1}(y) = \min\{0.6, \mu_{B_1}(y)\} = \begin{cases} 0, & \text{if } y \leq 10 \\ \frac{y-10}{20}, & \text{if } 10 < y \leq 22 \\ 0.6, & \text{if } 22 < y \leq 52 \\ 1 - \frac{y-40}{20}, & \text{if } 52 < y\,. \end{cases} \tag{6}$$

The output of all rules (the shaded portion of fig. 3) is the union of the output fuzzy sets of all rules, that is,

$$\mu_C(y) = \max\{\mu_{C_1}(y), \ldots, \mu_{C_k}(y)\}. \tag{7}$$

The overall output C is a fuzzy set, not a value. So, to use control, we must translate C to a scalar value. Most common method is center of gravity method. The overall output value y^* is

$$y^* = \frac{\int y \mu_C(y) dy}{\int \mu_C(y) dy}. \tag{8}$$

Above mentioned method is so-called min-max/center of gravity method. The algebraic product/addition/center of gravity method has also been used.

It is difficult for DYNAMO to calculate center of gravity, so we use simplified method. In this method, B_i is not fuzzy set, but a constant value b_i. Therefore, the equation(8) become

$$y^* = \frac{\sum_{i=1}^{k} w_i b_i}{\sum_{i=1}^{k} w_i}. \tag{9}$$

3 Design of Fuzzy Reasoning for SD

In [1], "design of fuzzy reasoning and rules is different for control and evaluation problems, since their natures are exactly opposite". SD has both natures. So, referencing [1], we study the design of fuzzy reasoning for SD.

In fuzzy control, "the purpose of fuzzy control is to set target controlling value v_o for variable v and have it stabilize at that value" [1]. Let v_k be observation at timepoint k , $E_k = v_k - v_o$ and $dE_K = E_k - E_{k-1}$. So, fuzzy control rules are depicted in tabular form.

		dE						
		NB	NM	NS	ZO	PS	PM	PB
	NB				NB			
	NM				NM			
	NS				NS			
E	ZO	NB	NM	NS	ZO	PS	PM	PB
	PS				PS			
	PM				PM			
	PB				PB			

```
E=Observation - target        dE=E-E(-1)
NB:NEGATIVE BIG               PS:POSITIVE SMALL
NM:NEGATIVE MIDDLE            PM:POSITIVE MIDDLE
NS:NEGATIVE SAMALL            PB:POSITIVE BIG
ZO:ZERO
```

Fig. 4: Fuzzy rules

In fig.4, total possible inputs are 49, but the control rules are 13 places. "The input for a region without a rule is interpolated by the existing rules" [1].

[1] points out 4 difference between control and evaluation case. So, we add SD case (Tables 1).

	Control	Evaluation	SD
1. Input space	Not homogeneous	Homogeneous	Homogeneous
2. Approach principle	Yes	No	Yes
3. Operation performance constraint	Exist	Not exist	Exist
4. Impossible input	Exist	Not exist	Not exist

Table 1: comparison among control, evaluation and SD

1. **Input space:** As the control has one target, so "emphasis during control is placed on reasoning (E, dE) when they deviate slightly from target values"[1]. Therefore, the control rules are set some places and the input space is not homogenous. In SD case, the target is often changed depending on input values. For example, the target of inventory level is changed depending on sales amount. So, the rules are set at all places.

2. **Approach principle:** In the control case, it is often impossible to closely approach target value with one computation, so operation is repeated many times to gradually reach the target value. Such the operational method is called "the approach principle". However, evaluation must reach the best value by one reasoning cycle. SD is the same as control.

3. **Operation performance constraint:** For control, the output is limited by the controller. Similarly, SD's rate variables are constrained by human ability, machine capacity or regulation and so on.

4. **Impossible Input:** In control, some input situation will never occur. For example, the room temperature is high and increasing shapely. In evaluation, all input situations may happen. Similarly, in the SD case, the situation that inventory level is low and decreasing shapely often happens.

Therefore, for SD, from homogeneous and no impossible inputs, every region must be covered by fuzzy rules like evaluation case. If every region is covered, duplication of rules is happen. To avoid the effect of the duplication of rules, we use min-max/center of gravity method. Because, if we use product and addition operator instead of max and min, output of rules has the same mean, so, its output is evaluated twice or more.

4 Application to Inventory Control Model

4.1 The outline of inventory control model

We now apply fuzzy reasoning to inventory control model as show in fig.5. Table. 2 show some fuzzy rules of this model and fig.6 show all rules.

Fuzzy rules are constructed as to avoid excess inventory, that is:

- If sales level is high then target inventory level is somewhat high.
- If sales level is low then target inventory level is very low.

Fig. 7 and 8 are fuzzy sets in this model. Table 3 show the output level b_i (constant) for the fuzzy labels.

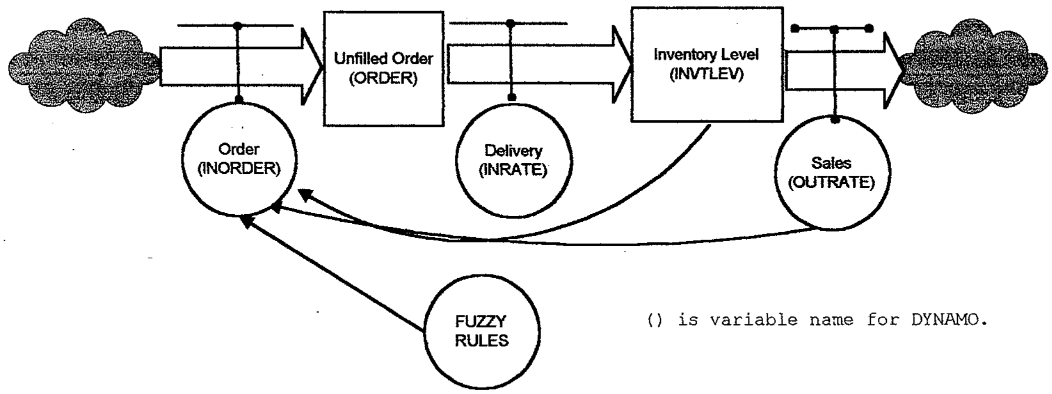

Fig. 5: inventory control model (fuzzy reasoning)

If the sales is very low and the inventory level is median, then the order is very low.
If the sales is median and the inventory level is median, then the order is median.

Table 2: Example of Fuzzy Rules

4.2 DYNAMO

Using the simplified method, we present a simulation method using DYNAMO. Fuzzy sets are described using DYNAMO's table functions. Following list is description of inventory level that corresponds fig. 7. INVENTLEV is inventory level and LEVMFV is $A_{i,1}$.

```
A    LEVMFV.K(ILEV)=TABHL(TLEVMFV(*,ILEV),INVTLEV.K,0,480,80)
T    TLEVMFV(*,LVL)=1,0,0,0,0,0,0
T    TLEVMFV(*,LLO)=0,1,0,0,0,0,0
T    TLEVMFV(*,LSL)=0,0,1,0,0,0,0
T    TLEVMFV(*,LMD)=0,0,0,1,0,0,0
T    TLEVMFV(*,LSH)=0,0,0,0,1,0,0
T    TLEVMFV(*,LHI)=0,0,0,0,0,1,0
T    TLEVMFV(*,LVH)=0,0,0,0,0,0,1
```

VL	0			SH	160
LO	40	MD	120	HI	200
SL	80			VH	240

Table 3: Output level for fuzzy labels

	SALES						
INVENTORY	VL	LO	SL	MD	SH	HI	VH
VL	VL	SL	MD	HI	HI	VH	VH
LO	VL	LO	MD	HI	HI	VH	VH
SL	VL	VL	SL	SH	HI	VH	VH
MD	VL	VL	LO	MD	SH	HI	VH
SH	VL	VL	VL	SL	MD	SH	HI
HI	VL	VL	VL	LO	MD	SH	SH
VH	VL	VL	VL	VL	LO	MD	SH

VL:Very Low　　MD:MeDium　　SH:Small High
LO:LOw　　HI:HIgh
SL:Small Low　　VH:Very High

Fig. 6: Fuzzy Rules

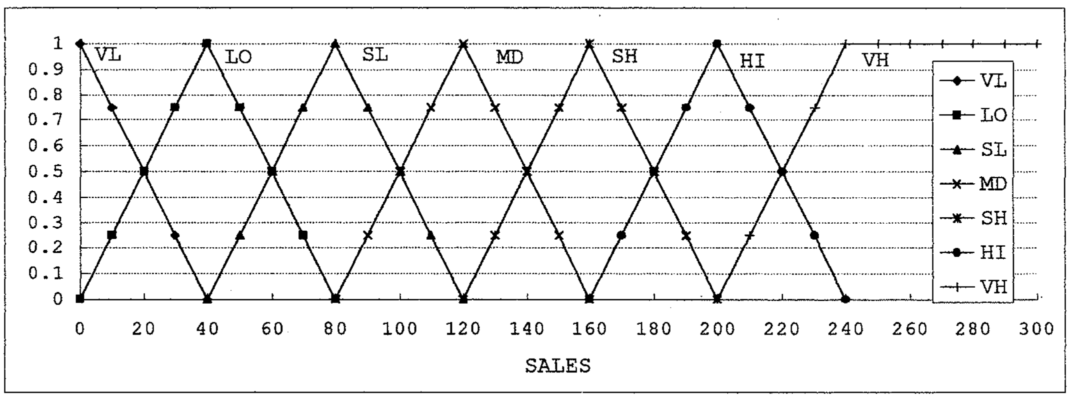

Fig. 7: SALES(fuzzy set)

Similarly, following is sales amount.

```
A    RATEMFV.K(IOUTRATE)=TABHL(TRATEMFV(*,IOUTRATE),OUTRATE.KL,0,240,40)
T    TRATEMFV(*,RVL)=1,0,0,0,0,0,0
T    TRATEMFV(*,RLO)=0,1,0,0,0,0,0
T    TRATEMFV(*,RSL)=0,0,1,0,0,0,0
T    TRATEMFV(*,RMD)=0,0,0,1,0,0,0
T    TRATEMFV(*,RSH)=0,0,0,0,1,0,0
T    TRATEMFV(*,RHI)=0,0,0,0,0,1,0
T    TRATEMFV(*,RVH)=0,0,0,0,0,0,1
```

As fuzzy rules are set for all combination of inventory level and sales amount fuzzy sets, fuzzy rules described as 2 dimension array of output amounts (b_i) .

```
I    KOKEN(*,LVL)=0,80,120,200,200,240,240
I    KOKEN(*,LLO)=0,40,120,200,200,240,240
```

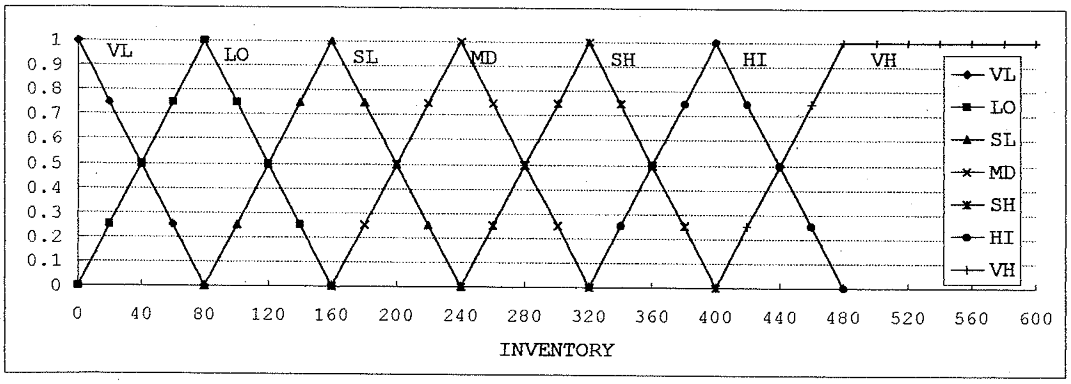

Fig. 8: INVENTORY(fuzzy set)

```
I    KOKEN(*,LSL)=0,0,80,160,200,240,240
I    KOKEN(*,LMD)=0,0,0,120,160,200,240
I    KOKEN(*,LSH)=0,0,0,80,120,160,200
I    KOKEN(*,LHI)=0,0,0,40,120,160,160
I    KOKEN(*,LVH)=0,0,0,0,40,120,160
```

MFV is w_i and WMFV is $w_i b_i$.

```
A    MFV.K(ILEV,IOUTRATE)=(MIN(LEVMFV.K(ILEV),RATEMFV.K(IOUTRATE))^
     *KOKEN(IOUTRATE,ILEV)
A    WMFV.K(ILEV,IOUTRATE)=(MIN(LEVMFV.K(ILEV),RATEMFV.K(IOUTRATE))
```

Lastly, the center of gravity is calculated.

```
R    INORDER.KL=SUM(SMFV.K)/SUM(WSMFV.K)
A    SMFV.K(ILEV)=SUM(MFV.K(ILEV,*))
A    WSMFV.K(ILEV)=SUM(WMFV.K(ILEV,*))
```

Complete list gives in supplement.

4.3 Simulation

For comparison, we make two models that are 1) "order = sales" model and "order = normal level - inventory - unfilled order" model.

As compared with fig.10 and 11, inventory level of fig. 9 looks like that of human's decision making. Because:

- When sales amount is high, inventory level is high (differently from fig. 10 and 11, the lack of inventory not happen).
- When sales amount decrease rapidly, excess inventory increases like human inventory control.

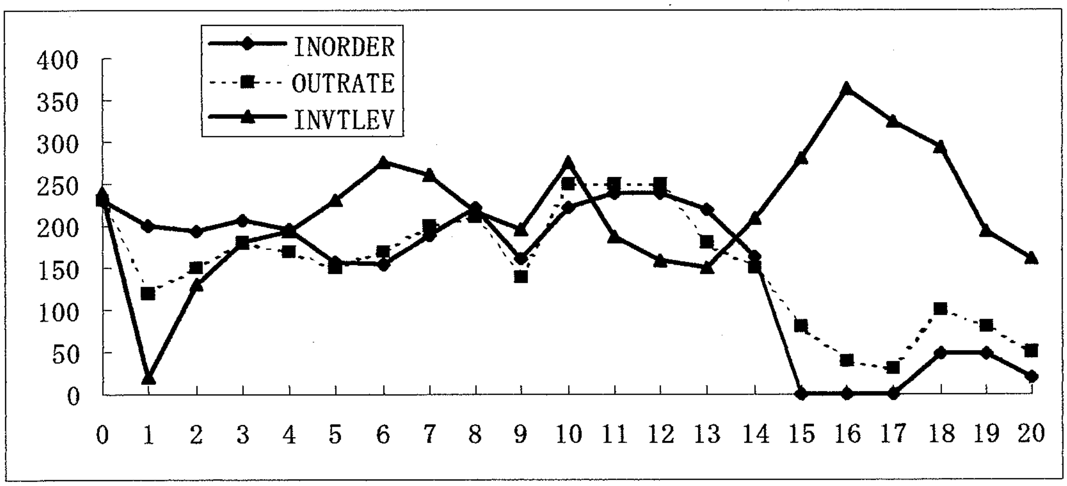

Fig. 9: Fuzzy Reasoning Model

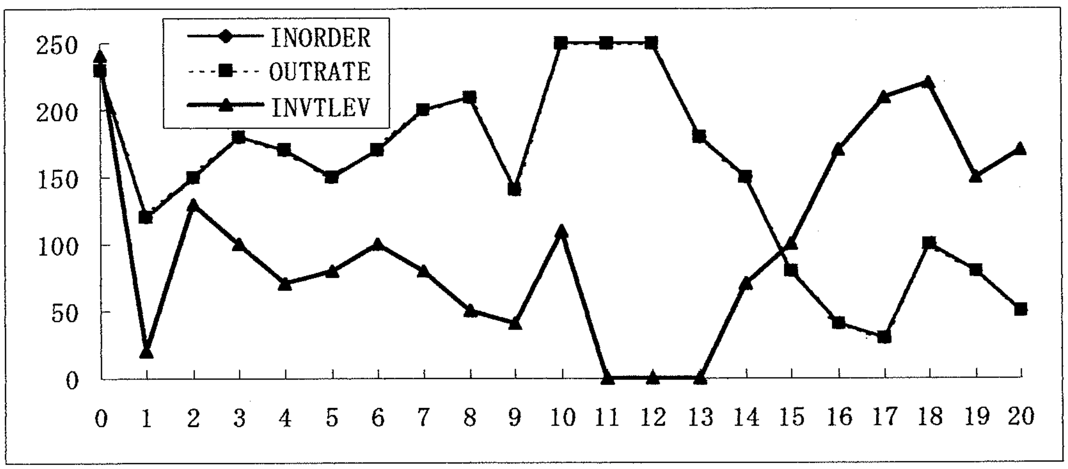

Fig. 10: "Order = Sales" Model

5 Conclusion

We present the way of using fuzzy reasoning in SD model, that are 1)some remarks of identify fuzzy rules, 2) the way of description using DYNAMO and 3) comparison with normal SD.

References

[1] Nakajima,N and Y.Hannya 1993. Fuzzy Evaluation by Fuzzy Reasoning and its Application. *Japanese Journal of Fuzzy Theory and Systems.* 5(4):517-531

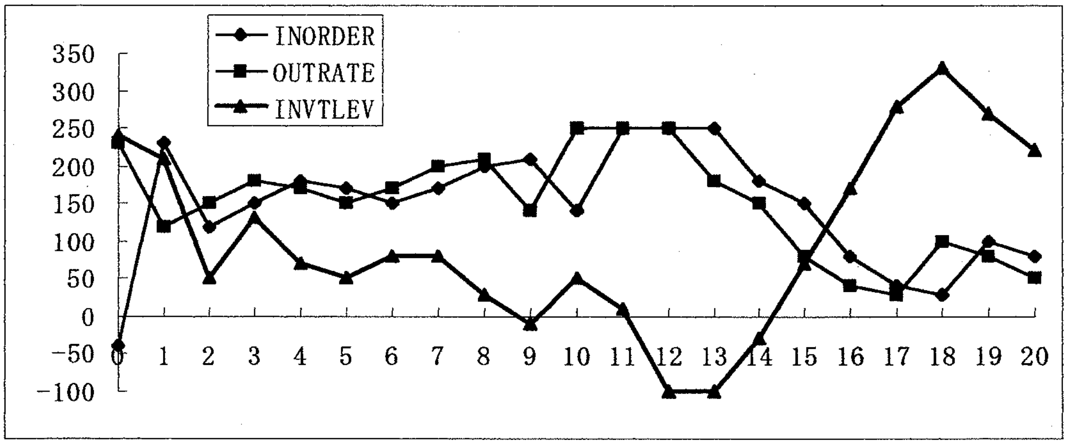

Fig. 11: "Order = Normal - Inventory - unfilled Order" Model

Supplement

```
NOTE     INVENTORY CONTOROL MODEL ( APPROXIMATE REASONING )
FOR ILEV=LVL,LLO,LSL,LMD,LSH,LHI,LVH
FOR IOUTRATE=RVL,RLO,RSL,RMD,RSH,RHI,RVH
L   INVTLEV.K=INVTLEV.J+DT*(INRATE.JK-OUTRATE.JK)
N   INVTLEV=240
R   OUTRATE.KL=TABHL(TOUTRATE,TIME.K,0,20,1)
T   TOUTRATE=230,120,150,180,170,150,170,200,210,140,^
     250,250,250,180,150,80,40,30,100,80,50
L   ORDER.K=ORDER.J+DT*(INORDER.JK-INRATE.JK)
N   ORDER=10
R   INRATE.KL=ORDER.K
R   INORDER.K=SUM(SMFV.K)/SUM(WSMFV.K)
A   SMFV.K(ILEV)=SUM(MFV.K(ILEV,*))
A   MFV.K(ILEV,IOUTRATE)=(MIN(LEVMFV.K(ILEV),RATEMFV.K(IOUTRATE)))^
     *KOKEN(IOUTRATE,ILEV)
A   WSMFV.K(ILEV)=SUM(WMFV.K(ILEV,*))
A   WMFV.K(ILEV,IOUTRATE)=(MIN(LEVMFV.K(ILEV),RATEMFV.K(IOUTRATE)))
I   KOKEN(*,LVL)=0,80,120,200,200,240,240
I   KOKEN(*,LLO)=0,40,120,200,200,240,240
I   KOKEN(*,LSL)=0,0,80,160,200,240,240
I   KOKEN(*,LMD)=0,0,0,120,160,200,240
I   KOKEN(*,LSH)=0,0,0,80,120,160,200
I   KOKEN(*,LHI)=0,0,0,40,120,160,160
I   KOKEN(*,LVH)=0,0,0,0,40,120,160
A   LEVMFV.K(ILEV)=TABHL(TLEVMFV(*,ILEV),INVTLEV.K,0,480,80)
T   TLEVMFV(*,LVL)=1,0,0,0,0,0,0
T   TLEVMFV(*,LLO)=0,1,0,0,0,0,0
T   TLEVMFV(*,LSL)=0,0,1,0,0,0,0
T   TLEVMFV(*,LMD)=0,0,0,1,0,0,0
T   TLEVMFV(*,LSH)=0,0,0,0,1,0,0
T   TLEVMFV(*,LHI)=0,0,0,0,0,1,0
T   TLEVMFV(*,LVH)=0,0,0,0,0,0,1
A   RATEMFV.K(IOUTRATE)=TABHL(TRATEMFV(*,IOUTRATE),OUTRATE.KL,0,240,40)
T   TRATEMFV(*,RVL)=1,0,0,0,0,0,0
T   TRATEMFV(*,RLO)=0,1,0,0,0,0,0
```