A MODEL OF CAPITAL ACCUMULATION, TECHNOLOGICAL PROGRESS AND LONG WAVES

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Abstract

In line with a previous research (Ryzhenkov 1993), a Goodwin-like model of fluctuating growth is represented by a three-dimensional competitive-co-operative system of non-linear ordinary differential equations. In particular, a labour income share enhances a rate of growth of a capital-output ratio. This ratio, in its turn, adversely affects the rate of growth of employment ratio. Under an appropriate constellation of coefficients and control parameters, this model is capable of generating long waves modelled by converging fluctuations in the vicinity of a dynamic equilibrium (steady-state growth path) or by closed orbits in the phase space. The analytical and experimental results seem to provide a new base for the conclusion that no intrinsic (exogenous) clustering of innovations is necessary to produce long period fluctuations of economic activity as the flow of invention and innovation is contingent upon the rate of capital accumulation. It is shown that the model is consistent with the Kaldor prominent stylised facts and the Valtukh information value hypothesis.

1. Introduction

The experts of the OECD have recently concluded: "...technological progress has become virtually synonymous with long-run economic growth and increases in productivity... A large research agenda lies ahead of us in defining, measuring, interpreting, and understanding not only productivity growth and technology change but also how they are related to each other. OECD can take the leadership in marshalling the necessary financial and intellectual resources to achieve significant and much needed improvements in this crucial area of far reaching policy importance" (OECD, 1991: 133, 135).

Goodwin's predator-prey growth cycle model (1972) mirrors social contradictions of income distribution as a cycle-generating factor. This model has been analysed and extended in different ways. The present paper, in the same channel of investigation, focuses on issues of stability, fluctuations and long run trends in connection with improving skills and growing capital intensity. The outlined approach, in Marxian tradition, connects the theory of value with the theory of economic growth and fluctuations.

2. The Model

The model omits the Goodwin assumption of constant capital-output ratio, but preserves his premise of the passivity of final demand (cf. Goodwin 1972). The other following most important assumptions are made:

(1) two social classes (capitalists and workers);

(2) only two factors of production, labour force and means of production, both homogenous and non-specific;
(3) only one good is produced for consumption and investment purposes, so the capital goods industries are not treated explicitly;
(4) production (supply) equals effective demand;
(5) all productive capacities are operated;
(6) all wages consumed, all profits saved and invested;
(7) steady growth in the labour force;
(8) real wage rate rises in the neighbourhood of full employment;
(9) a constant flow of new technological ideas over time;
(10) a constant labour intensity;
(11) a qualification of the labour force corresponds to technological requirements.

The model is formulated in continuous time. Time derivatives are denoted by a dot, while growth rates are indicated by a hat. The simplified version of the model consists of the following equations:

$$P = K/s \quad (2.1)$$
$$a = P/L \quad (2.2)$$
$$u = w/a \quad (2.3)$$
$$\hat{a} = m_1 + m_2(\widehat{K/L}), \quad m_1 \geq 0, \ m_2 > 0 \quad (2.4)$$
$$(\widehat{K/L}) = n_1 + n_2 u, \quad n_1 \geq 0, n_2 > 0 \quad (2.5)$$
$$v = L/N \quad (2.6)$$
$$N = N_0 e^{nt}, \ n = \text{const} \geq 0, N_0 = \text{const} > 0 \quad (2.7)$$
$$\hat{w} = -g_1 + rv, \qquad g_1 \geq 0, \qquad r > 0 \quad (2.8)$$
$$M = (1 - w/a)P \quad (2.9)$$
$$\dot{K} = (1 - w/a)P. \quad (2.10)$$

Equation (2.1) postulates a technical relation between the capital stock (K) and net output (P). The variable s is called capital-output ratio. (2.2) relates labour productivity (a), net output (P) and labour input or employment (L). Equations (2.3) and (2.6) describe the shares of labour in national income (u) and the ratio of employment (v), respectively. Equations (2.9) and (2.10) show that profit (M), savings, investment and incremental capital ($\dot{K}$) are equal. Workers do not save at all. Investment lag is set aside. An explicit consideration of the diffusion process goes beyond the scope of this paper.

Equation (2.4) is a linear form of the Kaldor technical progress function: the growth rate of labour productivity is assumed to depend linearly on the growth rate of capital intensity. "The use of more capital per worker... inevitably entails the introduction of superior techniques which require "inventiveness" of some kind... On the other hand, most...technical innovations which are capable of raising productivity of labour require the use of more capital per man - more elaborate equipment..." (Kaldor, 1957: 595). The Kaldor hypothesis of a constant flow of technological ideas over time and of a constant readiness with which they are adopted by industrialists underlies the assumption of the technical progress function with the constant parameters. Although exogenous bunching or clustering of innovations is not assumed, the model is capable of generating long waves (see section 4).

Equation (2.7) defines the exponential growth of the labour supply (N) with the rate n. Equation (2.8) represents the linear approximation of the real Phillips curve. A rising rate of employment is assumed to affect wage increases (in real terms). There is no money illusion.

A mechanisation function is introduced in (2.5). It relates the growth rate of capital per employee (of capital intensity) to income distribution. "Mechanisation is encouraged by a high wage share, i.e., high labour costs per unit of net product" (Glombowski and Krüger, 1984: 265). M. Porter has pointed out that rising wages "create pressure for innovation" and "spread prosperity to workers and sustain their motivation to improve their skills, speeding the rate at which companies can improve" (Porter, 1991: 686).

The Valtukh information value hypothesis evinces that the rate of surplus value, other things being equal, grows along with qualification. Real wage changes at a slower pace than the corresponding qualification. The higher the qualification, the higher is the capital intensity. The latter may be used as an indicator of qualification (Valtukh, 1991: 21-44). This hypothesis will help us to extend the model of capital accumulation.

Instead of assuming, as in the usual Phillips relation, that the rate of change of the wage rate depends only on the employment ratio, let this rate be additionally influenced by the rate of change of capital intensity:

$$\hat{w} = -g_1 + rv + g_2 + b(\widehat{K/L}), \quad g_2 \geq 0, \quad b > 0, b < m_2. \tag{2.11}$$

This modification also takes into consideration the historical or moral element in the value of labour power. The linear approximation is tentative and perhaps requires more precise definitions. Please note that we are abstracting from the institutional arrangements for the supply of skills. Equation (2.11) will be used in obtaining a convenient statement of the model.

3. A Nontrivial Equilibrium in the Modified Model

The central variables of the modified model are the employment ratio (v), the labour bill share (u) and the capital coefficient (s). Let us first consider the evolution of the latter. Substitution and logarithmic differentiation yield:

$$\hat{s} = (\widehat{K/P}) = (\widehat{K/L}) - (\widehat{P/L}) = -m_1 - m_2(\widehat{K/L}) + (\widehat{K/L}) = \\ = -m_1 + (1 - m_2)(n_1 + n_2 u). \tag{3.1}$$

The employment ratio (v) is defined by the equations:

$$\hat{v} = (\widehat{L/N}) = \hat{P} - \hat{a} - \hat{N} = (\widehat{P/K}) + \hat{K} - \hat{a} - \hat{N} = \\ -\hat{s} + (1 - w/a)/s - \hat{a} - n. \tag{3.2}$$

We have used in (2.11) $\hat{K} = (1 - w/a)P/K = (1 - w/a)/s$. Substitution and transformation allow us to write (2.11) in a simpler form:

$$\dot{v} = (1 - u)/s - (n_1 + n_2 u) - n. \tag{3.3}$$

The labour bill share (u) changes according to

$$\dot{u} = (\widehat{w/a}) = \hat{w} - \hat{a} = -g + rv + b(n_1 + n_2u) - (m_1 + m_2(n_1 + n_2u)) = -g + rv - m_1 + (b - m_2)(n_1 + n_2u), \text{ where } g = g_1 - g_2. \quad (3.4)$$

Thus, we have the following system of ordinary differential equations:

$$\dot{s} = -(m_1 + (m_2 - 1)(n_1 + n_2u))s \quad (3.5)$$
$$\dot{v} = ((1 - u)/s - (n_1 + n_2u) - n)v \quad (3.6)$$
$$\dot{u} = (-g + rv - m_1 + (b - m_2)(n_1 + n_2u))u. \quad (3.7)$$

It may be easily shown that the system (3.5) - (3.7) is an example of a non-linear competitive-co-operative network:

$$\partial f_1(s)/\partial v = 0, \quad \partial f_1(s)/\partial u = (1-m_2)n_2s \begin{cases} \leq 0, \text{ if } m_2 \geq 1 \\ > 0, \text{ if } m_2 < 1 \end{cases} \quad (3.8)$$

$$\partial f_2(v)/\partial s = -(1-u)v/s^2 < 0, \quad \partial f_2(v)/\partial u = -v/s - n_2v < 0 \quad (3.9)$$

$$\partial f_3(u)/\partial s = 0, \quad \partial f_3(u)/\partial v = ru > 0. \quad (3.10)$$

Notice that the labour bill share adversely affects growth of the employment ratio and, if $m_2 < 1$, it activates growth of the capital-output ratio. This ratio inhibits the growth of the employment ratio. The rising employment ratio is promoting the growth of the labour bill share.

The nontrivial singular point (equilibrium state) is given by:

$$E_2 = (s_2, v_2, u_2), \text{ where} \quad (3.11)$$

$u_2 = m_1/(n_2(1 - m_2)) - n_1/n_2$, $s_2 = (1 - u_2)/(n_1 + n_2u_2 + n)$ and
$v_2 = (g + m_1 + (m_2 - b)(n_1 + n_2u_2))/r$.

(s_2 follows from (3.6), v_2 - from (3.7) and u_2 - from (3.5).) This non-linear three-dimensional system could, potentially, display a very rich spectrum of behaviour. Our model is not only able to incorporate growth, but can also mimic the persistence of cycles arising in the negative feedback loops with the delayed corrective action. In this model, the integration of the flows causes a delay, although there are no explicit material or information delays.

If $m_1 = 0$ and $m_2 = 1$, then $\dot{s} = 0$ and the capital-output ratio is determined by the initial condition ($s = s_0 > 0$). The system degenerates into the two-dimensional system:

$$\dot{v} = ((1 - u)/s_0 - (n_1 + n_2u) - n)v \quad (3.12)$$
$$\dot{u} = (-g + rv + (b - 1)(n_1 + n_2u))u. \quad (3.13)$$

The nontrivial equilibrium in this case is given by (3.14):

$$u_e = (1 - s_0(n_1 + n))/(n_2s_0 + 1),$$
$$v_e = (g + (1 - b)(n_1 + n_2u_e))/r. \quad (3.14)$$

The Jacobian of the model evaluated at the nontrivial equilibrium is given by

$$J= \begin{vmatrix} 0 & 0 & n_2(1 - m_2)s_2 \\ -v_2(1 - u_2)/s_2^2 & 0 & -v_2/s_2 - n_2v_2 \\ 0 & ru_2 & (b - m_2)n_2u_2 \end{vmatrix}$$

Thus, the characteristic polynomial is $l^3 + a_2l^2 + a_1l + a_0 = 0$, where

$$a_0 = -\det(J) = (v_2(1 - u_2)ru_2n_2(1 - m_2))/s_2 > 0, \text{ if } m_2 < 1,$$
$$a_1 = ru_2v_2(1/s_2 + n_2) > 0,$$
$$a_2 = -\text{trace}(J) = (m_2 - b)n_2u_2 > 0, \text{ if } m_2 > b.$$

The Routh-Hourwitz conditions are necessary and sufficient for local stability and require that $a_0 > 0$, $a_1 > 0$ and $a_1a_2 > a_0$. The first and second inequalities are satisfied, whereas the third inequality corresponds to (3.15):

$$(n_2s_2 + 1)(m_2 - b) > ((1 - u_2)/u_2)(1 - m_2) > 0. \qquad (3.15)$$

Provided that the inequality (3.15) holds, the dynamics of the modified model (3.5), (3.6) and (3.7), in the neighbourhood of its equilibrium, are locally stable (cf. van der Ploeg 1987). Note that in (3.15), $(1-u_2)/u_2$ is equal to the rate of surplus value estimated at the equilibrium.

The inequality (3.15) is not true, if $b \geq m_2$. So $m_2 > b$ is a necessary condition of the local stability. The presence of coefficient $n_2 > 0$ on the left side of (3.15) shows that a distribution-induced change in the speed of technological progress produces a stabilising influence. The result gives support to the Valtukh information value hypothesis. We also agree with M. Porter that wage increases well ahead of productivity growth for a sustained period are a cause of concern (Porter, 1991: 643).

4. Trend and Long Waves in the Modelling Economy

We will abstract from short-term business cycles and from "random noise", which manifests itself in arbitrary variations in the growth rate in the short run. Extended reproduction along the steady state path displays the following properties. Output grows at its "natural rate", the sum of labour supply growth and productivity growth:

$$\hat{P}_2 = \hat{a}_2 + n = m_1/(1 - m_2) + n.$$

The growth of labour productivity and real wage is at the rate

$$\hat{a}_2 = \hat{w}_2 = m_1/(1 - m_2), \text{ if } m_1 \neq 0 \text{ and } m_2 \neq 1,$$
$$\hat{a}_e = \hat{w}_e = n_1 + n_2u_e \text{ (see (3.11) above), if } m_1 = 0 \text{ and } m_2 = 1,$$

whereas the profit share, the capital-output ratio and, therefore, the rate of profit itself are constant. Capital stock increases at the rate $\hat{K}_2 = \hat{P}_2$ ($\hat{K}_e = \hat{P}_e$) which is equal to the profit rate

$M_2/K_2 = \hat{K}_2 = \hat{P}_2$ (or $M_e/K_e = \hat{K}_e = \hat{P}_e$ in the degenerated case).

The labour income share, the rate of employment and capital-output ratio are constant.

There is a good agreement between the properties of the steady-state growth in our model and Kaldor's five stylised facts:

1. Output per worker shows continuing growth with no tendency for a falling rate of growth of productivity.
2. Capital per worker is rising more or less in proportion to productivity.
3. The rate of return on capital is steady.
4. The capital-output ratio is steady.
5. Labour and capital receive constant shares of total income.

We have determined the shares of wages and profits as well as the rate of profit on capital quite independently of the principles of marginal productivity. The rate of profit on capital depends on the coefficients of the technical progress function which, in turn, determine the rate of growth of labour productivity. Our model typically yields the persistence of unemployment. It will be rather simple to reflect in a similar model such a prominent feature of the data as a negative correlation of population growth rates with the level of income.

One can easily show that the higher the equilibrium rate of productivity growth $m_1/(1 - m_2)$, the higher are the equilibrium share of wages in the national income (u_2), employment ratio (v_2, if $b < 1$), rate of profit (M_2/K_2) and the lower is the capital-output ratio (s_2). Thus, for the model economy, a failure to innovate is detrimental to employment, real wages and profitability, whereas increasing technical dynamism is favourable for them.

To illustrate cyclical growth it is necessary to choose some appropriate values of coefficients and initial magnitudes of variables. We will provide examples of damping long waves. It is not implied that our illustrative constellation are in fact empirically accurate. We use vaguely plausible equilibrium values of the main variables relying on our predecessors: the equilibrium wage share, 0.75, coincides with that of van der Ploeg (1983: 259); the equilibrium employment ratio, 0.9, - with that of Goodwin (1990: 69, 87); the equilibrium capital-output ratio, 4.17, and growth rate of labour productivity, 4 %, are in a good agreement with the upper range of the Romer data (see Romer 1989).

The following values calibrate the model (see Table 1 that displays the relative speed of convergence for different magnitudes of the two significant parameters). If we set the initial magnitude of the rate of employment $v_0 = 0.89 \neq v_2$ without an initial displacement of other variables from their equilibrium values ($u_0 = u_2$, $s_0 = s_2$), a damping cyclical motion is obtained, the length of the cycle being approximately 60 years. Because the real parts of the eigenvalues are all negative, the fixed point E_2 is a sink. Moreover, it is locally asymptotically stable. The fluctuations are not strictly periodic. The amplitude and phasing of each variable are determined structurally (cf. in a linear case all variables oscillate with the same frequency and damping; only their amplitudes and phasing differ, these being parameters fixed for each variable separately by extraneous factors or initial conditions. See Allen, 1954: 161).

Table 1. The examples of the long waves

m_1	m_2	Steady state growth rate ($a_2 = w_2$, $a_e = w_e$)	Relative speed of convergence
0.02	0.5	0.04	low
0	1.	0.04	moderate

Notes:
1. The length of the fluctuations is about 60 years.
2. If $m_1 = 0.02$ and $m_2 = 0.5$ then the initial vector $(s_0, u_0, v_0) = (s_2, u_2, v_2 - 0.01) \cong (4.17, 0.75, 0.89)$; if $m_1 = 0$ and $m_2 = 1$ then the initial vector $(s_0, u_0, v_0) = (s_0, u_e, v_e - 0.01) \cong (4.17, 0.75, 0.89)$.
3. The magnitudes of the other parameters:
$n_1 = 0.01$, $n_2 = 0.04$, $r = 0.062$, $b = 0.1$, $g = 0.02$, $n = 0.02$.

The inequality (3.15) turns into equality if $b_0 \cong 0.357 < m_2 = 0.5$. The new nontrivial fixed point corresponding to this critical magnitude (x*) equals appoximately (4.17, 0.75, 0.74). When b is increased from $b < b_0$ to $b > b_0$, the system (3.5) - (3.7) loses its local stability at x* because the real part of the complex conjugate eigenvalues becomes positive. It may be shown that according to the Hopf bifurcation theorem (an existence part), there exist periodic solutions bifurcating from the new locally unstable fixed point at $b = b_0$ and the period of the solutions is close to 55-60 years. This theorem establishes only the existence of closed orbits in a neighbourhood of x* at $b = b_0$, and it does not clarify the stability of orbits, which may arise on either side of b_0. The stability properties of the closed orbits depend on the non-linear terms because in the Hopf bifurcation the real parts of the eigenvalues of Jacobian, i.e., of the linear approximation, vanish. I have simulated closed orbits in the phase space, which show the other possible pattern of long waves about the trend, using the program Gnans (Martensson 1993). A more rigorous mathematical analysis of the topological properties of these trajectories should be carried out in a further research.

We can distinguish the following four phases of the long wave.

Phase 1 (recession): the labour bill share is growing from its average magnitude to its maximum value; the employment ratio is decreasing from its maximum to the average level; the capital-output ratio is moving from its minimum to the average magnitude; profitability is falling from its average level to the minimum.

Phase 2 (depression): the labour bill share is decreasing from its maximum magnitude to its average value; the employment ratio is decreasing from its average level to the minimum; the capital-output ratio is moving from its average magnitude to its maximum; profitability is rising from its minimum to the average level.

Phase 3 (recovery): the labour bill share is falling from its average magnitude to its minimum; the employment ratio is growing from its minimum to the average level; the capital-output ratio is falling from its maximum to the average magnitude; profitability is increasing from its average level to the maximum.
Phase 4 (boom): the labour bill share is increasing from its minimum to its average value; the employment ratio is growing from its average level to the maximum; the capital-output ratio is moving from its average magnitude to its minimum; profitability is falling from its maximum to the average level.

Profitability and growth are at their nadir at the phase transition from recession to depression. At the end of the latter profitability is at its average value; when profitability is greatest (the beginning of the boom phase) employment is average. The improving profitability is engendering a too speedy expansion of output and employment, thus favouring labour's bargaining power and its share in national income, which could be only partially offset (in our example) by the drop of the capital-output coefficient during the boom phases. Substituting labour by capital and destroying employment (and workers' bargaining power) help to restore profitability during the depression.

The growing labour productivity (with the decreasing capital-output ratio) enable the simultaneous growth of employment and of the profit rate during recoveries and the simultaneous growth of employment and of the labour bill share during booms. Real wages must fall in relation to productivity during depressions (but not necessarily in absolute terms) in order to restore profitability.

In view of technological progress, a growing employment ratio not necessarily requires a reduction in real wage in order to restore profitability. The rates of growth of labour productivity and of the capital-output ratio are dependent upon the flow of new technological ideas and upon diffusion of innovations brought about by capital accumulation. In particular, the capital-output ratio tends to decrease in periods of accelerated growth (during the recovery and boom phases) fostering the rate of capital accumulation and vice versa.

Van der Ploeg writes: "The beauty of Goodwin-type models of cyclical growth is that they are able to explain short-run fluctuations around a long-run balanced growth trajectory" (van der Ploeg, 1987: 11). I have shown that they may explain long-run fluctuations as well.

The recent simulation work of Goodwin (1990) argues that business cycles and long waves can be generated simultaneously in market systems. Goodwin has obtained this results via a notion of a chaotic attractor adding exogenously a 50-year logistic of innovation in a Rössler-like system of three differential equations with only one non-linearity. We have shown that the long waves could be generated without such exogenous procedure. Their highly irregular interaction with shorter cycles has been not preserved in the above model yet.

5. Conclusion

In the model economy, technical progress and population growth tend to result in steady economic growth, while the long waves could represent important fluctuations about this trend, lasting some 50-60 years. Such a long-wave pattern of economic growth is determined by the internal structure of capital accumulation displayed above within the very stylised institutional setting. For this setting, the high technological dynamism benefits the both classes and mitigates the social contradictions in the process of income distribution. Our analytical findings and simulation experiments are supporting the conclusion that "no intrinsic clustering of innovations is necessary to produce long period fluctuations of economic activity" (Silverberg and Lehnert, 1992: 17) as the flow of invention and innovation is contingent upon the rate of capital accumulation. It is shown that the model is consistent with the Kaldor prominent stylised facts and the Valtukh information value hypothesis. The study of sustainable growth and (in)stability for more complex institutional, ecological and social structures of the industrialised market economies is to be done in a further research.

References

Allen, R. 1954. The Engineer's Approach to Economic Models. *Economica.* 21 (81-84): 158-168

Glombowski, J. and M. Krüger. 1984. Generalisations of Goodwin's Growth Cycle Model. In *Non-linear Models of Fluctuating Growth.* ed. R. Goodwin, M. Krüger, and A. Vercelli. Berlin: Springer, 260-290.

Goodwin, R. 1972. A Growth Cycle. In *Socialism, Capitalism and Economic Growth,* ed. C. Feinstein. Cambridge: Cambridge University Press, 165-170.

------. 1990. *Chaotic economic dynamics.* Oxford: Oxford University Press.

Kaldor, N. 1957. A Model of Economic Growth. *The Economic Journal.* 67: 591-624

Martensson, B. 1993. *Gnans: a Program for Stochastic and Deterministic Dynamical Systems.* Reference Manual. Report Nr. 300. Institut für Dynamische Systeme. Universität Bremen.

OECD. 1991. *Technology and Productivity.* Paris: OECD.

Phillips, A. 1958. The Relations between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957. *Economica.* 11: 283-299

Porter, M. 1991. *The Competitive Advantage of Nations.* London: The Macmillan Press LTD.

Romer, P. 1989. Capital Accumulation in the Theory of Long-Run Growth, in *Modern Business Cycle Theory.* ed. R. Barro. Cambridge (Massachusetts): Harvard University Press and Basil Blackwell, 51-127.

Ryzhenkov, A. 1993. An Application of System Dynamics Approach to the Modelling of Cyclical Capitalist Reproduction. *Jahrbücher für Nationalökonomie und Statistik.* 211 (5-6): 539-545

Silverberg, G., and D. Lehnert. 1992. *Long Waves and Evolutionary Chaos in a Simple Schumpeterian Model of Embodied Technical Change. Merit's Research Memorandum.* 92-023. Maastricht : MERIT.

Valtukh, K. 1991. The Theory of Value and Further Development of Macroeconomic Models. In *Technological Progress and Economic Development,* ed. E. Golland, and T. Rybakova. Novosibirsk: Nauka, 3-48 (in Russian).

Van der Ploeg, F. 1983. Economic Growth and Conflict over the Distribution of Income. *Journal of Economic Dynamics and Control.* 6: 253-79

-------. 1987. Growth Cycles, Induced Technical Change, and Perpetual Conflict over the Distribution of Income. *Journal of Macroeconomics.* 91: 1-12.

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1. In formula (2.4) on page 2, the variable a must be marked by a hat instead of a dot.
2. There should be (3.2) instead of (2.11) in lines 3 and 4 from below on page 3.
3. The formula (3.3) on page 3 (line 2 from below) is for variable v marked by the a instead of a dot.
4. The formula (3.4) on page 4 (line 1 from above) is for variable u marked by a hat instead of a dot.