System Dynamics Model For A Mixed Strategy Game Between Police and Driver

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Game theorists have recommended lots of reasonable strategies for resolving domestic and international policy problems. In general, they use a concept of equilibrium strategy for analyzing dynamic consequences of available policy options in game situations. One of the most famous policy recommendations suggested by game theorists is that of George Tsebelis (1989). He contends that an increase in penalty against law-violation is not a viable policy tool for decreasing the violation tendencies of drivers. That is because the interactions between police and drivers can be best represented as a mixed strategy game in which each player choose their alternative actions with a probability. In a mixed strategy game between police and driver, the probability of driver's law-violation cannot be decreased by increasing penalty against law-violation.

Tsebelis's suggestion seems to be contradictory to the common sense. For most policy makers, an increase in penalty is conceived as one of the most effective tools for policy implementation. In Korea, the increase in penalty have vastly reduced the number of drunken drivers. In many countries, the penalty management is a major policy implementation tool for inducing compliance from the people.

Our SD model for a mixed strategy game shows that it takes a very long time for an game-theoretic equilibrium to appear. Therefore, game players cannot and should not depend on the equilibrium state for choosing their actions. Furthermore, our mixed game model shows that an increase in penalty can induce a compliance from the people. Our model shows a behaviors which are contradictory to the game theoretic solution, but consistent to the real world behaviors. We have proposed that these gaps between SD model and a game theory come from the ambiguous conception of equilibrium state and the lack of dynamic and transient behavior analysis in the game theory.

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Feedback loops in the social world have been regarded as important research areas not only for system dynamists but also for various kinds of social scientists. Social psychologists find feedback loops in the self-fulfilling prophesy, group dynamics and social interactions (Weick 1979). Game theorists interpret feedback loops in terms of norms of reciprocity, interdependent choices and tit for tat (Komorita et al 1992; Elster 1989; Axerlord 1984). In particular, game theorists have developed their ideas on reciprocal decisions with a formal logic. They have recommended lots of reasonable strategies for resolving domestic and international policy problems (Snidal 1985, 1991).

However, the methodology and propositions of game theorists are different to those of system dynamists in many respects. While a building-block of system dynamics modelling is a decision-making activity, that of game theory is a game situation which is composed of pairs of players, preferences and strategies (Ordeshook 1986). In game situations, it is a general rule that more than two decision-makers interact to maximize their interests. While system dynamists focus their attentions on dynamic fluctuations of a system, game theorists make efforts to find equilibrium states in game situations. In general, game theorists use a concept of equilibrium strategy for finding out consequences of available policy options.

Game theory is contrasted to the decision theory. Within a decision theory, a decision maker is assumed to behave independently to the decision of others. In game theory, every players change their decisions in response to other players' actions. Players are supposed to exploit inferior decisions of others. Furthermore, if possible and desirable, players are assumed to violate laws and deceive other players. In these situations, explanations based on decision theory are prone to errors.

One of the most famous policy recommendations suggested by game theorists is that of George Tsebelis (1989). He contends that an increase in penalty against law-violation is not a viable policy tool for decreasing the violation tendencies of drivers. That is because the interactions between police and drivers can be best represented as a mixed strategy game in which each player choose their alternative actions with a probability. In a mixed strategy game between police and driver, the probability of driver's law-violation cannot be decreased by increasing penalty against law-violation.

In this paper, we have made a system dynamics model for a mixed-strategy game for investigating dynamic processes which are behind the game-theoretic equilibrium. Our SD model for a mixed strategy game shows that it takes a very long time for a game-theoretic equilibrium to appear. Therefore, game players cannot and should not depend on the equilibrium state for choosing their actions. Furthermore, our mixed game model shows that an increase in penalty can induce a compliance from the people. Our model shows behaviors which are contradictory to the game theoretic solution, but consistent to the real world behaviors. We have proposed that these gaps between SD model and a game theory come from the ambiguous conception of equilibrium state and the lack of dynamic and transient behavior analysis in the game theory.

1. Equilibrium of Mixed-strategy Game

For certain two-person games like prisoners' dilemma, there exist dominant strategies for each game players. For other kinds of games, dominant strategies are not present to the game players. In these situations, game players often adopt a mixed strategy in which they choose different actions at random. Runners in a baseball game have no dominant strategy for whether or not he should steal a base. Best runners would start to steal a base when anyone cannot anticipate it. This is a mixed-strategy game. Tsebelis describe it with the case of games between police and driver (Tsebelis 1989)

> You are driving your car and you are in a hurry...There are two state of the world: either the police are nearby or they are not. There are two acts to choose from: either to violate the speed limit or to abide by the law. Again, there are four possible outcomes: (a) you can get a ticket for speeding, (b) you can get to work on time without any incident, (c) you can arrive late and avoid a ticket, and finally (d) you can arrive late though there were no policemen on the streets.

Two states mean the acts of other players: decisions of a policeman to patrol or not. Two acts are available to the driver. Four outcomes in this game result from the combination of two states and two acts. They correspond to each cells in the payoff matrix of table 1.

		Decisions of the policeman	
		Patrol	Not Patrol
Decisions of the driver	Speed	a1 a2	b1 b2
	Not speed	c1 c2	d1 d2

<Table 1> Payoffs Matrix for policemen and driver

The entry in each cell is the vector of utilities for the policemen and the driver. In keeping with convention, the first entry in each cell (a1, b1, c1, d1) denotes the driver's utility, the second (a2, b2, c2, d2) denotes the policeman's utility. Note that a1 reflects the amount of penalty for the driver's law-violation.

For the utility of a driver, it is assumed that he would prefer not to speed if there is a policeman (c1 > a1) and he would prefer to speed if a policeman decides to patrol (b1 > d1). For the utility of a policeman, it is assumed that he would prefer to patrol if the driver decides to speed (a2 > b2), while he would prefer not to patrol if the driver abide by the speed law (d2 > c2).

In table 1, both of the driver and the policeman has no dominant strategy. If the policeman decide to patrol, the driver prefers not to speed, and then the policeman prefers not to patrol, and then the driver choose to speed. In order to maximize their expected utility, they should choose their acts with some probabilities. An equilibrium state in a mixed-strategy game means the state in which both players act with a pair of probabilities (p*, q*) that can give them greatest expected utilities (Ordeshook 1986). The p* is the probability with which the driver choose to speed, and the q* is the probability with which the policeman decide to patrol. The calculation of p* and q* gives (Tsebelis 1989).

$$p^* = (d2 - c2)/(a2 - b2 + d2 - c2) \quad (1)$$
$$q^* = (b1 - d1)/(b1 - d1 + c1 - a1) \quad (2)$$

Anyone can derive these formula quickly by maximizing the expected utilities of the driver and the policeman. Note that the probability of the driver's law-violation in (1) is not determined by the payoffs for the driver in table 1. This means that the penalty for the law-violation (a1 in table 1) has no effect on the probability of the driver's law-violation. From this logic, Tsebelis concludes following theorems.

Theorem 1. Under assumptions of c1 > a1, b1 > d1, a2 > b2 and d2> c2, the only equilibrium in the police-public game is in mixed strategies as specified by equations (1) and (2).

Theorem 2. An increase in the penalty leaves the frequency of violation for the law at equilibrium (p*) *unchanged.*

Theorem 3. An increase in the penalty *decreases* the frequency that the police enforce the law at equilibrium (q*).

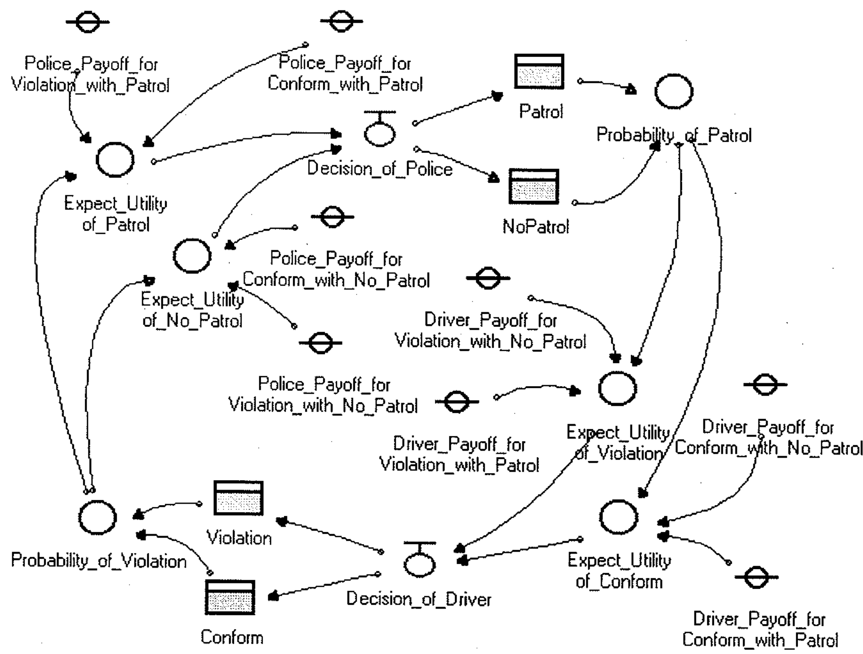
Theorems Tsebelis concluded seem to be contradictory to the common sense. For most policy makers, an increase in penalty is conceived as one of the most effective tools for policy implementation. In Korea, the increase in penalty have vastly reduced the number of drunken drivers. In many countries, the penalty management is a major policy implementation tool for inducing compliance from the people.

2. System dynamics model for a mixed strategy game

Why does a game theory produce theorems inconsistent to the common sense and to the real world? Two kinds of answers can be provided. First, we can reject the hypothesis that the interactions between a police and a driver is a kind of game. Second, if we accept their interactions as game, we can suspect the validity of a game theoretic solution. In this paper, we have a focus on the second possibility. We have made a system dynamics model for investigating the validity of a game theoretic solution in the mixed strategy game.

Figure 1 shows system dynamics diagram for a two-person mixed-strategy game. We have built the model to reflect decision-making processes of both players. In figure 1, two rate variables named as decision_of_police and decision_of_driver represent responses of players to others' acts. Four stock variables denote historical cumulations of their acts, from which probabilities of their acts can be calculated. Eight constant variables correspond to eight payoffs in table 1. They are summarized in table 2.

In table 2, cells in the third column display values of payoffs assumed in the model. Payoff values are assumed to satisfy conditions of mixed-strategy game; c1> a1, b1 > d1, a2 > b2 and d2 > c2.



<Figure 1> SD model for a two-person mixed-strategy game

Symbols for payoffs in table 1	Constant variables in SD model	Value assumed
a1	Driver_Payoff_for_Violation_with_Patrol	1
b1	Driver_Payoff_for_Violation_with_No_Patrol	4
c1	Driver_Payoff_for_Conform_with_Patrol	3
d1	Driver_Payoff_for_Conform_with_No_Patrol	2
a2	Police_Payoff_for_Violation_with_Patrol	4
b2	Police_Payoff_for_Violation_with_No_Patrol	1
c2	Police_Payoff_for_Conform_with_Patrol	2
d2	Police_Payoff_for_Conform_with_No_Patrol	3

< Table 2> Constant variables for payoffs in mixed-strategy game

In addition to these basic assumptions, we added some assumptions for making the model clear and similar to the real world. We have assumed that Driver_Payoff_for_Violation_with_Patrol (a1) is worst for the driver and Driver_Payoff_for_Violation_with_No_Patrol (b1) is best for the driver. Police_Payoff_for_Violation_with_No_Patrol (b2) is assumed worst for the police and Police_Payoff_for_Violation_with_Patrol (a2) is assumed best for the police. One can simulate the model without these additional assumptions and get results consistent to ours.

The equilibrium state in the model can be calculated by applying the equation (1) and (2) to the payoffs in table 2. We can get following probabilities for acts of game players.

p^* (probability of driver's violation) $= (3 - 2)/(4 - 1 + 3 - 2) = 0.25$
q^* (probability of policeman's patrol) $= (4 - 2)/(4 - 2 + 3 - 1) = 0.5$

We can examine three theorems presented by Tsebelis by comparing this equilibrium state to model behaviors.

3. Simulation results

1) Oscillation rather than equilibrium

The first theorem of Tsebelis says that a mixed strategy game has an equilibrium state where two players act with the probabilities calculated by equation (1) and (2). However, equation (1) and (2) say nothing about the processes by which two players move to the equilibrium state. Even if the equilibrium state is unique and stable, the path for reaching to it should be analyzed. If the path is filled with fluctuations and dissatisfactions, players will give up using the mixed strategy.

We have simulated our model until 1,000 simulation time with the time-interval of 1 (DT=1). In this simulation, DT means an event of repeated games between the police and the driver. If one want to interpret a simulation time as a real world time, one should know how many games are performed during one unit of a real world time. Figure 2 shows behaviors of the police and the driver during the simulation.

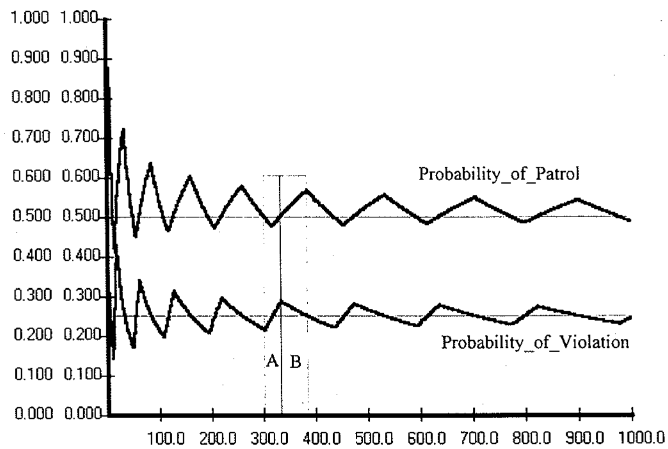
Figure 2 shows the tendency of game-players for moving to the equilibrium state. However, the paths for the equilibrium states are filled with oscillations. For getting to the equilibrium state, they should overcome the fluctuating behaviors of other players. An adjustment time necessary for the equilibrium state is so long that, if we assume one game per a day, it takes more than 3 years to get to the equilibrium state.

In order to examine the existence of the equilibrium states, we have simulated the model until 10,000 times. And we found that the fluctuations shown in figure 2 disappear in the long run. This long run equilibrium should not be interpreted as an structural attribute of a mixed strategy game. The essential cause of the long run equilibrium is the frequency of games itself. Only after players perform the game lots of times, probabilities of their acts reach a stable state. For example, after the games are repeated 5,000 times, their additional acts have little effects on the probabilities of violation and patrol. Speaking in a word, the long run equilibrium results from the frequency of the game and from the players' memory on past experiences. If we assumed that players forget ten percent of their memory, the long run equilibrium is replaced by endless oscillations.

Note that graphs in figure 2 denote the equilibrium pair of probabilities. The probability of driver's violation fluctuates around 0.25. However, the probability of patrol for the policeman fluctuates above the equilibrium probability of 0.5.

On the average, policeman patrols more frequently than the optimum level. Why this happens? This comes from the asymmetry between the optimal probability of violation and the optimal probability of patrol. The former is 0.25, while the latter is 0.5. In order to maintain the probability of violation at 0.25, the driver should violate the law much less frequently than abiding by it.

Therefore, the period for violations (area A in figure 2) is shorter than the period for conforming to the law (area B in figure 2). The period for driver's violations corresponds to the period during which the probability of patrol is below 0.5. As a result, the period when the probability of patrol is below 0.5 is shorter than the period when it is above 0.5. That is why the average probability of patrol for the policeman is above the equilibrium probability of 0.5. From these results and imbedded dynamics, we can propose a new proposition that in a mixed-strategy game those players whose equilibrium probability of certain act is higher than others' are destined to over-act.



<Figure 2> Behavior of players in a mixed-strategy game

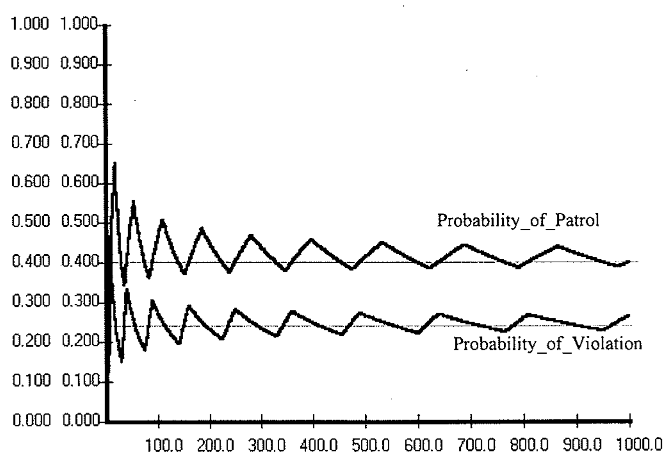
2) The effectiveness of penalty increase

In order to examine the second and third theorem of Tsebelis, we have increased the penalty for violating a speed limit. That is, we have reduced the value of Driver_Payoff_for_Violation_ with_Patrol from 1 to 0. The equilibrium state can be calculated by applying the equation (1) and (2) as follows.

p* (probability of driver's violation) = (3 - 2)/(4 - 1+ 3 - 2) = 0.25
q* (probability of policeman's patrol) = (4 - 2)/(4 - 2 + 3 - 0) = 0.4

An increase in the penalty reduces the equilibrium probability of patrol from 0.5 to 0.4 but does not change the equilibrium probability of violation as stated in the second and third theorem of Tsebelis.

Figure 3 shows results from our second simulation. In our second simulation, we have introduced the penalty-increase at initial time of simulation. As predicted by the second theorem, the average value of the probability of violation is not changed.



<Figure 3> Behaviors of players after increase in the penalty

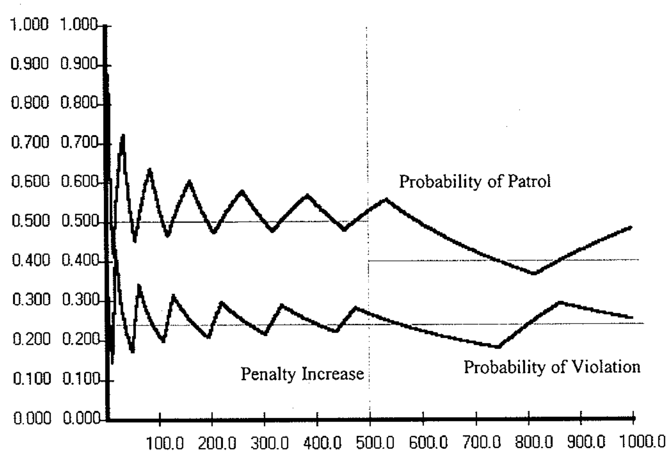
However, the number of cycles increased from 8 to 9 (compare figure 2 and figure 3). This means that the increase in penalty reduced the stability of a system. Game players change their behaviors more frequently with the increase in the penalty. This result is different to the second theorem of Tsebelis in the sense that the behavior of the driver is changed by the penalty increase.

On the other hand the probability of patrol is reduced by the penalty increase. Although the average value of probability of patrol is above the equilibrium of 0.4, it is reduced by 0.1. This result partly supports the third theorem of Tsebelis.

Now we have introduced the penalty-increase at the point of 500 simulation time to examine its effects on the evolutionary process of a mixed-game. Figure 4 shows time-behaviors of game players produced in our third simulation. A vertical line in figure 4 denotes the time of increasing the penalty. Horizontal lines shows the equilibrium probability of patrol and violation.

In figure 4, one can get two messages about the theorems of Tsebelis. First, a behavior of the driver is vastly changed by the increase in the penalty. The probability of violation is reduced under 0.25 for a considerable period. Second, after increasing the penalty, the width of response cycle is prolonged so that one can see less fluctuations. These messages raise a question on the second theorem of Tsebelis.

Of course, all of these do not continue forever. But the changing effects continue for a considerable period enough for the policeman to believe in the power of the penalty-increase. The fact that game-players come back to the equilibrium state after 750 time do not say that the increase in the penalty can't change the behavior of the driver. The fact is that an increase in the penalty decreased the violation tendency of the driver more than 250 times, which should be a critical period for policy makers.

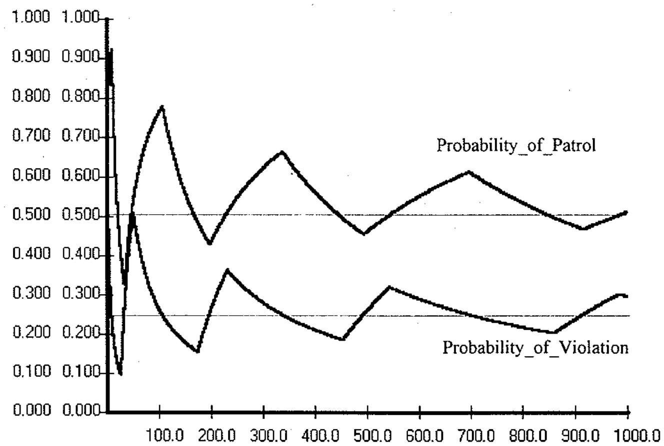


<Figure 4> The effects of increasing the penalty at 500 simulation time

3) Information Delay in a Mixed-strategy Game

We have not included information delays in the model for simplifying discussions about the game-theoretic equilibrium. It is reasonable to assume there are information delays between the police and the driver. We have assumed that it takes 6 times for game players to know the probability of other player's action. In the model two smooth functions were used to represent information delays of a driver and a policeman to figure out the probability of other players' action.

Figure 5 displays the simulation results. The response cycle of players are prolonged and have larger amplitude than before. That is, it takes longer time to reach the equilibrium state. It is due to the slow adaptiveness of players resulting from the information delay.



<Figure 5> Information delays and the time-behavior of mixed strategies

With the information delays, it is more difficult for the game players to reach equilibrium actions, and the penalty-increase will change the behavior of the driver for longer time. A set of experimental simulations shows that, in a mixed game, it takes long time for an equilibrium state to appear and that an increase in the penalty against violating the law decreases the probability of violation for a short time at least.

Our study shows that a game theoretic equilibrium is not a stable state. This comes from the fact that a game theoretic equilibrium lacks the transient behavior analysis. With the help of a game theory, one can find the equilibrium state. However, a game theory has its limit in showing various paths to the equilibrium state. In the real world where game players are myopic and limited resources, paths to the equilibrium state is more important to them than the equilibrium itself. We think that it is too dangerous to judge the effects of penalty-increase only on the basis of the game theoretic equilibrium. They should be analyzed within the evolutionary processes toward the equilibrium in which lots of unanticipated events arise.

4. Conclusions and Future Research

In this study, we developed a SD model for a mixed strategy game and showed that one should modify static theorems of a game theory for applying them to the dynamic world. Furthermore, our study showed that the penalty management can induce compliances from the people at least for a short time. Lacking the transient behavior analysis, most of game theories say little about how a system evolves to the equilibrium. The policy recommendations based only on the game theory seem to be dangerous.

We suggest that integrations of a game theory and system dynamics will provide a safe ground for describing and recommending policies in real world. System dynamics will simulate evolutionary processes of games toward (non) equilibrium states, while game theories provide frameworks for modelling the world which is filled with competitions and cooperations.

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[Appendix: Model Equations]

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Level Violation
        Init = 1.
        Violation = Violation + DT*((Decision_of_Driver  -  1) abs)
Level Conform
        Init = 1.
        Conform = Conform + DT*(Decision_of_Driver)
Level Patrol
        Init = 1.
        Patrol = Patrol + DT*(Decision_of_Police)
Level NoPatrol
        Init = 1.
        NoPatrol = NoPatrol + DT*((Decision_of_Police - 1) abs)
Rate Decision_of_Driver = IF Expect_Utility__of_Conform > Expect_Utility__of_Violation Then 1 Else 0
Rate Decision_of_Police = IF Expect_Utility__of_Patrol > Expect_Utility__of_No_Patrol Then 1 Else 0
Aux Probability_of_Patrol = Patrol / (Patrol + NoPatrol )
Aux Expect_Utility__of_Violation = Driver_Payoff_for__Violation_with_Patrol * Probability_of_Patrol +
        (Driver_Payoff_for__Violation_with_No_Patrol * (1 - Probability_of_Patrol ))
Aux Expect_Utility__of_Conform = Driver_Payoff_for__Conform_with_Patrol * Probability_of_Patrol +
        (Driver_Payoff_for__Conform_with_No_Patrol * (1 - Probability_of_Patrol ))
Aux Probability_of_Violation = Violation / (Violation + Conform )
Aux Expect_Utility__of_Patrol = Police_Payoff_for__Violation_with_Patrol * Probability_of_Violation +
        (Police_Payoff_for__Conform_with_Patrol * (1 - Probability_of_Violation ))
Aux Expect_Utility__of_No_Patrol =
        Police_Payoff_for__Violation_with_No_Patrol * Probability_of_Violation +
        ( Police_Payoff_for__Conform_with_No_Patrol * (1 - Probability_of_Violation ))
Constant Driver_Payoff_for__Violation_with_No_Patrol = 4
Constant Driver_Payoff_for__Conform_with_No_Patrol = 2
Constant Driver_Payoff_for__Violation_with_Patrol = 1
Constant Driver_Payoff_for__Conform_with_Patrol = 3
Constant Police_Payoff_for__Violation_with_No_Patrol = 1
Constant Police_Payoff_for__Violation_with_Patrol = 4
Constant Police_Payoff_for__Conform_with_No_Patrol = 3
Constant Police_Payoff_for__Conform_with_Patrol = 2
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