Feedback and Control in a Product Portfolio Management Model

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Abstract

An optimal control-based decision support model is developed which allows managers and future managers to gain hands-on experience with product portfolio management in a dynamic micro-world. In this micro-world, they study one of several scenarios, set objectives and importance hierarchies, create action plans, and control the system over time en route to their objectives. The system is demonstrated with data from an actual product portfolio management case.

Using the system, a manager or player can gain sophistication with decision-making as well as assess the capabilities of dynamic models for decision support. Working with the model gives players a feel for such important insights as the lag effects of response, the differential impact of various marketing tools, cross-elasticities, and the potential for cannibalization. A manager can devise promotional strategies to achieve specific sales results for a particular brand or product line, test those strategies, and learn as the system evolves. Backing up the micro-environment is a discrete-time optimal control model which allows the system to be optimized from the perspective of a decision-maker. Players can test their own strategies against those of the optimal control "shadow player".

Used as an optimization-based simulation tool, the model allows flexibility in the testing of alternative strategies and scenarios which affect achievement of product portfolio objectives. The model focuses on the overall objectives of the portfolio while recognizing the objectives and dynamics of the products within it.

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Introduction

The portfolio approach to product and market management has been recognized for a number of years and has become the subject of a significant body of research. In this framework, a product portfolio is generally defined as a group of business units, products, or other assets which are managed toward a single set of objectives and draw from the same stock of resources, even though each may have its own characteristic growth rate, profitability, and role within the portfolio.

Managers at several levels in a corporate hierarchy can be viewed in product portfolio management terms, each responsible for a portfolio of profit-making assets aggregated from lower levels in the organization. At the corporate level, management controls a portfolio of divisions, businesses, or strategic business units, with markets the typical unit of analysis and long term investment, risk, and payoff elements the major focus of decision-making. At the business unit, a portfolio manager typically has under his or her care a group of brands or product lines. One of the major questions at this level is how to allocate the marketing budget among products and aggregate marketing mix tools so as to best reach the objectives of the business unit. Each combination of these tools will have its own sales response function, and the response will differ across the set of products and over time. At the brand level, managers control portfolios of colors, flavors, sizes, brand extensions, and so on with short term objectives and tactical tools (e.g., promotions, prices, salesperson motivators, etc.). Their task is to blend the set of dollars allocated to a brand into detailed expenditure programs to achieve weekly and monthly objectives.

A Product Portfolio Management Micro-world

This paper introduces a dynamic model-based micro-world environment for teaching managers/players the skills and mindset of product portfolio management. Although it is recognized that even relatively simple dynamic systems have the potential for extremely erratic, complex outcomes, this paper follows Morrison (1991) in claiming that good research and teaching is still possible via careful model building and use. To be a useful teaching tool for enhancing actual product portfolio management in a dynamic world, this model-based micro-world must:

1. be centered around a realistic model of market response. This requires a set of parameterized response functions which equate performance over time with the decisions which bring it about. A corollary, of course, this that the model be built around the decision variables over which the system manager actually has control. The major cautions are oversimplification and overcomplexity.
2. be built so it can fit into and/or extend the capabilities of a company's existing planning process. This means that the model should be able to accept inputs from as well as complement the generic, box-type, or other planning tools (see Porter 1986 or Thompson and Strickland 1992, for example) used in today's multi-product companies.
3. be goal-driven to reflect the nature of business planning, i.e., be based on required profit, sales, stock price, or other objectives that must be achieved over a certain time period.
4. be able to deal with a hierarchy of objectives, where some product lines are more important than others and all are subordinate to the performance of the portfolio itself. Also included in this hierarchy are the different levels of importance and often conflicting objectives for different performance variables, say sales, costs and market share.
5. recognize the real-world importance of disciplined growth, which can be interpreted as the ability to maintain a controlled trajectory of performance over time.
6. be flexible enough to evaluate and redefine goals for the portfolio if necessary.
7. include a mechanism to reflect the parameter and other changes expected in market response over the planning period, especially if the model is to be used over a longer horizon than, say, one year.
8. generate outcomes which achieve the goals of the company in the "best" way possible, which requires an underlying optimization mechanism.
9. be able to serve as a focus for testing of alternative marketing- based product planning models, strategies, and research in general.

An Optimal Control Approach

A modeling framework with the ability to address all these issues exists under the heading of optimal control. The theories and methods of optimal control have been developed primarily in the last thirty years, although the methods are similar to the ones proposed in 1696 by Johann Bernoulli.

The impetus for the development of control theory emerged in the context of aerospace research, particularly problems of designing guidance systems for space vehicles. More recently, the concepts of optimal control have been directed toward the management of national economies over time (Otter 1984).

An optimal control problem consists essentially of a system of relationships among variables and time. The system consists of a vector of state variables (exogenous or output variables that define the "state" of the system at any one point in time, e.g., market share, sales, or profit levels), a vector of control variables (tools which can be purposely manipulated to effect a system's evolution, e.g., marketing expenditures), a system of equations of motion which relate the state and control variables over time, an objective function which reflects the goals of management, and external disturbances on the system.

In the marketing area, work by Fitzroy (1965) is an early effort at optimal control for marketing, although the initial foundation for marketing decision optimization was laid by authors like Nerlove and Arrow (1962) and Dorfman and Steiner (1957). Adaptive control methods for selecting optimal marketing policies while learning about the system dynamics have been proposed by Little (1977). Dolan and Jeuland (1981) present an optimal control methodology for choosing the optimal pricing strategy over the product life cycle. A list of optimal control applications in marketing is shown in Parsons, Hanssens, and Schultz (1990, pp280-83).

For whatever situation is being modeled, the creation of a realistic micro-world within an optimal control framework requires several tasks in advance of any manager-game interaction.

Construction and testing of the equations of motion.
- -specification of state and control variables
- -time series analysis, pre-whitening, and causal ordering tests
- -deciding the nature and shape of the response function
- -testing lag structures
- -specifying the nature of feedback
- -parameter estimation

Choosing the form and details of the objective function.
- -form
- -goals
- -penalty function and weights

Testing for and estimating time varying parameters.

Deciding on the form of system control and learning.

Setting up the form and details of interaction between the manager/player and the game/scenario to reflect real decision-making and stimulate generalization.

These tasks are certainly not trivial for any firm. However, although typically invisible to managers, this type information is the objective of much of the marketing and strategic research that already goes on in farsighted firms. It may just be called by another name.

The Setting

The product portfolio model from which this micro-world was developed was built for a large western wear specialty retailer in a Southeastern MSA in the United States. Many retail organizations now recognize their various department as elements in a portfolio (Rosenbloom 1980; Mason, Mayer, and Wilkenson 1993) and try to manage the departments so as to optimize overall store results. Product portfolio management involves allocation of resources to each department in the context of a set of overall store level objectives. This context provides a good teaching forum for product portfolio management: a natural department-centered product portfolio already exists, the products are familiar, and the planning cycle and time lags are relatively short.

There are eleven departments in the product portfolio model. They include **jeans** (one each for **Levi**'s, **Wrangler**, and **Lee** brands), **men's clothing, men's straw and casual western hats, men's "dress" western hats**, **men's western boots**, **women's boots**, **women's clothing** (including women's jeans) a **boy's department** (including boy's boots), and **moccasins**.

The objective in this retail micro-world is to manage the store's set of portfolio elements toward a set of storewide and department objectives over a specified planning horizon by (1) planning a time path of weekly marketing spending decisions, and (2) controlling the portfolio over time by reacting to feedback from store and department performance. Detailed information is written up for trainees explaining the product assortments, sales levels, and promotion histories of each department as well as specific problem scenarios.

Developing the Equations of Motion

For this scenario, the same linear econometric sales response models as were created for management form the basis for the equations of motion. The response equations which were used to build the control model are shown (without error terms) in Table 1 for six of the eleven departments. The problem was to take the set of predictive models of the form

$$y_k = c + A_1 y_{k-1} + A_2 x_k + A_3 x_{k-q} + B_1 u_k + B_2 u_{k-q}$$

where:

- y_k = a vector of department sales in week k
- c = a vector of constant terms
- A_1 = a matrix of estimated parameters on lagged sales
- y_{k-1} = a vector of previous week sales
- A_2 = a matrix of estimated parameters on state variables in week k
- x_k = a vector of state (non-controllable) variables in week k
- A_3 = a matrix of estimated parameters on state variables in previous weeks
- x_{k-q} = a vector of state (non-controllable) variables in previous weeks
- B_1 = a matrix of estimated parameters on control variables in week k
- u_k = a vector of managerial control (decision) variables in week k
- B_2 = a matrix of estimated parameters on control variables in previous weeks
- u_{k-q} = a vector of managerial control (decision) variables in previous weeks

and transform them into a form compatible with efficient control modeling.

The first step in adding the dynamic element necessary for equations of motion is to move the dependent state variable of major interest (y_k) ahead to period k+1 so that the planning effort concentrates on the future rather than the present. Next, it is more convenient from both the planning standpoint and the solution standpoint to redefine <u>actions</u> in period k as <u>decisions to act</u> in period k-1. This separates the time periods for decisions and their consequences. For example, newspaper inches advertised in week k+1 are modeled as promotion <u>decisions</u> made during week k. The result of these two transformations is a system:

$$y_{k+1} = c + A_1 y_k + A_2 x_k + A_3 x_{k-q} + B_1 u_k + B_2 u_{k-q}$$

The next step is to convert the system into a set of solely first order difference equations, which requires that the lagged relationships longer than one week be redefined for modeling purposes. This step transforms the overall problem into the minimization of a quadratic form subject to a set of first order difference equations, which is the classical quadratic-linear problem for which efficient solution techniques exist. Following Pindyck (1973, p.97), it is necessary to define new vectors for each period where a significant lagged relationship has been estimated. For this case where the maximum significant lag is two weeks, define:

$$w_k = u_{k-1} \text{ where } w_{k+1} = u_k \text{ for lagged control variables}$$
$$v_k = x_{k-1} \text{ where } v_{k+1} = x_k \text{ for lagged state variables.}$$

The system of equations is then rewritten:

$$y_{k+1} = c + A_1 y_k + A_2 x_k + A_3 v_k + B_2 w_k + B_1 u_k$$

where the only promotion decision variables controllable by management during period k are u_k. When the state vector is augmented by defining:

$$z_k = \begin{matrix} x_k \\ w_k \\ v_k \end{matrix}$$

the shortened form of the equation system becomes:

$$z_{k+1} = Az_k + Bu_k + c$$

which is a set of first order difference equations in z. The solution technique utilized in this research is the dynamic programming code DUAL (Kendrick 1983).

The sales response levels to the various promotional tools, as well as the lags, cross-elasticities, and other information captured in the set of parameter estimates is provided as decision assistance for the managers/players.

The Objective Function

The real differentiating element in an optimal control-based model compared to straight econometric or simpler Lotus-type simulations centers around the objective function. It is the objective function that provides the optimization underpinning of the model. Depending on the nature of the problem, management can try to maximize or minimize the value of the objective function over the time period (e.g., maximize sales, market share, ROI, or growth, minimize cost or time, or find an optimal combination of sales and cost). Another use of the objective function is in tracking, where the managerial goal is to minimize the discrepancy between desired and actual levels of both state and control variable levels at each time interval over a specified time horizon. This model employs a quadratic loss function and linear systems equations in a tracking problem context (see Kendrick 1981). The objective function is set up as:

Find $u_k \, {}_{k=0}^{n-1}$ to minimize J where J =

$$1/2(x_n-x_n)'W_n(x_n-x_n)+1/2 \, {}_{k=0}^{n-1} [(x_k-x_k)'W_k(x_k-x_k)+(u_k-u_k)'V_k(u_k-u_k)]$$

given that

x_n and x_n = realized and desired vectors of state variables during the last period of the planning process.

x_k and x_k = realized and desired vectors of state variables in each period k of the planning process.

W_n and W_k = penalty matrices on deviations of realized state variables from desired state variables at last period and at each period k.

u_k and u_k = realized and desired vectors of controllable variables in period k.

V_k = penalty matrix on deviations of realized controllable variables from desired control variables at each period k.

subject to:

$$x_{k+1} = Ax_k + Bu_k + c$$

which is the set of first-order equations of motion.

Manager-Game Interaction

Any model which is to be used for training management at product portfolio management needs to revolve around realistic portfolio management decisions:

1. establishment of goals for the portfolio as a whole and for each of its members, stated in terms of specific output variables (e.g., ROI, market share, growth, costs, sales, image rating, recall, and so forth). This set of objectives can be chosen by trainees or imposed from above depending on the role of that portfolio in the larger corporate portfolio.
2. selection of a set of action plans (controllable inputs) which drive the portfolio (and members) to overall goals by the end of the planning horizon. These take the form of planned allocations of available resources over the planning period.
3. dynamic control, which involves methods for (a) keeping the portfolio (or its members) on track over the course of the planning horizon, or (b) returning the portfolio (or its members) to their planned performance paths as the system moves through time.

Other keys to a successful micro-world model are to keep decision-making by players at the strategy level rather than the mathematical level and to keep mathematical complexities hidden in the background unless relevant to the decision-making process.

In the retail setting described above, the vector of decision variables which need to be set for every period of the planning process includes :

Print

-the number of newspaper column inches by newspaper devoted to each department (and cost).
-the price discount to featured in the advertisement. These discounts range from 0% to 50%.
-the placement of the ad (section of the newspaper)
-whether a coupon is to be included for a particular department and the amount.
-any special considerations (color, special section, etc.)

Radio

the number of radio advertisements, the time of day, the departments featured, and the cost by date.

Billboards

the number of billboards, cost (prorated), and whether the billboard is a generic store advertisement or features a particular department. (The number of billboards ranged from a low of two to a high of twelve at any one time for specific advertising campaigns in the database).

Television

television advertising by time, cost, and department featured (if any).

Other

Special sales, giveaways, contests, and so on.

In this case, external information about the product portfolio which would need to be communicated to players/managers would include:

-the varying levels of importance of performance for the different departments of the store. While an unconstrained control model might recommend de-emphasizing certain departments, such recommendations may not be managerially acceptable for various reasons.

-special situations for individual departments at different times during the planning period. For example, some external mechanism must tell the model that Father's Day sale planning should revolve around largely male-oriented departments.

Penalty Weights

In addition to promotion and spending paths, another important input which must be generated by the players is the set of weights, or penalties, on failure to attain or maintain the desired performance levels over the planning period. They are used in the objective function of the optimal control model and are set as dollar costs on the failure to achieve each of the goals set for the portfolio. These weights allow players to interact with the model and assign different levels of important to attainment of different objectives.

In addition to penalizing deviations of product line performance from planned performance, there are a number of subtleties possible in the use of penalty weights to affect the modeling process. Where a smooth growth path in sales and expenditures over the planned period is desired, for example, larger penalties are put on period to period deviations from plan. If final period performance (as in market share) is the critical factor, the major penalty will be on failure to achieve, not maintain objectives. It is at this penalty assignment process where numerous possible uses of control modeling exist, and it is here where much future research is needed.

Time-Varying Parameter Shifts

It has long been recognized that the parameters associated with dynamic models do not tend to remain constant over time. For the product portfolio problem, exogenous events will often have an impact on the effect of various marketing mix elements as the system moves through a planning period. Where such information is included in the model, optimal paths for the control variables can be chosen which will reflect the expected parameter changes. This is especially important for a longer planning process.

Control and Learning

A central issue in dynamic control, and by association a central issue in development of a product portfolio management model, involves the treatment of uncertainty in the modeling process. There are typically considerable uncertainties about the response of the system to specific levels of control variables and the length of time over which these effects are felt by the system. There may also be uncontrollable variables or unspecified variables which affect the transition of the state variables from period to period. There may be errors in actual measurement of state variables. There is certainly uncertainty about the future trends or events which can affect the state of the parameters of the equations in motion themselves.

If the effects of uncertainty about the transformation processes and possible disturbances on the system are ignored, the analysis approach is deterministic. In passive-learning control, the optimal controls are selected for a time period and applied to the system. After the system moves forward in time, a new measurement is taken. The random shocks and other unknown effects which influence the system thus influence the measurements. Using this information, new estimates for the state variables in the parameters are made. In active learning (also known as closed-loop, or adaptive control) mode, a control model makes decisions for each time period so as to both reach the desired paths for that time period and to gain information about the system which will make it easier to control in the future. Where there is substantial uncertainty about the response of the system and the dangers of poor control are greater, then the time spent "probing" the system will tend to be more rewarded.

An Example

Any number of specific scenarios can be designed for player/game interaction. The scena originally used to show the store manager the potential of the model was to match the shadow mod optimal performance against his actual decisions. The data for four similar sale periods w combined to create one ten week "storewide sale" period. As such, the managerial input vectors w taken from actual performance.

The objective established for the control model was to outperform actual store performance both the sales and cost variables over this hypothetical ten week period. The model was asked achieve for each of the ten weeks the highest level of storewide sales obtained in any of the f actual periods evaluated. The objectives for individual departments were established in the sa way, but were made subservient to the storewide objective via the penalty weights. This emphasi the importance of the store's overall "hard" goals but allowed the model to recommend major as w as minor reallocations of promotional effort among the departments (if warranted) based on th direct and cross elasticities of sales response to promotional tools. The desired promotion bud was minimized to emphasize cost containment.

Total Store Performance: Optimal Control Versus Actual

Figure 1A shows the total store's sales path and the model's sales path by week for the n "storewide sale" period, while Figure 1B shows the promotion spending paths week k-1). Wh actual sales and model-generated sales both increased substantially in weeks three, four, and five, t optimal control model predicted sales nearly $24,000 higher during those three weeks if the mod generated promotion strategy decisions were implemented. The difference in model-generated ver actual promotion spending paths is significant but not as spectacular, with the most noticea changes 1) a one week peak in spending rather than a three week spending blitz, and 2) a m gradual tailing off period than was exhibited in actuality. Overall, the model-generated sales tc exceeds actual sales by more than $55,000 or 29% over the 10 week period, while model-genera promotional expenditures are nearly $1,700 or 13% lower than the average of the four ma storewide sales events studied. In general, the model was able to significantly improve upon overall performance of the store by improving the allocation of resources among the departments generate higher sales levels at lower promotion costs.

Department-Level Analysis

Figures 2A and 2B show the model-generated sales paths for four departments and promoti paths for four controllable variables. Tables 2 and 3 show how actual sales levels and promoti levels compare to model-generated results for six selected departments. Major differences in composition of the promotion mix across the ten week period are apparent. For example, Tabl shows that the model recommended the use of over 7 billboards per week during the ten week per as compared to the average of just over 4 per week actually purchased. Billboard advertising sho have been substantially higher during weeks three, four and five, based on the model, because of blend of multi-department response and relatively low cost. At the same time, the mo recommends significant cuts in Levi's advertising relative to what management actually spent. shown in Table 2, the model generates improved performance in each of the store's departme compared to actual sales levels generated by management.

Overall, the major reasons the superior performance of the model appear to be that 1) optimal control model generates a promotional mix with the highest response elasticities over ti for each department, and 2) the model concentrates on maximizing synergies by allocating funds the promotional mix which generate the highest multi-department sales response. Once There nothing magical about this finding. Once such an insight is ingrained in the decision-mak mentality of management, it is expected that significant improvements can be made there as well.

Summary

The optimal control-based product portfolio management model shown here appears to h significant potential to teach managers to make better dynamic planning and control decisions emulating the decision processes of the model. The model's ability to blend management-genera objectives with optimal decision paths, its ability to address the issue of tradeoffs among compet objectives, and its emphasis on period-by-period plan adjustments are all innovations that manag can adopt. This type micro-world also allows management to develop "what if" scenarios determine in advance the results of alternative resource allocation decisio

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Table 1
Selected Sales Response Parameters within Equations of Motion

Variables (sig at .05 or better)	Levi's Jeans	Men's Clothing	Men's Boots	Women's Boots	Women's Clothing
Constant	246.8		1810.2	294.7	361.9
Sales -1 wk lag	.055	.85	.42	.67	.63
Ad Inches Levi's Jeans	9.4				
Ad Inches M-Clothing	10.4	12.5			
Ad Inches M-Boots -25%	12.6		12.5		
Ad Inches M-Boots -35%	7.7		41.4		
Ad Inches W-Clothing -0%		10.6	30.9		13.5
Ad Inches W-Clothing -25%				14.7	15.6
Seconds - Radio	.41				
Billboards - per wk	75.2	35.3			
Coupons -1 wk lag			701.7		
Adjusted R-Square	.89	.90	.83	.73	.73

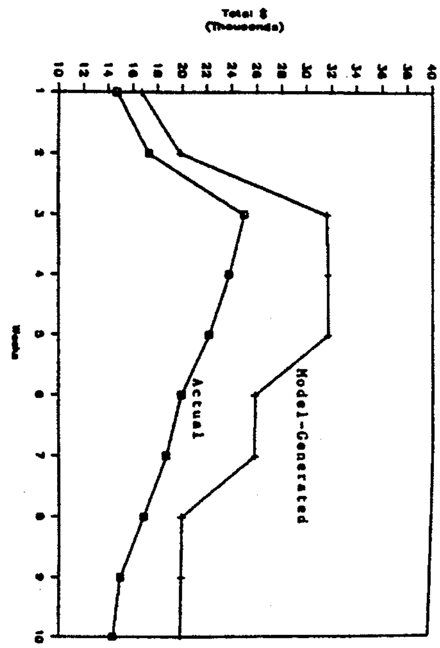
Table 2
Actual versus Model-Generated Sales for Selected Departments

Depts	LEVI'S		WRANGLER		MEN'S CLOTHING		MEN'S BOOTS		WOMEN'S BOOTS		WOMEN'S CLOTHING	
Week	Actual	Model	Actual	Model	Actual	Model	Actual	Model	Actual	Model	Actual	Model
1	$2654	$3051	$150	$318	$1637	$2559	$3382	$4264	$1196	$1392	$989	$1868
2	2807	3573	188	363	1955	2998	4192	4629	1310	1614	1446	2446
3	2986	4682	146	524	2522	3838	8369	7542	1182	1852	1698	3879
4	4090	4711	324	538	2503	4166	6503	7212	1160	1894	2329	3885
5	4306	4472	156	547	2433	4261	6452	7377	1004	1825	1649	3656
6	3408	3766	167	456	2058	3983	5544	5909	946	1682	1689	2777
7	2818	3504	167	441	3778	3820	3821	6299	986	1572	1420	2528
8	2702	2910	129	352	1586	3417	4391	4816	841	1449	1355	1754
9	2885	2837	255	333	1373	3238	3652	4994	800	1399	908	1676
10	2325	2836	391	319	1474	3117	3567	5042	696	1365	1217	1678
Total	$30.981	$36.343	$2.074	$4.191	$21.320	$35.396	$49.873	$58.083	$10.120	$16.044	$14.700	$26.146

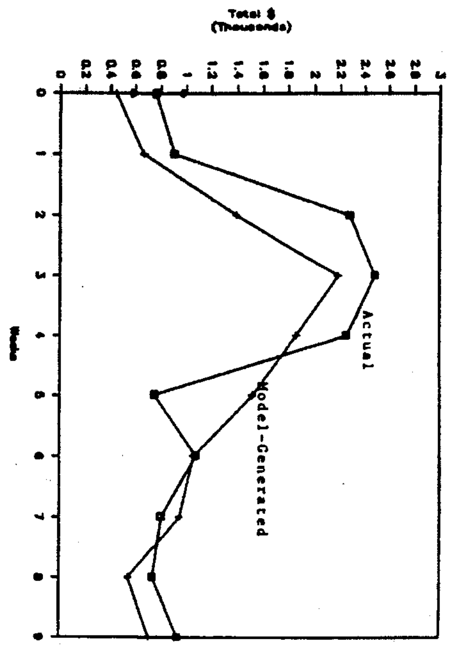
Table 3
Actual versus Model-Generated Promotion Expenditures

					Newspaper Column-Inches of Advertising at Regular Price						Newspaper Column-Inches at Reduced Price						Newspaper Column-Inches			
	Number of Billboards		Coupon Program		Levi's		Wrangler		Men's Clothing		25% Off Men's Boots		30% Off Men's Boots		35% Off Men's Boots		Women's Clothing at Regular Price		Women's Clothing at 25% Off	
Week	Actual	Model	Actual	Model	Actual	Model	Actual	Model	Actual	Model	Actual	Model	Actual	Model	Actual	Model	Actual	Model	Actual	Model
0	3	7	0	1	55	4	0	8	0	16	0	19	8	0	0	0	0	1	0	18
1	3	12	0	1	47	5	3	9	3	22	0	28	7	1	4	1	0	1	18	26
2	3	12	1	1	44	11	9	21	17	34	19	43	32	1	13	1	0	1	30	32
3	6	11	1	1	52	11	6	18	12	21	23	33	25	1	9	1	0	0	22	24
4	6	9	0.	0	47	13	2	18	6	15	0	28	16	1	3	1	0	0	9	17
5	6	5	0	1	39	11	0	11	0	5	0	17	0	1	13	1	0	1	0	11
6	4	6	0	0	51	13	5	12	0	8	0	18	0	1	8	1	0	0	7	10
7	5	3	0	0	44	10	4	6	0	1	0	10	0	0	8	0	0	0	5	7
8	5	5	0	0	52	11	0	6	0	6	0	14	8	0	0	0	0	0	0	9
9	2	5	0	0	40	10	26	6	3	7	0	14	7	0	4	0	0	0	18	9
Total	42	74	2	7	471	98	54	116	39	135	42	223	101	6	60	6	0	5	108	163

Figure 1

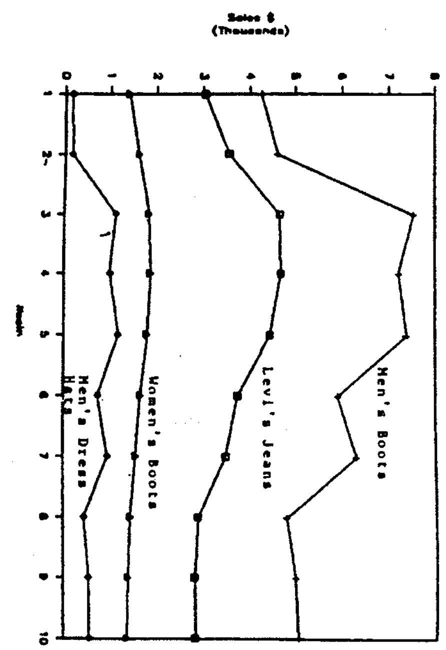


a. Total Sales Paths: Model-Generated Versus Actual

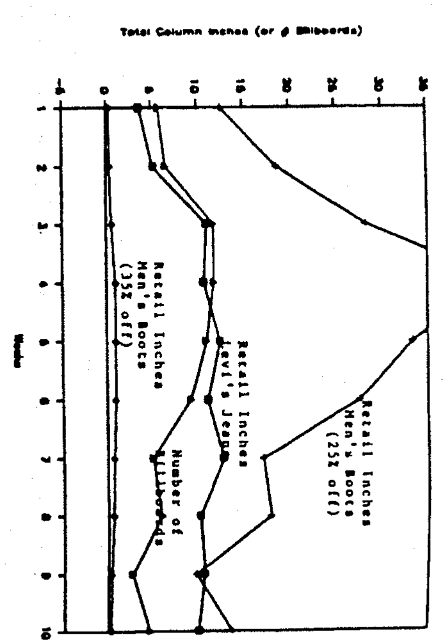


b. Total Promotion Mix Spending Decisions: Model-Generated Versus Actual

Figure 2



a. Model-Generated Sales Paths for Selected Departments



a. Model-Generated Paths for Selected Promotional Variables