A MODERN CONTROL ENGINEERING APPROACH TO SYSTEM DYNAMICS

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ABSTRACT

The paper reviews, briefly, the development of system dynamics (SD) and presents a modern control engineering approach. It formulates and solves the SD policy design problem as a model-following control system design problem in an adaptive control framework. A computationally simple policy algorithm based on variable-structure system theory is used in an illustrative example of the stabilisation of the dynamic characteristics of a production/raw materials system. Computer simulation results are given for the modern control approach as well as the classical SD techniques. Directions in which the modern control approach could be developed are indicated.

1. Introduction

Over the past two decades, system dynamics (SD), initiated by Forrester [1], has developed into an effective technique for mathematically modelling and analysing the dynamics of diverse socio-economic phenomena. The flow diagram of system dynamics in Fig.1 shows three paths of development of the subject. Starting with a given real-world socio-economic process, Forrester showed, perhaps for the first time, how the basic interactions between the variables of such a system can be captured as a tissue of feedback loops in an influence (or causal-loop) diagram. With the notions of flows, accummulations and delays of variables, he provided a nomenclature for setting up flow diagrams from the influence diagram. He further developed a computational procedure, DYNAMO, which is a numerical integration procedure directly suitable for digital computer simulation.

In the digital computer simulations, variations of the classical three-term controller have been exclusively used as policy functions.

In studying certain socio-economic problems, what is sometimes required is a simple and systematic method of thinking a problem through, and documenting it. The emphasis is on qualitative analysis of a complex problem and a heuristic appreciation of the consequences of various decision actions. A methodology for qualitative SD analysis has recently been developed [4].

The third approach is that of modern control engineering. It consists of firstly translating the influence diagram into an analogue computer flow diagram and setting up state-space equations from it.

The second stage is to bring all relevant results in modern system theory and modern controller design techniques to bear on the policy design problem. Digital computer simulation is used as an aid.

A departure may be made from Forrester's precept that flows and accummulations in socio-economic systems are continuous-time processes. Or even if they are trully so we may choose to take a sampled-data view of these processes. Either way, the influence diagram may be translated into a digital filter diagram from which discrete-time state-space equations may also be easily developed.

This paper reports some initial results in following the (continuous-time) modern control engineering path in system dynamics. It has been chosen because Forrester's Path has the following short comings:

1) it has led to little attention being paid to the very important issue of policy design;

2) it has not benefitted from powerful results in feedback theory, and in modern system theory based on the structure of state-space models, e.g. stability, controllability, etc; and

3/.....

3) it has not benefitted from modern controller design techniques.

The authors view their efforts here as an attempt to place SD in its proper setting of system and control engineering.

Managed systems constitute one class of socio-economic problems to which the SD methodology has been widely applied [2]. We illustrate the modern control engineering approach with an example from this field.

In a recent paper [3], the stabilisation of the dynamic characteristics of a production/raw materials system was used to elucidate some problems of system dynamics modelling of managed systems. The central points of that paper may be re-stated briefly as follows:

a) The problem is the interaction between the production manager and the raw materials manager in a "typical" consumer goods manufacturing firm. Orders for goods are received from distributors and accummulated into an orders backlog which the production manager attempts to control to a reasonable level by varying the production rate. Production uses up raw materials and the raw materials manager re-orders them so as to keep stocks to an adequate level.

b) The pattern of orders is generally cyclical and the pattern of production was also cyclical with an amplitude roughly one-half that of orders. Raw material stocks, however, fluctuated in an explosive manner causing complaints from raw materials suppliers and problems in the works and the raw materials manager's competence was generally in question. See Fig.3 produced by the model of Fig.2 representing the original system.

The problem is to bring this behaviour under control.

c) In Reference 2, system dynamics is defined to be the application of the attitude of mind, and some of the approaches, of a control engineer to the design of regulatory policies for managed systems. The purpose of Ref.3 was to demonstrate that viewpoint.

d) It is recognised, however, that there are appreciable differences between the design requirements for managed and engineering systems so that the system dynamicist must, perforce, proceed by a mixture of common sense, experience, and rules of thumb derived from control engineering practice in the search for improved policies. This approach will usually be applied via a simulation model.

5/.....

In Ref.3, that approach was applied to generate three options for alternative control strategies and it was shown not only that they enabled stabilisation of the system to be achieved but that the performance of the system differed significantly between the options.

This paper addresses the central problem of managed system analysis, viz policy design, from a modern control engineering viewpoint. Specifically, it formulates and solves the SD policy design problem as a model-following control problem in an adaptive control framework.

The analysis starts in section 2 with the influence diagram of the production/raw materials system in Fig.2 translated into a continuous-time state-space model. The policy design problem is formulated in section 3. Linear model-following control interpretations are discussed in section 4, together with a policy algorithm derived from variable-structure system theory.

In the course of the exposition, some observations are made to highlight some differences of emphasis between system dynamics and control engineering. The authors view the differences as posing a potentially rewarding challenge to the control engineer who wishes to contribute to the analysis of managed systems.

6/.....

2. Analogue model

The steps followed in forming a system dynamics model are clearly illustrated in Ref.4. In this exposition we extend the procedure somewhat to facilitate the development of the control engineering viewpoint. Thus starting with the influence diagram of Fig.2, we proceed to draw an analogue computer flow diagram for the model using deviational system variables about some nominal operating values. Then we develop state-space equations from the flow diagram.

To develop dynamical equations in terms of deviational variables, let

$NO = \overline{NO} + w$	$PR = \overline{PR} + u_1$
$BLOG = \overline{BLOG} + x_{p1}$	$PMOR = \overline{PMOR} + u_2$
$DBLOG = \overline{DBLOG} + x_{m1}$	$RMAR = \overline{RMAR} + u'_2$
$RMS = \overline{RMS} + x_{p2}$	$SO = \overline{SO} + e_1$
$DRMS = \overline{DRMS} + x_{m2}$	$SRM = \overline{SRM} + e_2$
$AOR = \overline{AOR} + y_1$	$APR = \overline{APR} + y_2$

Where the bar $(\overline{\ })$ denotes a nominal steady-state operating value. As Ref.3 shows, it is easy to choose nominal values for the system variables to obtain equilibrium, characterised by SO = SRM = O. About equilibrium, the dynamics are described in terms of the (deviational) state vector, $\underline{x}_p$; desired state vector, $\underline{x}_m$; error vector, $\underline{e}$; policy (or control) vector, $\underline{u}$; and the exogenous input vector, $\underline{w}$.

7/.....

It is a basic tenet of system dynamics that the resource and information flows and accummulations in a socio-economic system are continuous in time so that the deviational variables defined above are all analogue.

A procedure is now given for translating an influence diagram into an analogue computer flow diagram, using only standard symbols. Level: A level (or state variable) is the output of an integrator whose inputs are flows, signed + if they increase the level or - if they deplete it. See the Table.

Smoothed level: From the influence diagram, Fig.2, a typical smoothed level is Average Order Rate (AOR) having the DYSMAP [5] equation:

L AOR.K = AOR.J + (DT/AOR)(NO.JK - AOR.J).

The corresponding integral equation is

$$AOR = \int \alpha (NO - AOR)\, dt \ , \ \alpha = {}^{1}/TAOR$$

with analogue computer circuit as shown in the Table.

Delay: Delay in resource or information flow is often modelled in SD by an exponential delay, typically by a third-order delay. The DYSMAP equations for a first-order exponential delay with time delay, DDEL, are:

L LEV.K = LEV.J + DT * (INPUT.JK - OUTPUT.JK)

R OUTPUT.KL = LEV.K/DDEL

8/.....

The corresponding integro-algebraic equations are

$$LEV = \int (INPUT - OUTPUT)\, dt$$

$$OUTPUT = \gamma_* LEV$$

$$\gamma = {}^{1}/DDEL.$$

An nth-order exponential delay is a cascade of n first-order delays each with

$$\gamma = {}^{n}/DDEL,$$

DDEL being the overall time delay.

The Table shows analogue computer circuits for first- and third-order exponential delays.

As delay in information flow is a time displacement, it would seem that a Pade approximant would be a more appropriate model in such cases. Circuits of Pade approximants are, however, not given here.

For a linear SD model the units described above, together with standard analogue computer symbols for potentiometer and summing amplifier, are sufficient to translate an influence diagram into an analogue computer flow diagram. The Table summarises these "building blocks" for ease of reference. Nonlinearities are easily accommodated in analogue computer diagrams.

9/.....

In view of the above, then, the influence diagram of Fig.2 easily translates into the analogue computer flow diagram of Fig.4, starting with the integrations in the physical flow modules and adding on the "building blocks".

<u>Plant and reference model</u>

Controller design techniques in control engineering require knowledge, in the form of a mathematical model, of the open-loop system to be controlled, here called the plant. In a state-space continuous-time formulation a linear plant is typically described by

$$\dot{\underline{z}}(t) = A\, \underline{z}(t) + B\, \underline{u}(t) + H\, \underline{v}(t) \qquad (1)$$

$$\underline{\mu}(t) = C\, \underline{z}(t) \qquad (2)$$

where the matrices A, B, H and C may be constant or time-varying; $\underline{z}$ is the state vector; $\underline{u}$ the control vector; $\underline{v}$ a vector of disturbances; and $\underline{\mu}$ the output vector.

To bring control theory to bear on a managed system, then it is necessary to identify the plant. In managed systems this identification is not as obvious as in engineering. We consider as plant the aggregate of parts of the system, apart from parts generating the desired state variables, that generate information necessary for policy design. This will subsume the resource flow modules and parts of the information modules.

10/.....

Fig.3 specifies the plant with a third-order pipeline delay, and a reference model (in adaptive control terminology) corresponding to the production/raw materials system in open loop.

It must be noted that the system dynamicist has a flexibility that the control engineer rarely has in being able to re-structure the system, in addition to designing policies to obtain a satisfactory overall model. Given the great complexity, and sometimes severe non-linearity, of managed systems, it is probably only this freedom to change the structure of the system which makes policy design a practical proposition.

Dynamical equations for the units of the system shown in Fig.3 may be written as follows. Plant equations:

$$\begin{aligned}\dot{x}_{p1} &= w - u_1\\ \dot{x}_{p2} &= \theta_1 - u_1\\ \dot{\theta}_1 &= -\gamma\theta_1 + \gamma\theta_2\\ \dot{\theta}_2 &= -\gamma\theta_2 + \gamma\theta_3\\ \dot{\theta}_3 &= -\gamma\theta_3 + u_2.\end{aligned} \qquad (3)$$

11/.....

Reference model equations:

$$\begin{aligned}\dot{x}_{m1} &= -\alpha_1 x_{m1} + \alpha_1 \beta_1 w\\ \dot{x}_{m2} &= -\alpha_2 x_{m2} + \alpha_2 \beta_2 u_1.\end{aligned} \qquad (4)$$

Note that desired values are specified only for the levels x_{p1} and x_{p2}, but not for the levels in the delay, θ_1, θ_2 and θ_3. Thus the ideal plant (without delay) is described by:

$$\begin{aligned}\dot{x}_{p1} &= w - u_1\\ \dot{x}_{p2} &= u_2 - u_1\end{aligned} \qquad (5)$$

These sets of first-order differential equations may be written more elegantly in the vector-matrix notation of state-space description as follows.

Plant equations:

$$\dot{\underline{x}} = A\,\underline{x} + B\,\underline{u} + D'_p\,\underline{w} \qquad (6)$$

where

$$\underline{x} = \begin{bmatrix} \underline{x}_p \\ \underline{\theta} \end{bmatrix}, \quad A = \begin{bmatrix} A_p & A_{12} \\ 0 & A_d \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix},$$

$$D'_p = \begin{bmatrix} D_p \\ 0 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -1 & 0 \\ -1 & 0 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad A_d = \begin{bmatrix} -\gamma & \gamma & 0 \\ 0 & -\gamma & \gamma \\ 0 & 0 & -\gamma \end{bmatrix}, \quad A_p = 0 .$$

12/.....

Without the delay, plants equations are:

$$\dot{\underline{x}}_p = A_p \underline{x}_p + B_p \underline{u} + \underline{D}_p \underline{w} \qquad (7)$$

where

$$A_p = \underline{O} \ , \ B_p = \begin{bmatrix} -1 & o \\ -1 & 1 \end{bmatrix}, \quad \underline{D}_p = \begin{bmatrix} 1 \\ o \end{bmatrix}, \quad \underline{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}.$$

Reference model equations are:

$$\dot{\underline{x}}_m = A_m x_m + B_m \underline{u} + D_m \underline{w} \qquad (8)$$

where

$$A_m = \begin{bmatrix} -\alpha_1 & o \\ o & -\alpha_2 \end{bmatrix}, \quad B_m = \begin{bmatrix} o & o \\ \alpha_2\beta_2 & o \end{bmatrix}, \quad D_m = \begin{bmatrix} \alpha_1\beta_1 \\ o \end{bmatrix}.$$

The state generalised error is

$$\underline{e} = \underline{x}_p - \underline{x}_m \ . \qquad (9)$$

Delays necessarily increase the dimension of the plant necessitating some sort of model reduction. The control engineering literature is replete with linear model reduction techniques [6,7]. As is generally well-known socio-economic phenomena exhibit multiple-time-scale properties and the method of singular perturbation [8,9] is attractive for dealing with such problems.

As the simulation results show later on, the reduced model, eqn. 7, is adequate in this case for designing policies.

13/.....

2.2 Controllability and stabilisability

Policy design using modern control engineering techniques demands that the plant satisfies controllability and/or stabilisability conditions.

The system

$$\dot{\underline{z}} = A \ \underline{z} + B \ \underline{u} \qquad (10)$$

is said to be completely controllable if there exists a piecewise continuous control vector, $\underline{u}(t)$, which steers the system from an arbitrary initial state, $\underline{z}(t_o)$, to any terminal state, $\underline{z}(t_f)$, in finite time, $t_f - t_o$. Complete controllability of a system is characterised by the algebraic condition that the controllability matrix

$$\ell = \left[B, \ AB, \ \ldots \ , \ A^{n-1} \ B \right] \qquad (11)$$

must be of full rank, i.e.

$$\text{rank } (\ell) = n \qquad (12)$$

where n is the order of the system.

The system is said to be stabilisable if there exists a matrix, G, such that all eigenvalues of the modified system matrix (A + BG) are in the left half of the complex plane. A system is stabilisable if it is completely controllable; but the converse is not necessarily true.

It is easy to show that the plant with or without delay is completely controllable.

14/.....

3. Policy in system dynamics

So far in system dynamics, the structure of policy functions has been no more than variations of the classical three-term controller

$$u_i = K\left[e_i + \frac{1}{T_R}\int e_i\,dt + T_D\frac{de_i}{dt}\right],$$

$$i = 1, \ldots, m. \qquad (13)$$

But the state variables of system dynamics models are all accessible, except apparently for some state variables of pipeline delays. Even here, however, there is no limit to the detailed information which management can collect, if they wish to do so. It is, therefore, always possible to re-write the equations simulating the pipeline, to give access to information about all, or any part of, the contents. An example of this was Option II in [3] in which a large measure of stability can be induced by allowing the Raw Material Manager to have access to the Raw Material On Order, which is the content of a delay.

Policies can therefore be functions of as many relevant state variables as required to achieve a robust performance. Robustness in this context means that the system always behaves as well as possible for any member of a specified set of input functions - the input space, Ω. And the system is said to be vulnerable if it behaves unsatisfactorily for at least one member of this input space.

15/.....

It must be observed here that in managed systems (as opposed to engineering systems) exogenous inputs are not necessarily disturbances to be rejected; neither are they necessarily desired states or outputs to be followed. A central requirement of a social system is that it should be able to adapt itself to benefit from favourable changes in exogenous inputs, and to reject or cope with unfavourable changes. Such adaptivity must derive from the policies of the system, or from changes to the structure.

For the production/raw materials system, the policy design problem may be stated as follows:

<u>Policy design problem</u>: Choose $\underline{u}(t)$ such that

$$\lim_{t \to \infty} \underline{e}(t) = \underline{0} \qquad \forall\ \underline{w}(t) \in \Omega\,. \qquad (14)$$

Conventional system dynamics has used linear policy functions of the form;

$$\underline{u} = G\,\underline{\Psi}\,, \qquad (15)$$

where G is a constant policy matrix and $\underline{\Psi}$ is the information vector of system errors and smoothed variables defined as

$$\underline{\Psi}^T = \left[\underline{e}^T, \underline{y}^T\right]. \qquad (16)$$

16/.....

The structure of G for the original system, Fig. 2, is

$$G = \begin{bmatrix} 1/_{TABL} & 0 & 1.0 & 0 \\ 0 & -1/_{TARMS} & 0 & 1.0 \end{bmatrix} \qquad (17)$$

and for Option III (Ref 3)

$$G = \begin{bmatrix} 1/_{TABL} & -1/_{TARMS} & 1.0 & 0 \\ 0 & -1/_{TARMS} & 0 & 1.0 \end{bmatrix} \qquad (18)$$

Another possible structure, Option IV, is

$$G = \begin{bmatrix} 1/_{TABL} & -1/_{TARMS} & 1.0 & 0 \\ 1/_{TABL} & -1/_{TARMS} & 0 & 1.0 \end{bmatrix} \qquad (19)$$

Figs. 3, 5 and 6 show the simulation results using eqns. 17, 18 and 19 respectively.

In the following section, policy design from a modern control engineering viewpoint is presented.

17/.....

4. Linear model-following control

We recognise the SD policy design problem stated in section 3 as a linear model-following control (LMFC) problem assuming only slight system parameter uncertainty. With poor knowledge of parameter values and/or severe parameter variations it becomes an adaptive model-following control (AMFC) problem.

The adaptive control approach is appropriate in such systems because: 1) parameter estimates in SD models are often only engineering estimates, ie parameters are not identified to a high degree of accuracy; 2) parameters of the model may vary, reflecting changes in the real-world situation that the model is trying to capture; and 3) compared with the more obvious approach of optimal control theory, the adaptive control approach addresses the sensitivity problem directly and also yields computationally simpler algorithms.

A brief introduction to LFMC is given in this section, together with a computationally simple algorithm. A simulation example using the algorithm is also given.

4.1 Linear model-following control system

In the linear model-following control, the control system design specifications are embodied in a reference model whose dynamical behaviour the plant is forced to

18/.....

reproduce. The LMFC scheme [10] is shown in Fig. 7, in which the plant is described by

$$\dot{\underline{x}}_p = A_p \underline{x}_p + B_p \underline{u} , \tag{20}$$

the reference model by

$$\dot{\underline{x}}_m = A_m \underline{x}_m + B_m \underline{u}_m . \tag{21}$$

The required control is given by

$$\underline{u} = -K_p \underline{x}_p + K_m \underline{x}_m + K_u \underline{u}_m \tag{22}$$

or equivalently by

$$\underline{u} = -\tilde{K}_p \underline{x}_p + K_m \underline{e} + K_u \underline{u}_m \tag{23}$$

where $\tilde{K}_p = K_p - K_m$; $\underline{x}_m$ and $\underline{x}_p$ are of dimension n, and $\underline{u}_m$ and $\underline{u}$ are respectively of dimensions r and m. It is a combination of feedback and feedforward controls.

The reference model must be stable, and both plant and reference model must be stabilisable.

A fundamental problem of LMFC is:

<u>"Perfect model following"</u>:

Given a set of matrices $\{A_m, B_m, A_p, B_p,\}$, what conditions guarantee the existence of the gain matrices K_p, K_m, and K_u in order that the states of the plant and of the reference model are the same, ie such that

$$\lim_{t \to \infty} \underline{e}(t) = \underline{0} \ ?$$

19/.....

It is easily shown [11] that "perfect model following" is only achieved if the gain matrices satisfy the following equations

$$A_p - A_m + B_p (K_m - K_p) = 0 \tag{24}$$

$$B_p K_u - B_m = 0 \tag{25}$$

And solution of these equations for K_m, K_p, K_u, is possible if and only if

$$(I - B_p B^*_p) B_m = 0 \tag{26}$$

$$(I - B_p B^*_p) (A_p - A_m) = 0 \tag{27}$$

ie if the matrix $(I - B_p B^*_p)$ is either

(i) null, or

(ii) orthogonal to B_m and to $(A_p - A_m)$.

B^*_p is the Penrose generalised inverse of B_p given by

$$B^*_p = (B_p^T B_p)^{-1} B_p^T . \tag{28}$$

These conditions were first derived by Erzberger [12]. Equivalent conditions for "perfect model following" are [13]:

$$\text{rank } B_p = \text{rank } (B_p , B_m) = \text{rank } (B_p , A_m - A_p) . \tag{29}$$

Several methods of determining K_m, K_p, and K_u for perfect model following have been studied in the control literature [14].

20/.....

4.2 SD model as an LMFC system

The information vector of conventional system dynamics, eqn 13, may be modified into

$$\underline{\psi}^T = \left[\underline{x}_p^T , \underline{e}^T , \underline{w}^T \right] . \tag{30}$$

Thus the closed-loop SD model may be represented as shown in Fig.8. It differs from the conventional LMFC system only in the following respects:

(i) the exogenous inputs are reference inputs to the reference model, $\underline{u}_m = \underline{w}$, and

(ii) some policy functions may also drive the reference model.

The SD model-following system may thus be described by

$$\dot{\underline{x}}_p = A_p \underline{x}_p + B_p \underline{u} + D_p \underline{w} \tag{31}$$

$$\dot{\underline{x}}_m = A_m \underline{x}_m + B_m \underline{u} + D_m \underline{w} \tag{32}$$

with

$$\underline{u} = - \tilde{G}_p \underline{x}_p - G_m \underline{e} + G_w \underline{w} . \tag{33}$$

where

$$\tilde{G}_p = G_p - G_m . \tag{34}$$

Let $\tilde{B}_p = (B_p - B_m)$, and $\tilde{D}_m = (D_m - D_p)$ (35)

21/.....

Then it is easy to show that for "perfect model-following" G_m , G_p and G_w must satisfy the equations

$$A_m - A_p + \tilde{B}_p (G_p - G_m) = 0 \tag{36}$$

$$\tilde{D}_m - \tilde{B}_p G_w = 0 \tag{37}$$

Hence the Erzberger conditions for "perfect SD model-following"

$$(I - \tilde{B}_p \tilde{B}_p {}^*) D_m = 0 \tag{38}$$

$$(I - \tilde{B}_p \tilde{B}_p {}^*) (A_m - A_p) = 0 \tag{39}$$

SD model-following may be further recognised as corresponding to the so-called real model-following in control engineering. In such LMFC schemes, a reference model is made physically part of the control system.

4.3 Policy algorithm

The various methods of determining K_m, K_p and K_u satisfying the Erzberger conditions that have been developed in control engineering also apply to the SD policy design problem. In particular we present the "linear equivalent policy" method [15] based on variable-structure system theory.

This choice reflects our desire to exploit high-gain feedback design for maximum "passive" adaptivity - the additional complexity of "active" adaptive control can

22/.....

only be justified after the full capabilities of the high-gain feedback design have been explored.

4.3.1 Variable-structure system [16, 17]

The basic idea of variable-structure control systems is to steer a system

$$\dot{\underline{z}}(t) = A(t)\ \underline{z}(t) + B(t)\ \underline{u}(t) + H(t)\ \underline{v}(t) \qquad (40)$$

from some arbitrary initial state, $\underline{z}(t_o) = \underline{z}_o$, to the origin of state space, using controls which change their structure depending on the values of some switching functions, $s_i(\underline{z})$:

$$u_i = \begin{cases} u_i^+ & \text{if } s_i(\underline{z}) > 0 \\ u_i^- & \text{if } s_i(\underline{z}) < 0 \end{cases} \qquad (41)$$

$i = 1, \ldots, m.$

Each $s_i(\underline{z}) = 0$ is a switching plane.

A striking property of such systems is that under certain conditions, the state can be driven to the intersection of the switching planes and then slide along the intersection to the origin. This behaviour is depicted in Fig. 9 for a second-order, single-input system. The latter motion is known as sliding motion.

It has been shown that during sliding motion the system satisfies

$$s_i(\underline{z}) = 0, \quad \dot{s}_i(\underline{z}) = 0, \quad i = 1, \ldots, m. \qquad (42)$$

23/.....

It is claimed that variable-structure control systems with sliding motion are insensitive to plant parameter variations and to noise disturbances.

4.3.2 Linear equivalent policy

It must be noted that the variable-structure control of eqn 41 is a fast-switching control which may be intolerable in certain situations. However, a smooth control, called "linear equivalent control", ensuring sliding motion for a time-invariant system, exists, and is derived as follows:

Consider

$$\dot{\underline{x}}_p = A_p\ \underline{x}_p + B_p\ \underline{u} + D_p\ \underline{w} \qquad (43)$$

$$\dot{\underline{x}}_m = A_m\ \underline{x}_m + B_m\ \underline{u} + D_m\ \underline{w} \qquad (44)$$

and $\underline{e} = \underline{x}_p - \underline{x}_m$. (45)

Then

$$\dot{\underline{e}} = A_m\ \underline{e} + (A_p - A_m)\ \underline{x}_p + \tilde{B}_p\ \underline{u} - \tilde{D}_m\ \underline{w}\ . \qquad (46)$$

Switching planes in error space are

$$\underline{s}(\underline{e}) = C\ \underline{e} = \underline{0} \qquad (47)$$

where C is known as switching matrix.

In the sliding mode the system satisfies

$$\dot{\underline{s}}(\underline{e}) = \underline{0} \qquad (48)$$

i.e. $C\ \dot{\underline{e}} = \underline{0}$ (49)

24/.....

or

$$C\left[A_m \underline{e} + (A_p - A_m)\,\underline{x}_p + \tilde{B}_p \underline{u} - \tilde{D}_m \underline{w}\right] = \underline{0}\,. \tag{50}$$

The smooth linear equivalent policy, $\underline{u}_s(t)$, is defined as the value of $\underline{u}(t)$ which satisfies eqn.50 ensuring sliding motion.

Thus

$$\underline{u}_s = -\,(C\,\tilde{B}_p)^{-1}\,C\left[A_m \underline{e} + (A_p - A_m)\,\underline{x}_p - \tilde{D}_m \underline{w}\right] \tag{51}$$

i.e.

$$\underline{u}_s = -\,\tilde{G}_p \underline{x}_p - G_m \underline{e} + G_w \underline{w} \tag{52}$$

where

$$\tilde{G}_p = P(A_p - A_m) \tag{53}$$

$$G_m = P\,A_m \tag{54}$$

$$G_w = P\,\tilde{D}_m \tag{55}$$

$$P = (C\,\tilde{B}_p)^{-1}\,C \tag{56}$$

This policy is analytically straight forward and computationally simple. However, there are no systematic methods available so far for choosing the switching matrix, C. Such methods are presently being studied.

25/.....

6.3 Example and discussions

Consider the production/raw materials system with numerical values as in Ref. 3. The policy matrices are determined here using the reduced system, eqn. 7.

$$A_p = \underline{0},\; B_p = \begin{bmatrix} -1 & 0 \\ -1 & 1 \end{bmatrix}, \quad D_p = \begin{bmatrix} 1 \\ 0 \end{bmatrix};$$

$$A_m = \begin{bmatrix} -0.25 & 0 \\ 0 & -0.25 \end{bmatrix}, \quad B_m = \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}, \quad D_m = \begin{bmatrix} 1.5 \\ 0 \end{bmatrix}.$$

$$\tilde{B}_p = B_p - B_m = \begin{bmatrix} -1 & 0 \\ -3 & 1 \end{bmatrix}.$$

$$\tilde{D}_m = D_m - D_p = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}.$$

The Erzberger conditions, eqns. 38 and 39 are satisfied.

In the general case there is as yet no systematic method of choosing the switching matrix, C. In this particular example, though, the following observations lead to an algorithm which is independent of C, if C is non-singular:

a) $\tilde{B}_p$ and C are square matrices of the same order, and

b) $\tilde{B}_p$ is involutory, i.e. it is its own inverse.

Hence

$$P = (C\,\tilde{B}_p)^{-1}\,C = \tilde{B}_p^{-1}\,C^{-1}\,C = \tilde{B}_p\,,$$

26/.....

and the policy matrices are:

$$G_m = \begin{bmatrix} 0.25 & 0 \\ 0.75 & -0.25 \end{bmatrix}, \quad G_p = \begin{bmatrix} -0.25 & 0 \\ -0.75 & 0.25 \end{bmatrix}, \quad \text{and}$$

$$G_w = \begin{bmatrix} -0.5 \\ -1.5 \end{bmatrix}.$$

Fig. 10 shows the resulting influence diagram, and Fig. 11 the simulation results for the plant without delay. It should be noted that "perfect model following" is achieved in this case - something that none of the previous methods achieved. Note that in the noiseless case the overall model may be considerably simplified by removing the SURPLUS ORDER terms in the policies. Fig. 12 shows the results for the plant with delay. In this case "perfect model following" is achieved only in the production sector - due to the decoupled nature of the reference model, the plant approximation affects only the raw materials sector.

However, the production policy that gives zero surplus order is sinusoidal but, in practice, less oscillatory policies are to be preferred. The raw materials ordering policy is also sinusoidal, as in options III and IV.

All this indicates that perhaps the SD policy design problem would be more appropriately re-defined as an LMFC problem with constrained policies. This is the subject of another study.

27/.....

6. Conclusions

It has been shown how, starting with the influence diagram, an SD model can be transformed into an analogue computer flow diagram and then into a continuous-time state-space mathematical model. The SD policy design problem was then formulated as a model-following problem in modern control engineering, thereby providing policy design a proper and firmer basis in an adaptive control framework. A variable-structure policy algorithm was derived. The numerical simulation results shows that "perfect model following" is achieved when the plant is accurately modelled. Deterioration of performance results with plant model approximation.

There are several other directions in which the work outlined in this paper could be developed: The feedback (i.e. sensitivity) properties of the solution have yet to be investigated. Model-following with nonlinear reference models is an area completely unexplored - intuitively one feels some difficult policy problems in managed systems could be treated as such. And, of course, the adaptive control framework adopted here is by no means the only set of results that the vast and rich subject of automatic control offers.

Acknowledgement

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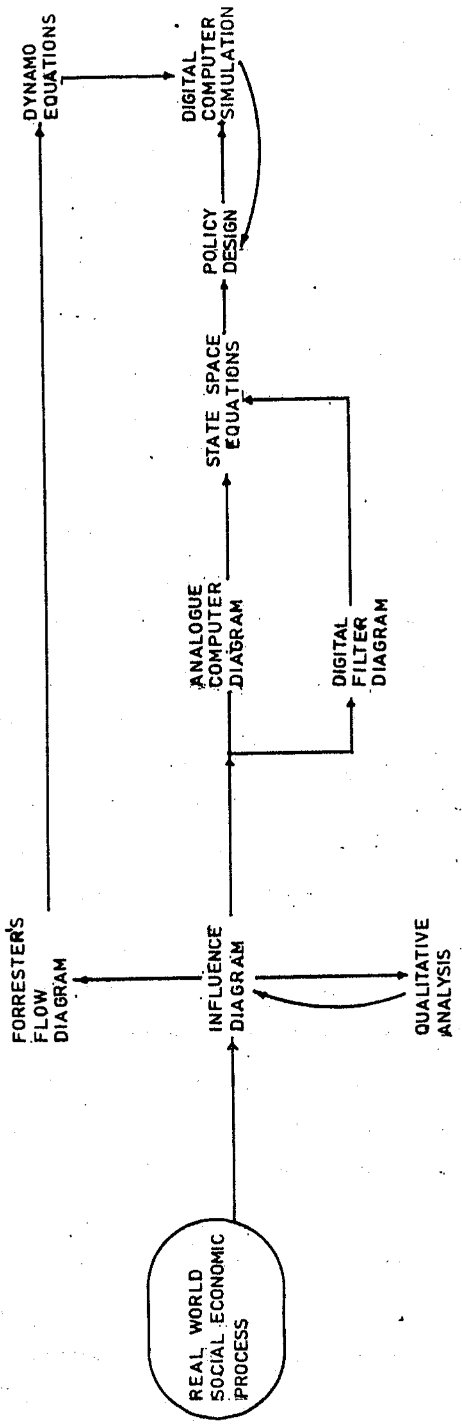


Fig. 1 FLOW DIAGRAM OF SYSTEM DYNAMICS

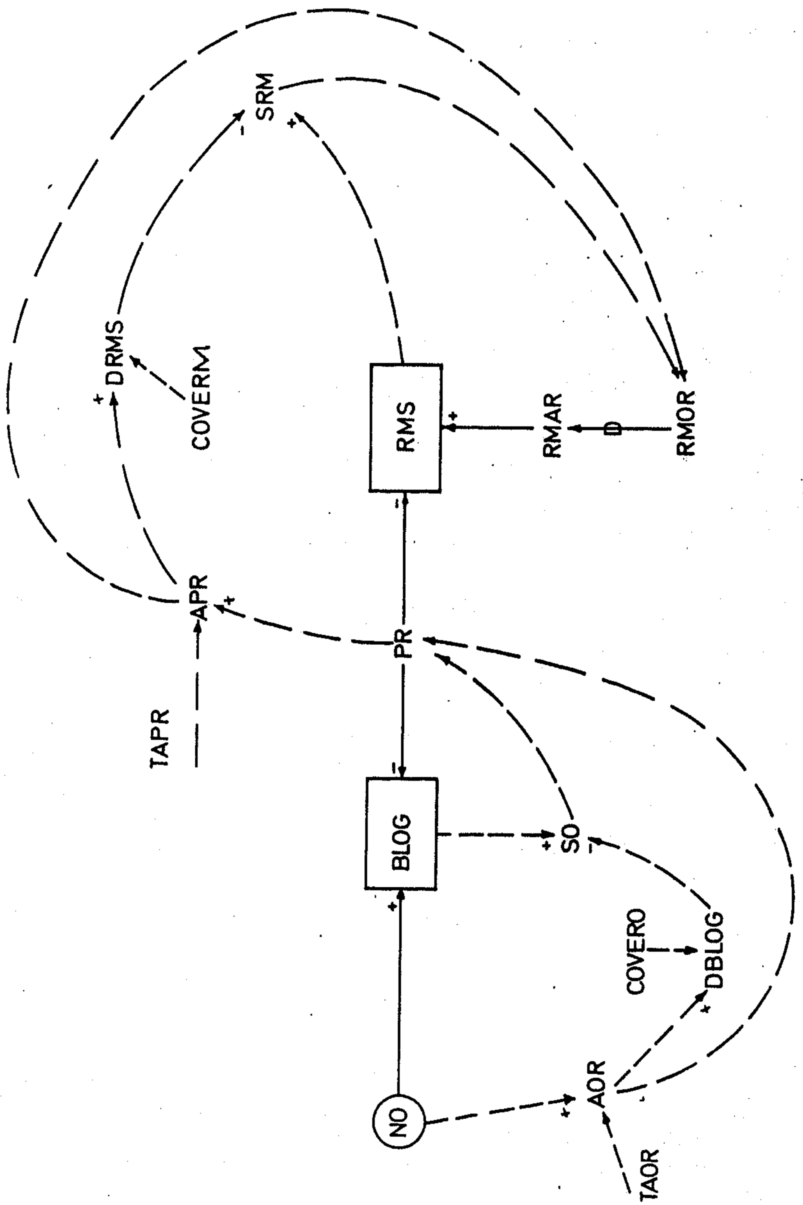
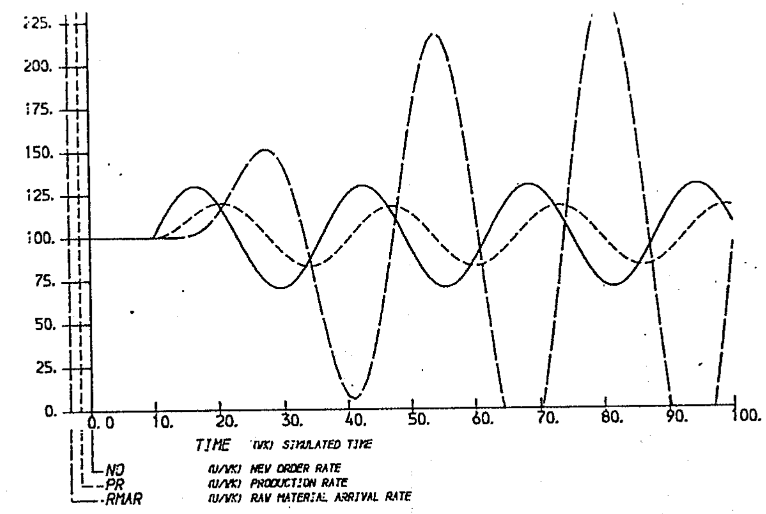
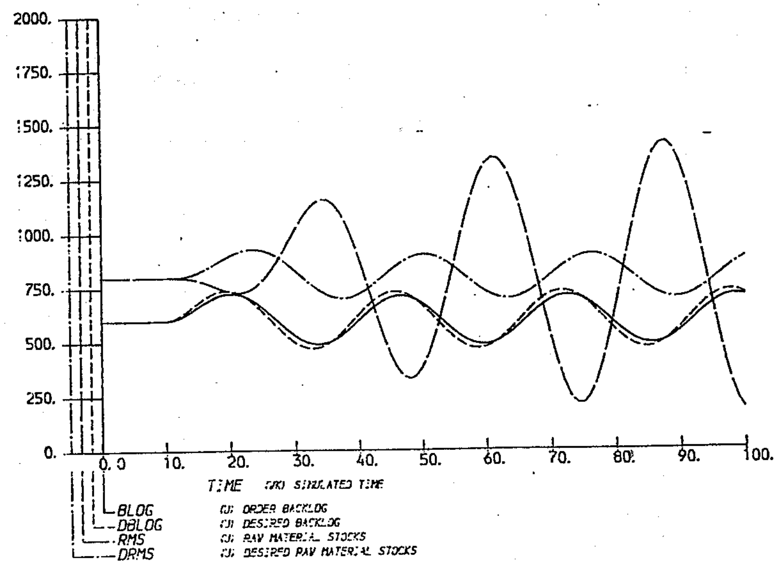


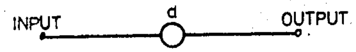
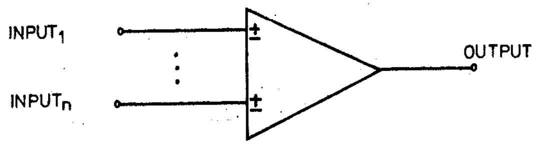
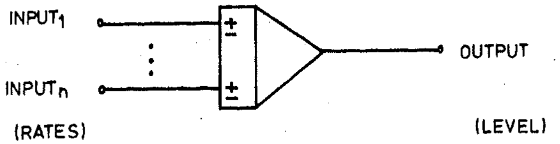
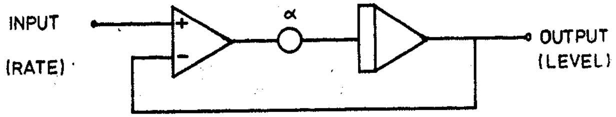
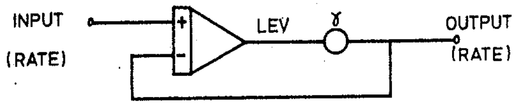
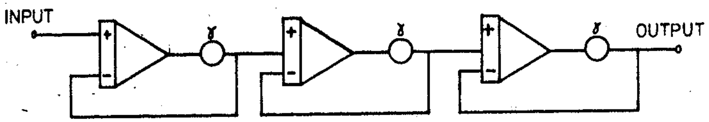
Fig.2 ORIGINAL SYSTEM INFLUENCE DIAGRAM





26 WEEK SINE WAVE FOR ORIGINAL SYSTEM

MODEL OF PRODUCTION/RAW MATERIAL STABILISATION

FUNCTION	ANALOGUE COMPUTER CIRCUIT
COEFFICIENT MULTIPLIER	 OUTPUT = a * INPUT, a ≥ 0
SUMMING AMPLIFIER	 OUTPUT = $\sum_{1}^{n} \pm \text{INPUT}_i$
INTEGRATION (ACCUMMULATION)	
SMOOTHED LEVEL	 α = 1/AVERAGING TIME
FIRST ORDER EXPONENTIAL DELAY	 γ = 1/DDEL
THIRD ORDER EXPONENTIAL DELAY	 γ = 3/DDEL

TABLE

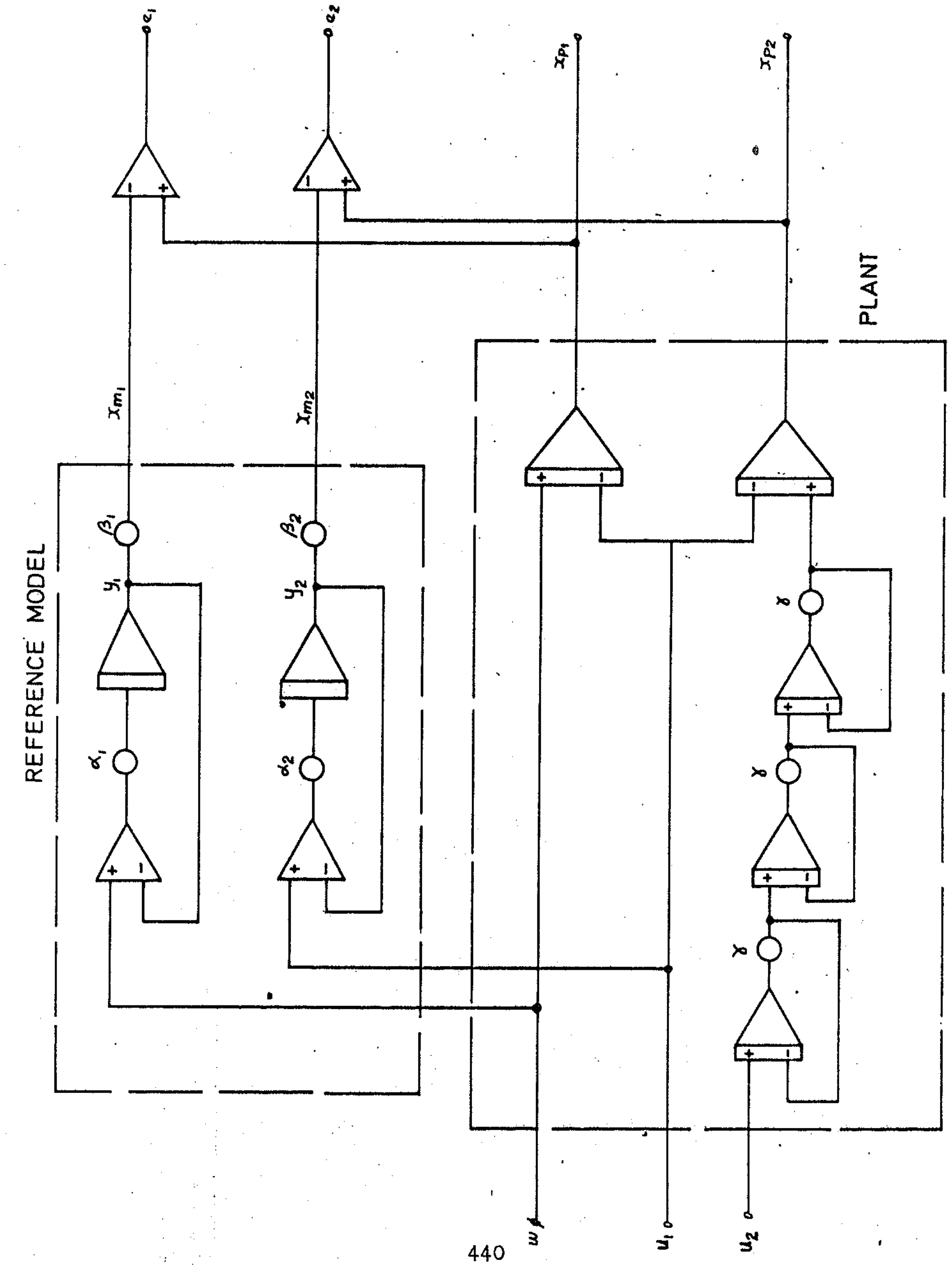
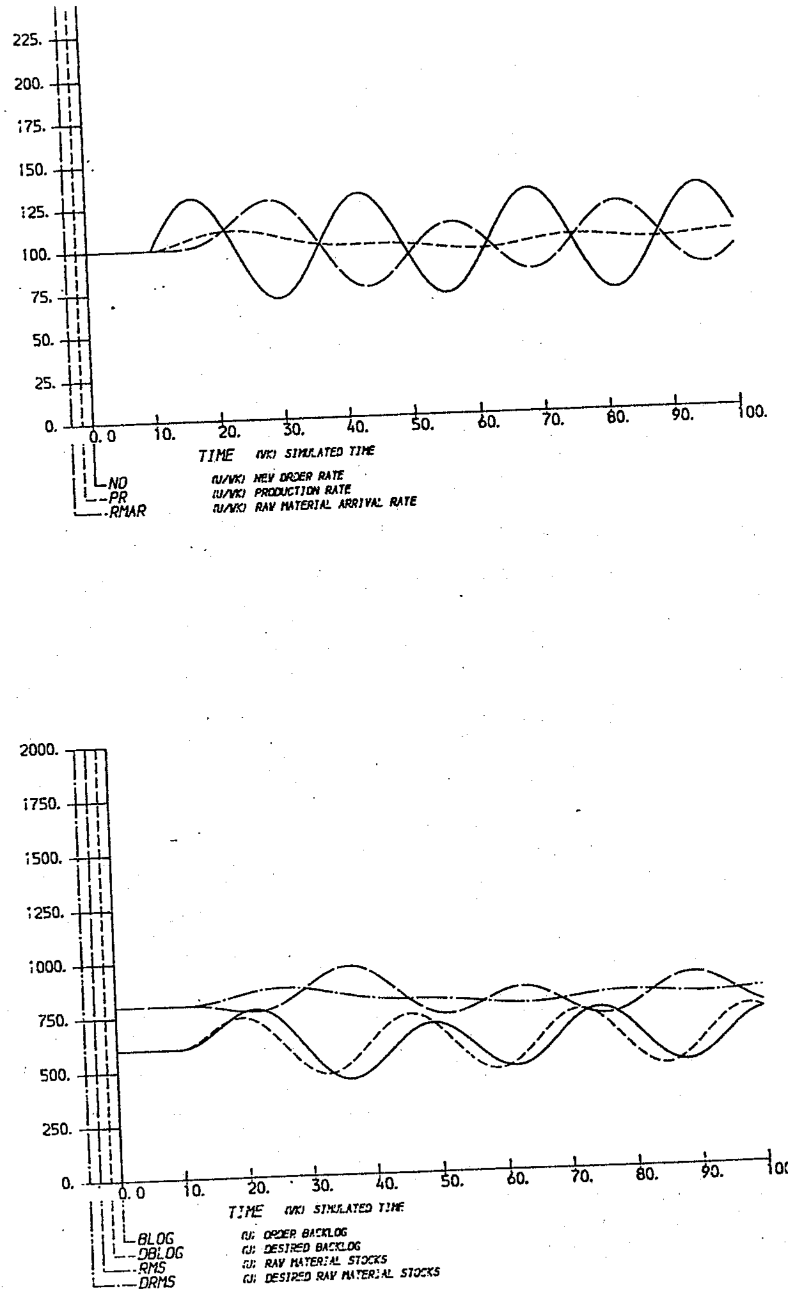


FIG. 4: OPEN-LOOP SD MODEL



OPTION III

FIG. 5: MODEL OF PRODUCTION/RAW MATERIAL STABILISATION

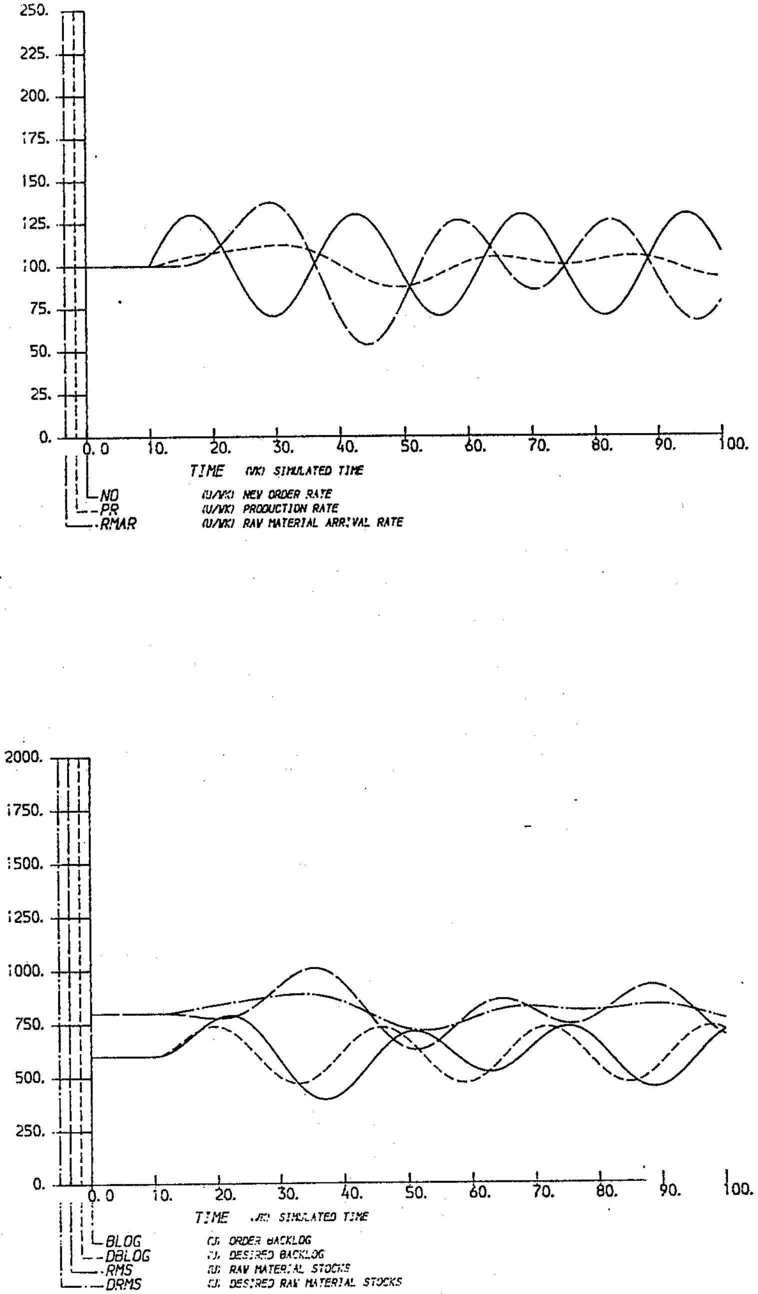


FIG. 6:

OPTION IV MATRIX 1

MODEL OF PRODUCTION/RAW MATERIAL STABILISATION

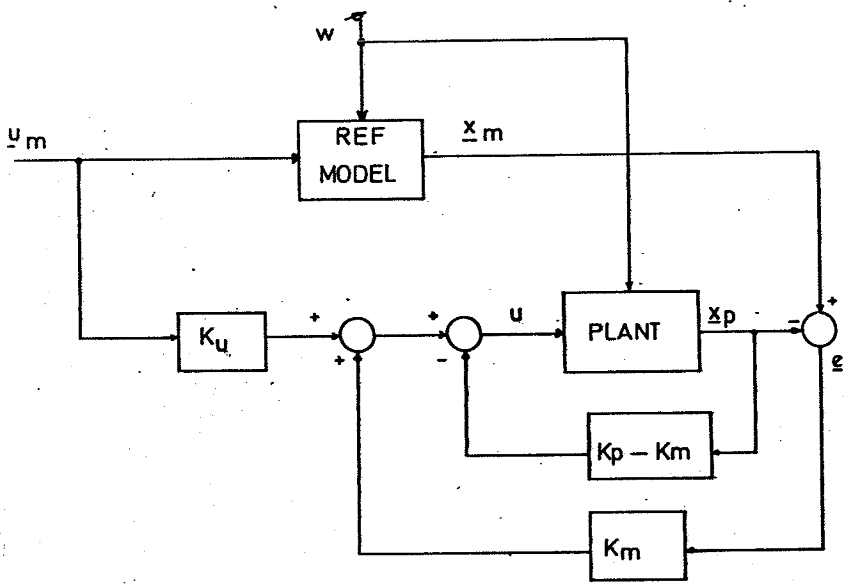


Fig. 7: LINEAR MODEL-FOLLOWING CONTROL (LMFC) SYSTEM

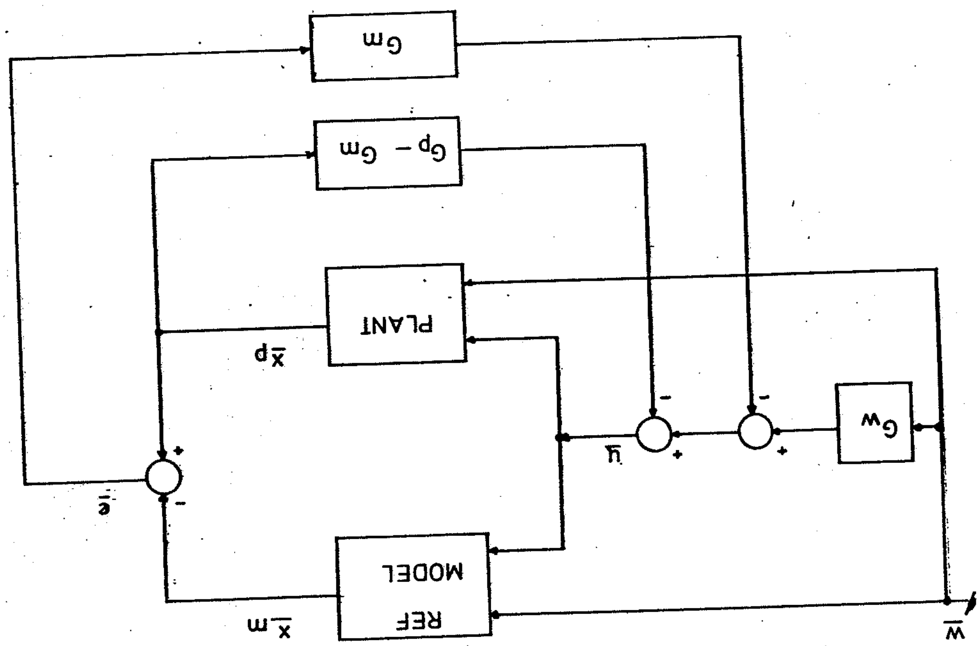


Fig. 8: SYSTEM DYNAMICS MODEL AS AN LMFC SYSTEM

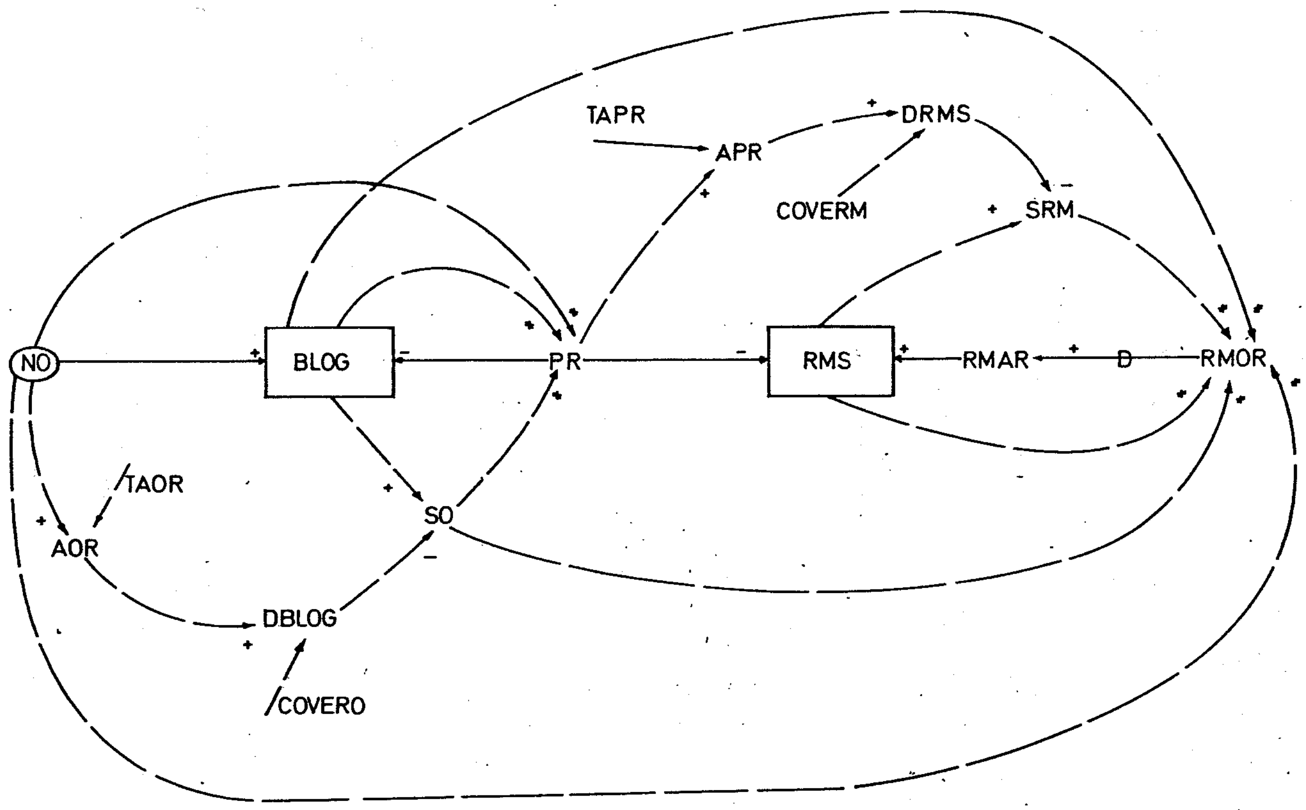
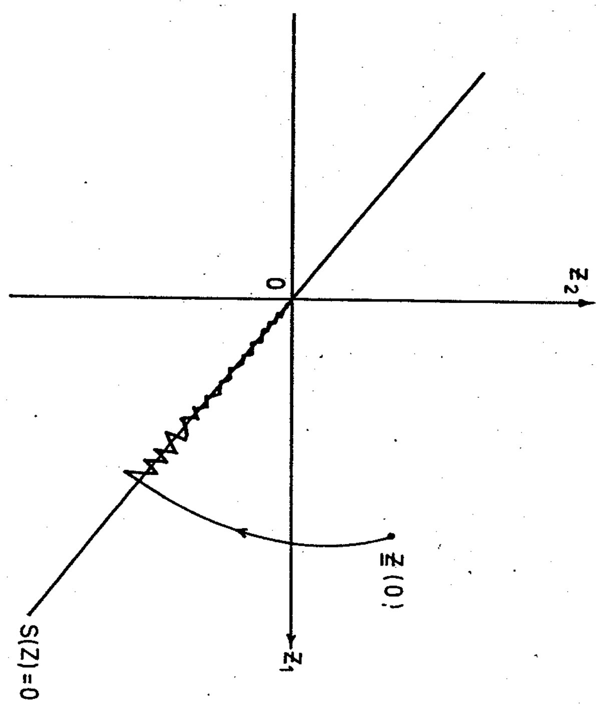


Fig. 10 LINEAR MODEL-FOLLOWING SD MODEL

446



g.9 SLIDING MOTION IN STATE SPACE

445

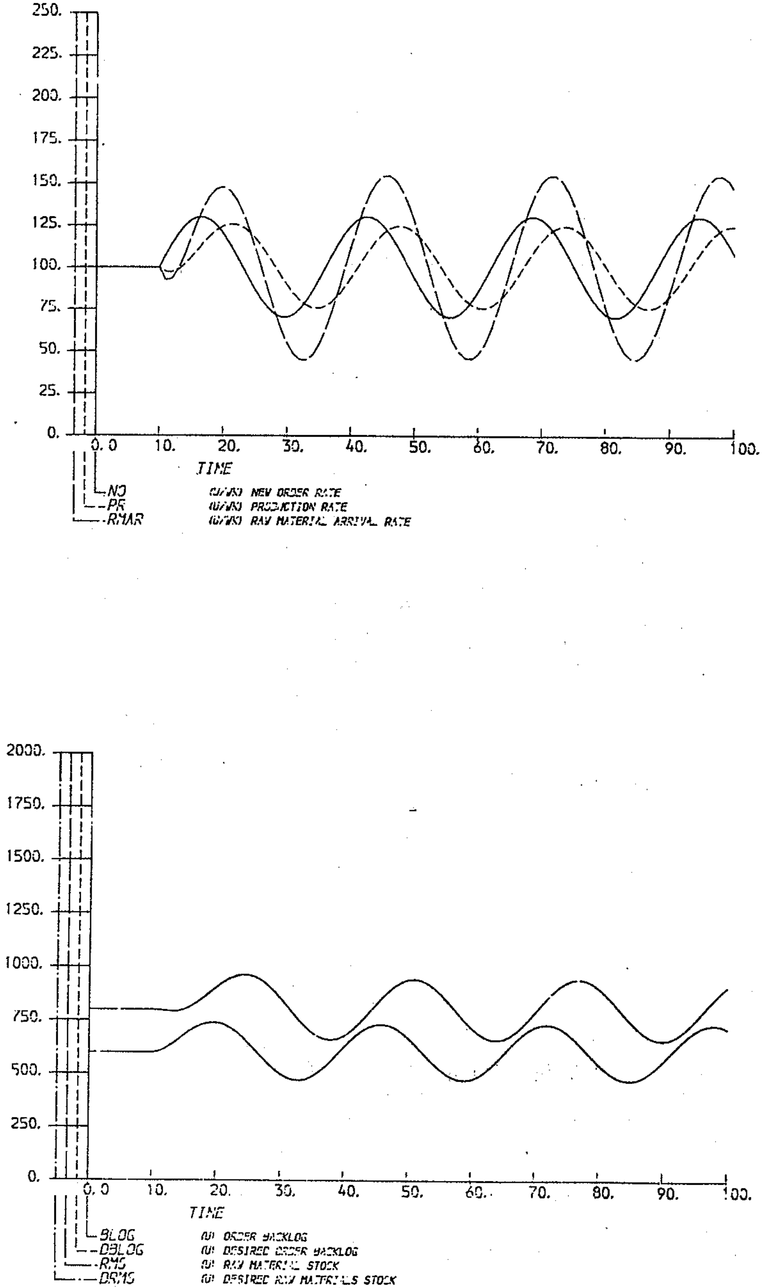


FIG. 11 LINEAR EQUIVALENT POLICY: PLANT WITHOUT DELAY

MODEL OF PRODUCTION/RAW MATERIAL STABILISATION

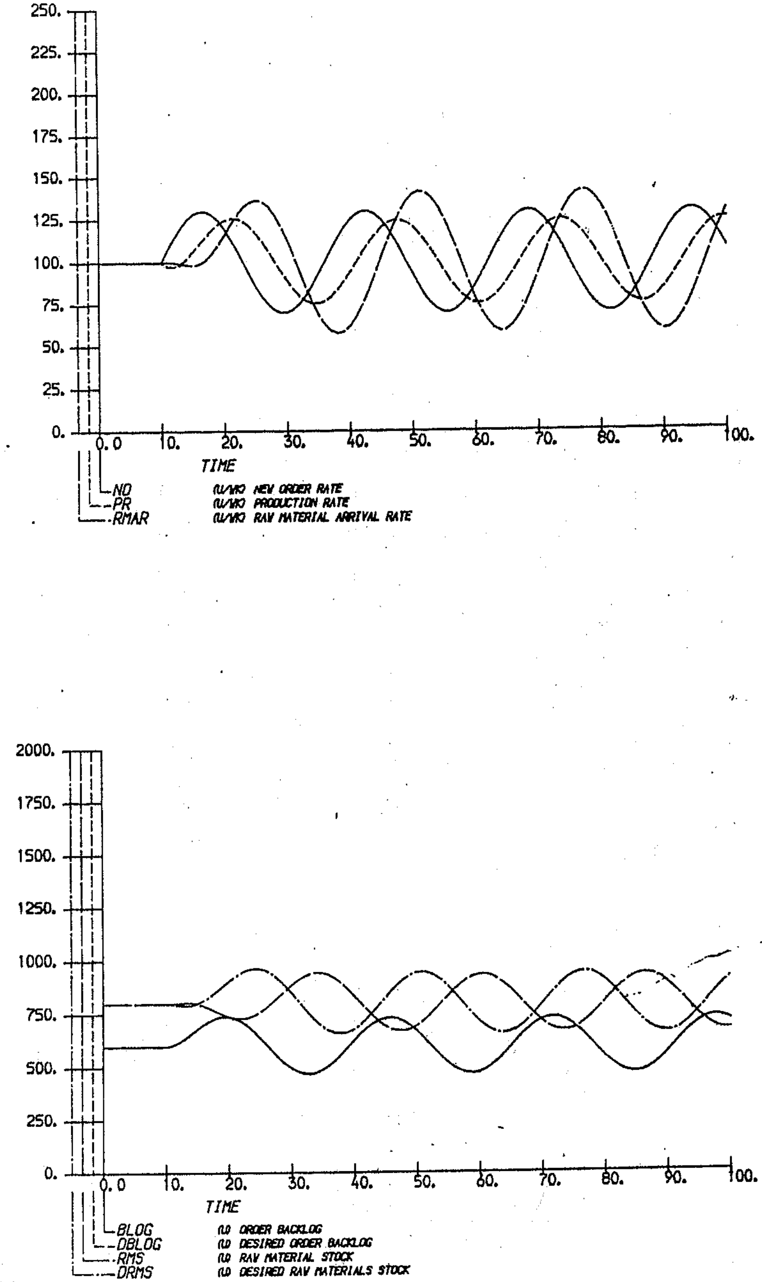


FIG 12: LINEAR EQUIVALENT POLICY

MODEL OF PRODUCTION/RAW MATERIAL STABILISATION

List of principal symbols

NO	=	NEW ORDER RATE
BLOG	=	ORDER BACKLOG
PR	=	PRODUCTION RATE
DBLOG	=	DESIRED ORDER BACKLOG
AOR	=	AVERAGE ORDER RATE
TAOR	=	ORDER AVERAGING TIME
COVERO	=	WEEKS OF AVERAGE ORDER IN DBLOG
SO	=	SURPLUS ORDER
RMS	=	RAW MATERIAL STOCK
RMOR	=	RAW MATERIALS ORDER RATE
RMAR	=	RAW MATERIAL ARRIVAL RATE
DDEL	=	RAW MATERIAL DELAY
APR	=	AVERAGE PRODUCTION RATE
TAPR	=	PRODUCTION AVERAGING TIME
COVERM	=	WEEKS OF AVERAGE PRODUCTION IN DRMS
DRMS	=	DESIRED RAW MATERIALS STOCK
SRM	=	SURPLUS RAW MATERIALS
TABL	=	TIME TO ADJUST BACKLOG
TARMS	=	TIME TO ADJUST RAW MATERIAL STOCK
$\underline{x}_p$	=	plant state vector
$\underline{x}_m$	=	reference model or desired state vector
$\underline{e}$	=	state generalised error vector
$\underline{w}$	=	exogenous inputs vector
$\underline{u}$	=	policy vector
$\underline{y}$	=	vector of smoothed levels